

14. Introduction to Photonics

Absorption, Spontaneous and Stimulated Emission

a. Black-body radiation. Planck's Radiation Theorem

3.2. A fekete test sugárzása

Általános tapasztalat, hogy minden gáz, folyadék és szilárd anyag -273 Kelvinnél magasabb hőmérsékleten elektromágneses hullámokat sugároz, másnéven sugárzást emittál.

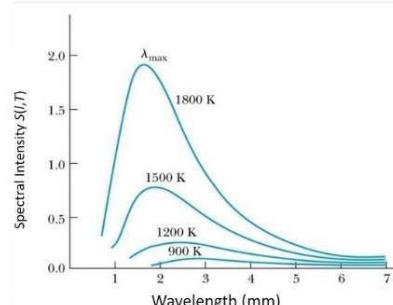
A minden sugárzást elnyelő (abszorbáló), úgy nevezett fekete test (13. ábra) sugárzása független a sugárzó test anyagától, illetve alakjától.



13. ábra.

Azért nyel el minden sugárzást, mert a fényforrás csak egy nagyon kis résen tud bejutni a test belsejébe, és ott a folyamatos visszaverődésekkor energiát veszít, és matematikailag nagyon kicsiny az esélye, hogy a hullám bármikor pontosan úgy érkezzen a réshez, ahogyan befele érkezett, tehát ponyolán megfogalmazva "sosem jön ki".

A kísérletekben rögzített hőmérsékleten mérték a felületegységen (vagy hullámhosszon) hullámhosszegységre eső sugárzott intenzitást, amit $S(\lambda, T)$ -vel jelölünk. A mérések után a következő görbét kapták:



14. ábra.

A mérésekkel kapcsolatban több megfigyelés, illetve törvény született

1. Wien-törvény
2. Stefan-Boltzmann-törvény
3. Planck-törvény

Wien megfigyelése, vagy törvénye kimondja, hogy a görbe maximumhelyének és a hőmérsékletnek (amin a kísérletet elvégeztek) szorzata állandó, és

$$\lambda_{max}T = 2.898 \cdot 10^{-3} mK.$$

Stefan-Boltzmann-törvény azt mondja ki, hogy a felületegységen sugárzott összteljesítmény az abszolút hőmérséklet negyedik hatványával nő, tehát

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$$\frac{P(T)}{A} = \sigma T^4 = \frac{\int_0^\infty S(\lambda, T) d\lambda}{A},$$

ahol a paraméter értéke

$$\sigma = 5.6705 \cdot 10^{-8} \frac{W}{m^2 K^4}$$

Planck a méréseket próbálta zárt alakban leírni. Tudták, hogy az ideális fekete test, minden színt tökéletesen elnyel, és és minden színt sugároz. Az intenzitás csak a hőmérséklettől függ, a test alakjától, illetve anyagától nem.

Planck talált rá egy formulát, amely egyszerű, és csupán egyetlen paramétertől függ, amelyet a mért görbükre való illesztéssel határozott meg, ez a Planck állandó.

Feltette, hogy a sugárzást az üregben lévő "oszcillátorok", az üreg rezonáns módusai nyelték el, és sugározottak vissza a kis nyílásom át.

Adott T hőmérsékleten, egyenstílusban, minden szabadságfokra $\frac{1}{2}kT$ energia jut, ahol k a Boltzmann-állandó. A módusok száma a frekvencia négyzetével arányosan nő, ez az ultraibolya paradox.

A mérésekkel egyező formula azonban egy komoly "gondra" mutatott rá, ugyanis csak úgy sikerült megegyeznie a mérésekkel, hogy Planck feltette, hogy az oszcillátorok csak "kvantált" energiájuk lehetnek, mégpedig

$$E = nh, n = 0, 1, 2..$$

Ezek után pedig a formula:

$$S(\lambda, T) = \frac{2\pi ch}{\lambda^5} \frac{1}{e^{\frac{hc}{\lambda k_B T}} - 1}.$$

b. Photon gas. Bose-Einstein statistics.

In physics, a **photon gas** is a **gas-like** collection of **photons**, which has many of the same properties of a conventional gas like hydrogen or neon - including pressure, temperature, and entropy. The most common example of a photon gas in equilibrium is **black body radiation**.

A massive **ideal gas** with only one type of particle is uniquely described by three state functions such as the **temperature**, **volume**, and the **number of particles**. However, for a black body, the energy distribution is established by the interaction of the photons with matter, usually the walls of the container. In this interaction, the number of photons is not conserved. As a result, the **chemical potential** of the black body photon gas is zero. The number of state functions needed to describe a black body state is thus reduced from three to two (e.g. temperature and volume).

A very important difference between a gas of massive particles and a photon gas with a black body distribution is that the number of photons in the system is not conserved. A photon may collide with an electron in the wall, exciting it to a higher energy state, removing a photon from the photon gas. This electron may drop back to its lower level in a series of steps, each one of which releases an individual photon back into the photon gas. Although the sum of the energies of the emitted photons are the same as the absorbed photon, the number of emitted photons will vary. It can be shown that, as a result of this lack of constraint on the number of photons in the system, the **chemical potential** of the photons must be zero for black body radiation.

