

Given $X = \{-1, 1\}^N$; $S = \{s^\alpha, \alpha = 1, \dots, M\}$

↑
input space

↑
stored samples

AM: $X \rightarrow S$; $\psi(\bar{x}) = \bar{s}^\beta$, $d(\bar{x}, \bar{s}^\beta) < d(\bar{x}, \bar{s}^\alpha) \quad \forall \alpha = 1, \dots, M, \alpha \neq \beta$

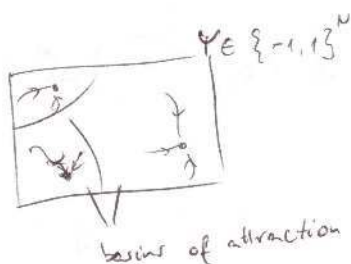
Dynamic approach

$\bar{y}(k+1) = \text{sgn} \{ \bar{w} \bar{y}(k) - \bar{b} \}$

we associate to the nearest stored element

↑
 $\bar{x} = \bar{y}(0)$

↑
initial state



stable: ✓

convergence speed ✓ $O(N^2)$

learning rule: if we are given $S = \{s^\alpha, \alpha = 1, \dots, M\}$

✓
 $\bar{w}_{ij} = \frac{1}{N} \sum_{\alpha=1}^M s_i^\alpha s_j^\alpha$

H: $\bar{s}^\beta = \text{sgn} \{ \bar{w} \bar{s}^\beta \} \quad \forall \bar{s}^\beta \in S \quad \leftarrow \bar{s}^\beta \text{ is a fix point}$

$\bar{w} = \frac{1}{N} \sum_{\alpha=1}^M \bar{s}^\alpha \bar{s}^{\alpha T}$

$y_k(k+1) = \text{sgn} \left\{ \sum_{j=1}^N w_{kj} y_j(k) \right\} \quad k = \text{mod}_N \quad \leftarrow \text{cyclic sequential update rule}$

$w_{ij} = \frac{1}{N} \sum_{\alpha=1}^M s_i^\alpha s_j^\alpha$

for max M: $\bar{s}^\beta = \text{sgn} \{ \bar{w} \bar{s}^\beta \}$

Deterministic static capacity: $M \leq N$

Deterministic dynamic capacity: $M \leq \frac{N^2}{(N-2)^2}$

$d(\bar{y}) = \bar{y}^T \bar{w} \bar{y}$

Random coding:

$P(s_i^\alpha = 1) = P(s_i^\alpha = -1) = 0.5$

← we generate the stored elements randomly

$\bar{s}^{(1)} = (-1 \ -1 \ 1 \ -1 \ 1 \ \dots)$

$\bar{s}^{(2)} = (1 \ 1 \ 1 \ -1 \ 1 \ \dots)$

IT capacity of HM: $\bar{s}^\beta = \text{sgn} \{ \bar{w} \bar{s}^\beta \}$

if it is true or false occurs with a given probability

lim
 $N \rightarrow \infty$

$P(\bar{s}^\beta = \text{sgn} \{ \bar{w} \bar{s}^\beta \}) = 1$

← informational theoretical capacity

$$M: \lim_{N \rightarrow \infty} P(\bar{s}^R = \text{sgn}\{\bar{w} \bar{s}^R\}) = 1 \quad \forall \bar{s}^R \in S$$

↑ max # of stored elements

independent events

$$\begin{aligned} P(\bar{s}^R = \text{sgn}\{\bar{w} \bar{s}^R\}) &= P\left(\bigcap_{i=1}^N s_i^R = \text{sgn}\left\{\sum_{j=1}^N w_{ij} s_j^R\right\}\right) = \\ &= \prod_{i=1}^N P\left(s_i^R = \text{sgn}\left\{\sum_{j=1}^N w_{ij} s_j^R\right\}\right) = \prod_{i=1}^N P\left(s_i^R = \text{sgn}\left\{\sum_{j=1}^N \left(\frac{1}{N} \sum_{\alpha=1}^M s_i^\alpha s_j^\alpha\right) s_j^R\right\}\right) \\ &= \prod_{i=1}^N P\left(s_i^R = \text{sgn}\left\{\sum_{\alpha=1}^M s_i^\alpha \frac{1}{N} \sum_{j=1}^N s_j^\alpha s_j^R\right\}\right) = \\ &= \prod_{i=1}^N P\left(s_i^R = \text{sgn}\left\{s_i^R \cdot \underbrace{\frac{1}{N} \sum_{j=1}^N (s_j^R)^2}_1 + \underbrace{\sum_{\alpha=1}^M s_i^\alpha \frac{1}{N} \sum_{j=1}^N s_j^\alpha s_j^R}_{v_i}\right\}\right) = \\ &= \prod_{i=1}^N P\left(s_i^R = \text{sgn}\left\{s_i^R + v_i\right\}\right) = \\ &= \prod_{i=1}^N \left[\frac{1}{2} P\left(s_i^R = \text{sgn}\left\{s_i^R + v_i\right\} \mid s_i^R = 1\right) + \frac{1}{2} P\left(\dots \mid s_i^R = -1\right)\right] = \\ &= \prod_{i=1}^N \frac{1}{2} \left[P(1 + v_i > 0) + P(-1 + v_i < 0)\right] = \\ &= \prod_{i=1}^N \frac{1}{2} \left[P(v_i > -1) + P(v_i < 1)\right] = \\ &= \prod_{i=1}^N \frac{1}{2} \left[2\Phi\left(\frac{1}{\sqrt{\frac{M-1}{N}}}\right)\right] = \Phi^N\left(\sqrt{\frac{N}{M-1}}\right) \approx P(\bar{s}^R = \text{sgn}\{\bar{w} \bar{s}^R\}) \end{aligned}$$

