
SEMINAR NOVEMBER 20 2014
1. In a monochromatic, 600 nm wavelength coherent plane wave the stream of photons is moving into the positive z direction. The transmitted power is 1 pico-watt. Is it true or false that
a. The energy of a photons is about 2.07 eV;
b. The average number of photons are about 6.02 million photons/sec;
c. If the signal is the average number of photons, the coherent noise level is about 0.33 pico-watt, and the SNR is about 3.02;
d. The direction of the momentum vector is $-z$,
e. In case of a collision with a standing still hydrogen atom, the atom will gain a velocity of about 0.66 m/s.
2. In a cavity the classical electromagnetic field can always be represented by the linear combination of the fields of its cavity modes. Is it true or false that
a. The state variable in cavity is the vector potential satisfying the wave equation $\Delta \mathbf{A}(\mathbf{r},t)=\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\mathbf{A}(\mathbf{r},t)$;
b. From the known vector-potential the electrinc field of the cavity can be derived $\mathbf{E}(\mathbf{r},t)=\nabla\times\mathbf{A}(\mathbf{r},t)$;
c. Also the magnetic field, which is $\mathbf{B}(\mathbf{r},t)=-\frac{\partial}{\partial t}\mathbf{A}(\mathbf{r},t)$;

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    d. The general solution of the wave equation is $\mathbf{A}(\mathbf{r},t)=\sum_k f_k(t)\mathbf{A}_k(\mathbf{r})$. , where the

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   d. The general solution of the wave equation is $\mathbf{A}(\mathbf{r},t)=\sum_k f_k(t)\mathbf{A}_k(\mathbf{r})$, where the eigenvalue problem $\Delta\mathbf{A}_k(\mathbf{r})=-k^2\mathbf{A}_k(\mathbf{r})$ specifies the spatial structure of the mode functions;
   e. The time-dependent f_k functions determine the Lagrangian of the cavity modes $L_k=E_{Ek}-E_{Mk}=\frac{1}{2}\varepsilon_0\left[\frac{df_k}{dt}\right]^2-\frac{k_k}{2\mu_0}f_k^2$

3. Is it true or false that in a closed ideal cavity
   a. The energy of each cavity mode is proportional to the frequency of the mode;
   b. If there were no photons in a mode the energy would be zero;
   c. If the number of photons in a mode was doubled the energy stored in that mode would double;

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a. The state variable in cavity is the vector potential satisfying the wave equation $\Delta \mathbf{A}(\mathbf{r},t) = \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\mathbf{A}(\mathbf{r},t)$;
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   d. If the ground-state energy of a cavity mode was E_0 eV, the energy of that mode could be E_0+2nE_0, where n is the number of photons in the cavity mode;
   e. If we increase the number of photons in a mode by one, the energy of the mode would increase by $2E_0$.
4. Using the analogy between a classical harmonic oscillator and the time-dependence of a classical cavity mode the Hamiltonian operator of a cavity mode can be

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4. Using the analogy between a classical harmonic oscillator and the time-dependence of a classical cavity mode, the Hamiltonian operator of a cavity mode can be established, and the quantum behavior of the cavity can be explained

a. The energy of a cavity mode does not change continuously with the intensity of the field, but it is quantized as $E_{k\alpha} = \hbar\,\omega_k\left(n+\frac{1}{2}\right)$, where n is referring to the number of photons in a cavity mode ;
b. For $n=0$ (no photons), there is no energy in the cavity;
c. For $n=0$ (no photons), there is energy in the cavity; We experience the zero-point fluctuation of the field;
d. Both the electric and the magnetic fields can be zero in this zero-point state;
e. The expectation values of the both the electric and magnetic fields can be zero, however, the variances can´t be both zero, and thus the energy content of the zero point state fluctuations is finite.

5. In a cubic ideal cavity resonator of size $a\times a\times a = 10^{-3}\mathrm{m}\times 10^{-3}\mathrm{m}\times 10^{-3}\mathrm{m}$ there are 100 photons in a mode (1,1,1) and 14 in the next excited mode. The system is cooled down to 0.01 K. Is it true or false that

a. The first mode resonance frequency is $\omega_{111} = \frac{\pi\cdot c}{a}\sqrt{3} = 2\pi\cdot 260\ \mathrm{GHz} \to 1.08\cdot 10^{-3}\ \mathrm{eV}$
b. The resonance of the next excited mode is $\omega_{211} = \frac{\pi\cdot c}{a}\sqrt{6} = 2\pi\cdot 366\ \mathrm{GHz} \to 1.52\cdot 10^{-3}\ \mathrm{eV}$
c. The total EMAG energy is $E \approx 0.13\ \mathrm{eV}$

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   d. The cavity state in occupation number representation is $|100,14\rangle$

   e. EMAG vacuum fluctuation is three orders of magnitude smaller than the classical noise level of the environment.

6. We build a closed composite quantum system from two two-state atoms. Is it true or false that

   a. The time dependent state can be represented as $|\psi(t)\rangle \rightarrow \begin{bmatrix} c_1(t) \\ c_2(t) \end{bmatrix}$; $|c_1(t)|^2 + |c_2(t)|^2 = 1$

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c. The time-evolution of the state satisfies the time-dependent Schrödinger equation as $j\hbar \frac{\partial |\psi(t)\rangle}{\partial t} = \mathbf{H}|\psi(t)\rangle$
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e. The time-evolution of the state can be expressed as $|\psi(t)\rangle = e^{i\frac{H}{\hbar}t} \cdot |\psi(0)\rangle$.