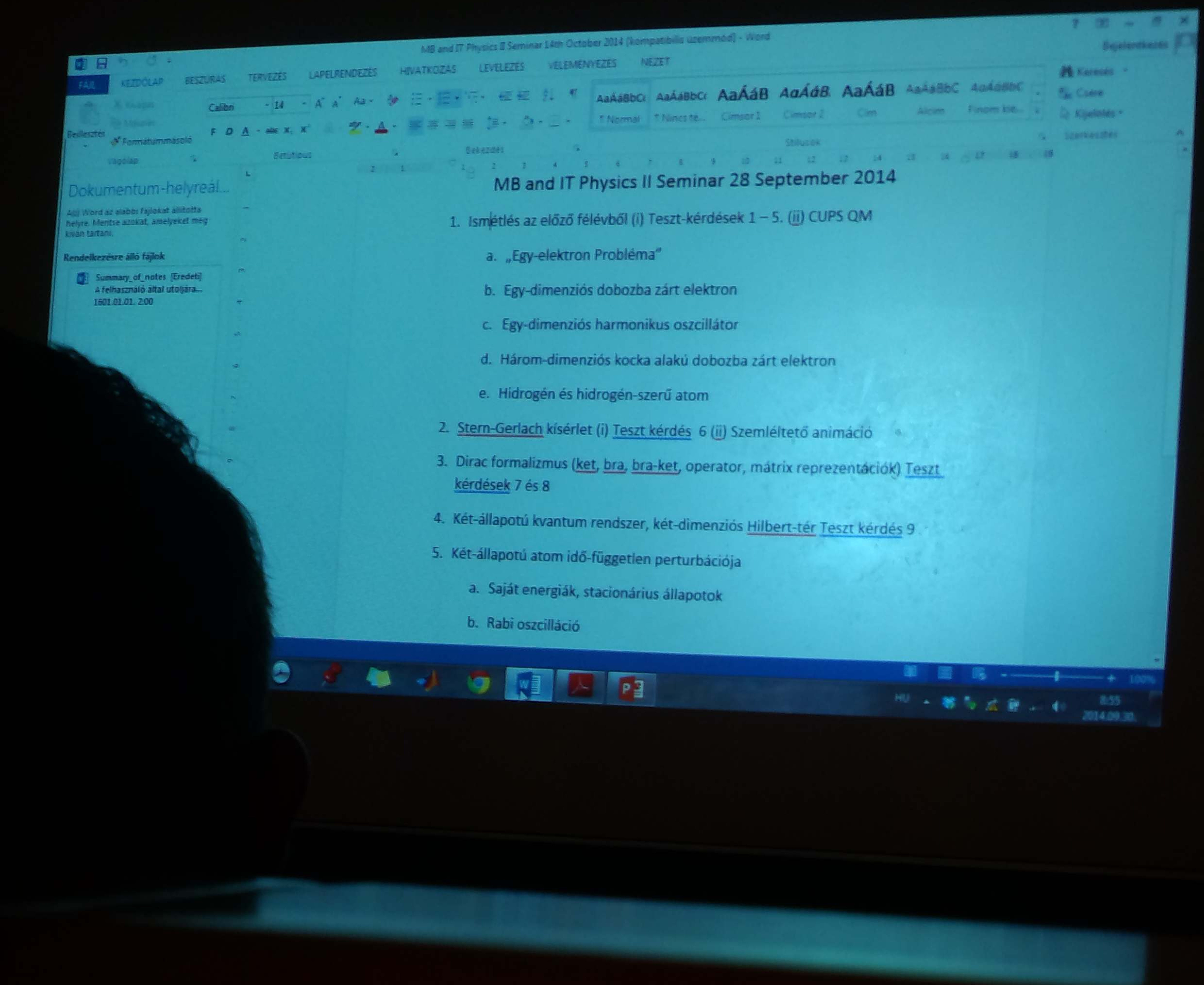

MB and IT Physics II Seminar 28 September 2014

1. Ismétlés az előző félévből (i) Teszt-kérdések 1 – 5. (ii) CUPS QM
   a. „Egy-elektron Probléma"
   b. Egy-dimenziós dobozba zárt elektron
   c. Egy-dimenziós harmonikus oszcillátor
   d. Három-dimenziós kocka alakú dobozba zárt elektron
   e. Hidrogén és hidrogén-szerű atom
2. Stern-Gerlach kísérlet (i) Teszt kérdés 6 (ii) Szemléltető animáció
3. Dirac formalizmus (ket, bra, bra-ket, operator, mátrix reprezentációk) Teszt kérdések 7 és 8
4. Két-állapotú kvantum rendszer, két-dimenziós Hilbert-tér Teszt kérdés 9
5. Két-állapotú atom idő-független perturbációja
   a. Saját energiák, stacionárius állapotok
   b. Rabi oszcilláció

-z direction (spin −z).