equivalent with the above written

$$M: \lim_{N \rightarrow \infty} \log P(\bar{s}^R = \text{sgn}\{\bar{w} \bar{s}^R\}) = \infty$$

$$\lim_{N \rightarrow \infty} N \cdot \log \Phi\left(\sqrt{\frac{N}{M-1}}\right)$$

$$\approx \lim_{N \rightarrow \infty} N \cdot \log\left(1 - \sqrt{\frac{N}{M-1}} e^{-\frac{N}{2(M-1)}}\right)$$

$$\approx \lim_{N \rightarrow \infty} N \cdot \sqrt{\frac{N}{M-1}} e^{-\frac{N}{2(M-1)}}$$

$$M: \lim_{N \rightarrow \infty} e^{-\frac{N}{2M} + \log N - \frac{1}{2} \log \frac{N}{M}} = \infty$$

asymptotics: $\lim_{u \rightarrow \infty} \phi(u) = 1 - \frac{1}{u} e^{-\frac{u^2}{2}}$

$\lim_{u \rightarrow 0} \log(1-u) \approx -u$

if $\frac{N}{2M} \gg \log N$

$M \leq \frac{N}{2 \log N} \Rightarrow M \leq \frac{N}{2 \log N - \log \log N}$

IT capacity!

Designing a HN: given: $N \rightarrow M < \frac{N}{2 \log N}$

(11) nicht nur ja es könnte möglich?

$$M: \lim_{N \rightarrow \infty} P(\bar{s}^P = \text{sgn} \{ \bar{W} \bar{s}^P \}) = 1 \quad M \leq \frac{N}{2 \log N}$$

we have not taken into account the dynamic ~~capacity~~ of the HN

statistik
es mag sein ~~dimensional~~
kapazität voll

Statistical neurodynamics

$$\bar{x} = \bar{y}(0) \xrightarrow{??} \bar{s}^P \quad N \rightarrow \infty$$

Micro

$\bar{y}(k)$

get
micro transitional
rule

$$\bar{y}_i(k+1) = \text{sgn} \left\{ \sum_{j=1}^N w_{ij} \bar{y}_j(k) \right\}$$

$$\bar{y}(k+1) = \text{sgn} \{ \bar{W} \bar{y}(k) \}$$

Macro

average

$$a(k) = A_v \{ \bar{y}(k) \}$$

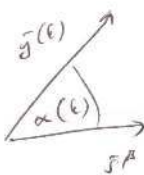
$$a(k+1) = \Upsilon(a(k))$$

macro state transitional
rule

if we pick $\bar{s}^P = (1, 1, 1, \dots, 1)$

define
$$a(k) = \frac{1}{N} \sum_{i=1}^N s_i^P \bar{y}_i(k) = \frac{1}{N} \sum_{i=1}^N \bar{y}_i(k)$$

??



$$a(k) \sim \cos(\alpha(k)) < a(k+1) < a(k+2) \Rightarrow \alpha(k) \rightarrow 0, \bar{y}(k) \rightarrow \bar{s}^P$$

normalised shall converge

statistical average

$a(k+1) = \Upsilon(a(k))$ what is the state transition rule between the macro states?