Massless Bose–Einstein particles (e.g. black body radiation) [edit]

For the case of massless particles, the massless energy distribution function must be used. It is convenient to convert this function to a frequency distribution function:

$$P_\nu \, d\nu = \frac{h^3}{N} \left(\frac{Vf}{\Lambda^3} \right) \frac{1}{2} \frac{\beta^3 \nu^2}{e^{(h\nu-\mu)/kT} - 1} \, d\nu$$

where Λ is the thermal wavelength for massless particles. The spectral energy density (energy per unit volume per unit frequency) is then

$$U_\nu \, d\nu = \left(\frac{N h\nu}{V} \right) P_\nu \, d\nu = \frac{4\pi f h\nu^3}{c^3} \frac{1}{e^{(h\nu-\mu)/kT} - 1} \, d\nu.$$

Other thermodynamic parameters may be derived analogously to the case for massive particles. For example, integrating the frequency distribution function and solving for N gives the number of particles:

$$N = \frac{16\pi V}{c^3 h^3 \beta^3} \text{Li}_3 \left(e^{\mu/kT} \right).$$

The most common massless Bose gas is a photon gas in a [black body](#). Taking the "box" to be a black body cavity, the photons are continually being absorbed and re-emitted by the walls. When this is the case, the number of photons is not conserved. In the derivation of [Bose–Einstein statistics](#), when the restraint on the number of particles is removed, this is effectively the same as setting the chemical potential (μ) to zero. Furthermore, since photons have two spin states, the value of f is 2. The spectral energy density is then

$$U_\nu \, d\nu = \frac{8\pi h\nu^3}{c^3} \frac{1}{e^{h\nu/kT} - 1} \, d\nu$$

which is just the spectral energy density for [Planck's law of black body radiation](#). Note that the [Wien distribution](#) is recovered if this procedure is carried out for massless Maxwell–Boltzmann particles, which approximates a Planck's distribution for high temperatures or low densities.

In certain situations, the reactions involving photons will result in the conservation of the number of photons (e.g. [light-emitting diodes](#), "white" cavities). In these cases, the photon distribution function will involve a non-zero chemical potential. (Hermann 2005)

c. Thermal equilibrium of a system of matter and photon gas. Photon absorption, spontaneous and stimulated emission. The Einstein coefficients.

Photon gas equilibrium most common example: Black body radiation

Thermal equilibrium of a system of matter:

Two physical systems are in **thermal equilibrium** if no heat flows between them when they are connected by a path permeable to heat. Thermal equilibrium obeys the [zeroth law of thermodynamics](#). A system is said to be in thermal equilibrium with itself if the temperature within the system is spatially and temporally uniform.

Systems in [thermodynamic equilibrium](#) are always in thermal equilibrium, but the converse is not always true. If the connection between the systems allows transfer of energy as heat but does not allow transfer of matter or transfer of energy as work, the two systems may reach thermal equilibrium without reaching thermodynamic equilibrium.

Relation of thermal equilibrium between two thermally connected bodies [edit]

The relation of thermal equilibrium is an instance of a contact equilibrium between two bodies, which means that it refers to transfer through a selectively permeable partition, the contact path.^[1] For the relation of thermal equilibrium, the contact path is permeable only to heat; it does not permit the passage of matter or work; it is called a diathermal connection. According to Lieb and Yngvason, the essential meaning of the relation of thermal equilibrium includes that it is reflexive and symmetric. It is not included in the essential meaning whether it is or is not transitive. After discussing the semantics of the definition, they postulate a substantial physical axiom, that they call the "zeroth law of thermodynamics", that thermal equilibrium is a transitive relation. They comment that the equivalence classes of systems so established are called isotherms.^[2]

Photon absorption, spontaneous and stimulated emission: 15.tétel

Einstein coefficients are mathematical quantities which are a measure of the probability of absorption or emission of light by an atom or molecule.^[1] The Einstein A coefficient is related to the rate of **spontaneous emission** of light and the Einstein B coefficients are related to the **absorption** and **stimulated emission** of light.

An atomic spectral line refers to emission and absorption events in a gas in which n_2 is the density of atoms in the upper energy state for the line, and n_1 is the density of atoms in the lower energy state for the line.

The emission of atomic line radiation at frequency ν may be described by an **emission coefficient** ϵ with units of energy/time/volume/solid angle. $\epsilon dt dV d\Omega$ is then the energy emitted by a volume element dV in time dt into solid angle $d\Omega$. For atomic line radiation:

$$\epsilon = \frac{h\nu}{4\pi} n_2 A_{21}$$

where A_{21} is the Einstein coefficient for spontaneous emission, which is fixed by the intrinsic properties of the relevant atom for the two relevant energy levels.

The absorption of atomic line radiation may be described by an **absorption coefficient** κ with units of 1/length. The expression $\kappa' dx$ gives the fraction of intensity absorbed for a light beam at frequency ν while traveling distance dx . The absorption coefficient is given by:

$$\kappa' = \frac{h\nu}{4\pi} (n_1 B_{12} - n_2 B_{21})$$

where B_{12} and B_{21} are the Einstein coefficients for photo absorption and induced emission respectively. Like the coefficient A_{21} , these are also fixed by the intrinsic properties of the relevant atom for the two relevant energy levels. For thermodynamics and for the application of Kirchhoff's law, it is necessary that the total absorption be expressed as the algebraic sum of two components, described respectively by B_{12} and B_{21} , which may be regarded as positive and negative absorption, which are, respectively, the direct photon absorption, and what is commonly called stimulated or induced emission.^{[8][9][10]}

d. Light Amplification by Stimulated Emission of Radiation (LASER)

15. tétel