7. In an orthonormal basis linear Hermitian operators have matrix representations. Are the following operators with matrix representations Hermitian or not?

a. $\hat{A} = \begin{bmatrix} a & c \cdot e^{j\varphi} \\ c \cdot e^{j\varphi} & b \end{bmatrix}$

b. $\hat{B} = \begin{bmatrix} a & c \cdot e^{j\varphi} \\ c \cdot e^{-j\varphi} & b \end{bmatrix}$

c. $\hat{C} = \begin{bmatrix} a + jb & c \cdot e^{j\varphi} \\ c \cdot e^{-j\varphi} & b + ja \end{bmatrix}$

d. $\hat{D} = \begin{bmatrix} 2.12 & 1+j \\ 1-j & 3.14 \end{bmatrix}$

e. $\hat{E} = \begin{bmatrix} 2 & 1+j \\ 1+j & 3 \end{bmatrix}$

8. Are the following formulas true or false?

   a. Outer product $|a\rangle\langle b| = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 4 \end{bmatrix} = \begin{bmatrix} 3 & 6 & 12 \\ 2 & 4 & 8 \\ 1 & 2 & 4 \end{bmatrix}$

   b. For self-adjoint (Hermitian) operators $\langle a|\hat{X}|b\rangle = \langle b|\hat{X}|a\rangle^*$

   c. Eigen kets constitute a complete orthonormal set: $|a\rangle = \sum_n c_n |n\rangle = \sum_n |n\rangle\langle a|n\rangle$

   d. $\sum |c_n|^2 = \sum_n |\langle n|a\rangle|^2 = 1$

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e. Elements of the matrix representation of operator $\hat{X}$ are $x_{nm} = \langle n|\hat{X}|m\rangle$

9. .Let us consider a two-dimensional Hilbert-space $|\psi\rangle \in \mathcal{H}$. In this Hilbert-space the self-adjoint operator of a two-state quantum system is $\hat{H}_0 = \begin{bmatrix} E_2 & 0 \\ 0 & E_1 \end{bmatrix}$, which

9. .Let us consider a two-dimensional Hilbert-space $|\psi\rangle \in \mathcal{H}$. In this Hilbert-space the self-adjoint operator of a two-state quantum system is $\hat{H}_0 = \begin{bmatrix} E_2 & 0 \\ 0 & E_1 \end{bmatrix}$, which generates an orthonormal basis $|1\rangle, |2\rangle$, i.e. $\hat{H}_0|1\rangle = E_1|1\rangle, \hat{H}_0|2\rangle = E_2|2\rangle$. Check whether the following statements are true or false.

   a. If there is no external interaction, and at initial time $t = 0$ the state of the system is $|\psi(0)\rangle = c_1|1\rangle + c_2|2\rangle$, where $|c_1|^2 + |c_2|^2 = 1$, then at a later time $t > 0$ the state evolves as $|\psi(t)\rangle = c_1 \cdot e^{-j\frac{E_1}{\hbar}t}|1\rangle + c_2 \cdot e^{-j\frac{E_2}{\hbar}t}|2\rangle$;
   b. If at $t = 0$ the state is one of its eigen-states, i.e. $|\psi(0)\rangle = |1\rangle$, then the system remains in its initial eigen-state forever, i.e. $|\psi(t)\rangle = e^{-j\frac{E_1}{\hbar}t}|1\rangle$, i.e. the probability of finding the system in state $|1\rangle$ is $|\langle 1|\psi(t)\rangle|^2 = \left|e^{-j\frac{E_1}{\hbar}t}\langle 1|1\rangle\right|^2 = 1$

c. Let the initial state be $|\psi(0)\rangle = 0.6|1\rangle + 0.8|2\rangle$. The probability that in a measurement the state is found to be $|1\rangle$ will remain constant in time, namely $P_1 = |\langle 1|\psi(t)\rangle|^2 = 0.36$.

d. Is it true or false that in c) the probability of finding the system in state $|2\rangle$ is also constant and $P_2 = |\langle 2|\psi(t)\rangle|^2 = 0.64 = 1 - P_1$.

e. The two eigen-states evolve independently, thus $|\psi(t)\rangle = 0.6 \cdot e^{-j\frac{E_1}{\hbar}t}|1\rangle + 0.8 \cdot e^{-j\frac{E_2}{\hbar}t}|2\rangle$.

10. Let the $\hat{H}_0$ system be perturbed by a constant interaction, thus the Hamiltonian of the system changes to $\hat{H} = \begin{bmatrix} E_2 & W_{12} \\ W_{12}^* & E_1 \end{bmatrix}$, where $E_2 = 4\,\mathrm{eV}$, $E_1 = 1\,\mathrm{eV}$, $W_{12} = 1\,\mathrm{eV}$., and the initial state of the system is $|\psi(0)\rangle = |1\rangle$.

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Is it true or false that

a. The eigen-energies of the perturbed system remain the same as in the unperturbed case, namely E_2, and E_1 ;

b. The evolution of the perturbed system can be written as $|\psi(t)\rangle = c_1(t)\cdot e^{-j\frac{E_1}{\hbar}t}|1\rangle + c_2(t)\cdot e^{-j\frac{E_2}{\hbar}t}|2\rangle$ with unknown $c_1(t)$ and $c_2(t)$.

c. The perturbed system's eigen energies will be $E_+ = 4.3\,\text{eV}$, $E_- = 0.7\,\text{eV}$

d. The probability that the perturbed system starting from state $|\psi(0)\rangle = |1\rangle$ will evolve, and after time t the it will be found in state $|2\rangle$ will change periodically with the Rabi frequency as $P_{1\to 2} = 0.308\cdot\sin^2\left(\frac{E_+ - E_-}{\hbar}t\right)$