$$a(k+1) = \frac{1}{N} \sum_{i=1}^N \bar{y}_i(k+1) = \mathbb{E}(\bar{y}_i(k+1)) = 1 \cdot P(\bar{y}_i(k+1) = 1) + (-1) \cdot P(\bar{y}_i(k+1) = -1) =$$

$$= P\left(\text{sgn} \left\{ \sum_{j=1}^N w_{ij} \bar{y}_j(k) \right\} = 1\right) - P\left(\text{sgn} \left\{ \sum_{j=1}^N w_{ij} \bar{y}_j(k) \right\} = -1\right) =$$

$$= P\left(\text{sgn} \left\{ \sum_{j=1}^N \frac{1}{N} \sum_{\alpha=1}^M s_i^\alpha s_j^\alpha \bar{y}_j(k) \right\} = 1\right) - P(\dots = -1) =$$

$$= P\left(\text{sgn} \left\{ \sum_{\alpha=1}^M s_i^\alpha \frac{1}{N} \sum_{j=1}^N s_j^\alpha \bar{y}_j(k) \right\} = 1\right) - P(\dots = -1) =$$

$$= P \left(\text{sgn} \left\{ s_i^\beta \frac{1}{N} \sum_{j=1}^N s_j^\beta y_j(k) + \sum_{\substack{\alpha=1 \\ \alpha \neq \beta}}^M s_i^\alpha \frac{1}{N} \sum_{j=1}^N s_j^\alpha y_j(k) \right\} = 1 \right) - P(\dots = -1) =$$

$$v_i \sim \mathcal{N}(0, \sqrt{\frac{M}{N}})$$

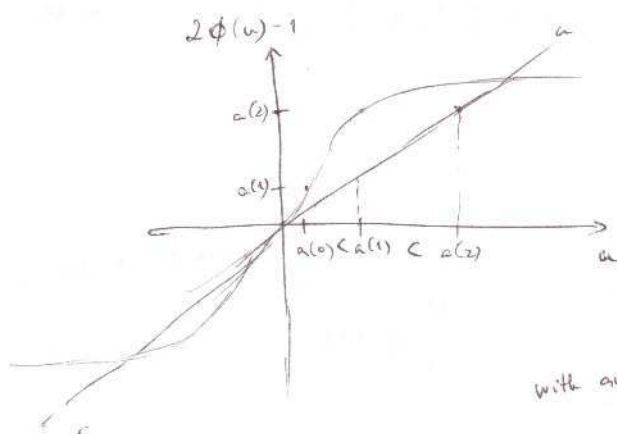
$$= P \left(\underbrace{\frac{1}{N} \sum_{j=1}^N y_j(k)}_{a(k)} + v_i = 1 \right) - P(\dots = -1) =$$

$$= P \left(\text{sgn} \{ a(k) + v_i \} = 1 \right) - P \left(\text{sgn} \{ a(k) + v_i \} = -1 \right)$$

$$a(k+1) = P(a(k) + v_i > 0) - P(a(k) + v_i < 0) =$$

$$a(k+1) = P(v_i > -a(k)) - P(v_i \leq -a(k)) =$$

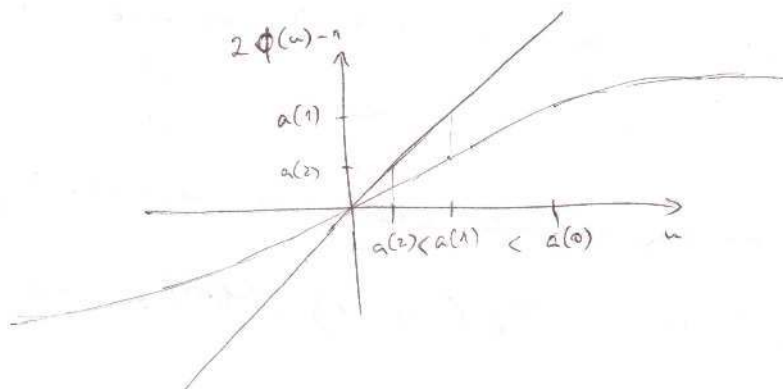
$$a(k+1) = 2 \phi \left(\frac{a(k)}{\sqrt{\frac{M}{N}}} \right) - 1$$



with another variable

stable because $a(k)$ increases

↓
cosine increases



unstable

doesn't converge

if the derivative > 1 at 0 $\rightarrow$ we go above the identity function

$$\frac{1}{\sqrt{2\pi \frac{M}{N}}} \geq 1 \rightarrow \boxed{M \leq \frac{N}{2\pi}} \leftarrow \text{this capacity is guaranteed for } w$$

conventional way to calculate the capacity

$$\min \left\{ \frac{N}{2\pi}, \frac{N}{2 \log N} \right\}$$

dynamic

static