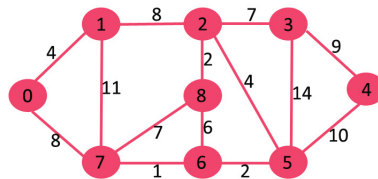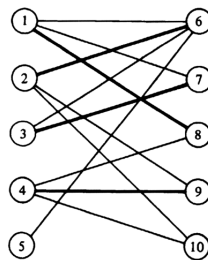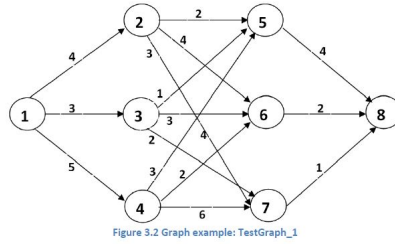# Optimization methods

Please show all your work!

1. What is a minimum weight spanning tree? On the following weighted graph, first run Kruskal's and then Prim's algorithm from the vertex 3. What is the difference between the two algorithms?
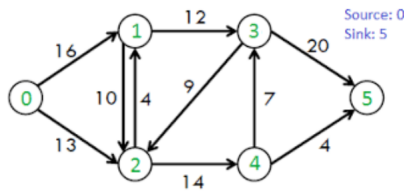


2. What is a matching? State König's Theorem! Find a maximum matching in the following graph and prove that it is, indeed, maximum!



3. How Dijkstra's algorithm finds the shortest path from a vertex $u$ to a vertex $v$ in a labelled directed graph? When does it fail? Run Dijkstra's algorithm on the following graph to obtain the shortest path from vertex 1 to 8.

Figure 3.2 Graph example: TestGraph_1

4. What is a network? What is a (feasable) flow? What is the value of a flow $F$? What is an augmenting $(s, t)$-path ($s$ is the source, $t$ is the sink of a network) and how can we improve a flow along its egdes? State the max flow min cut theorem! Find a maximum flow in the following network and show that it is, indeed, maximum.



5. What are the eigenvectors and the corresponding eigenvalues of a matrix $A$? Why do we consider unit norm eigenvectors? What is the eigen decomposition of a matrix $A$? Find the eigenvalues and the corresponding eigenvectors of the matrix

$$A = \begin{pmatrix} 5 & 7 \\ -3 & -5 \end{pmatrix}.$$

6. Define the conditional probability! Explain why 'switch' is a better strategy then 'stay' in the Monty Hall Problem.