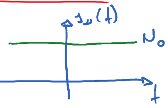
Noisy transmission

2016. december 14., szerda 16:41

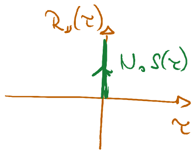
Main questions:

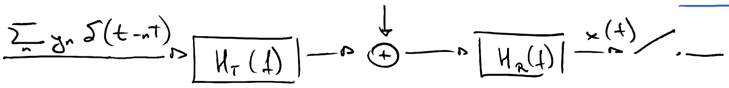
Resources	?? $\Longrightarrow$	QoS
→ bandwidth → transmission power		→ dataspeed → P_b (BEER)

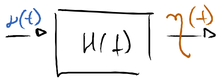
$\nu(t) \leftarrow$ AWGN channel:



$R(\tau) = \mathcal{F}^{-1}\{\tau\} = \int s_\nu(f)\, e^{j2\pi f\tau}\, df$







$\left(s_\eta(t) = \int_{-\infty}^{\infty} |H(f)|^2 s_\nu(f)\, df\right)$

Output noise energy (expressed by its spectral density):

$$\min \int_{-\infty}^{\infty} |H_R(f)|^2 N_0\, df$$

→ constrained optimization problem:

$$H_T(f)\, H_R(f) = H(f)$$

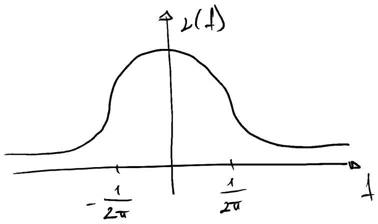
Nyquist equation: $\sum_n H_T\left(f + \frac{e}{T}\right) H_R\left(f + \frac{e}{T}\right) = 1$

→ $|f| \le \frac{1}{2T}$

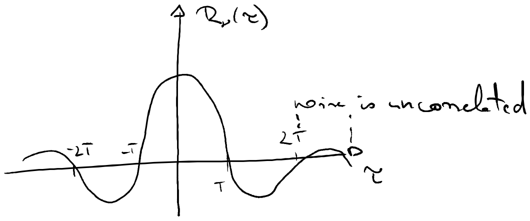
$H_T(f) = H_R^*(f)$

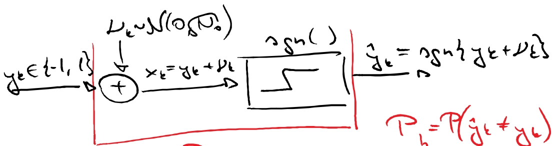
raised cosine

$s_\nu(f) = N_0 |H_R(f)|^2 = N_0$ Raised cosine



$\Longrightarrow$





$P_b = P(\hat{y}_k \ne y_k)$

BSC

Binary Symmetric Channel

$$P_b = P(\hat{y}_k \ne y_k) = P(\hat{y}_k = 1 \mid y_k = -1) P(y_k = -1) + P(\hat{y}_k = -1 \mid y_k = 1) P(y_k = 1) =$$

$$= P(\mathrm{sgn}\{y_k + \nu_k\} = 1 \mid y_k = -1)\tfrac{1}{2} + P(\mathrm{sgn}\{y_k + \nu_k\} = -1 \mid y_k = 1) = \tfrac{1}{2}\left[P(-1 + \nu_k > 0) + P(1 + \nu_k < 0)\right] =$$

$$= \tfrac{1}{2}\left[P(\nu_k > 1) + P(\nu_k < -1)\right] = \tfrac{1}{2}\Big[\underbrace{1 - \Phi\left(\tfrac{1}{\sqrt{N_0}}\right)}_{\Phi\left(-\frac{1}{\sqrt{N_0}}\right)} + \Phi\left(-\tfrac{1}{\sqrt{N_0}}\right)$$

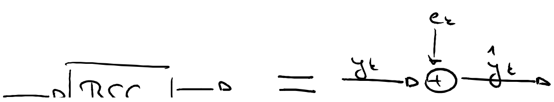
$\Longrightarrow P_b = \Phi\left(-\frac{1}{\sqrt{N_0}}\right)$ → $1^2 = 1$: transmission power

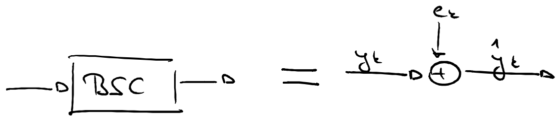
N_0 : noise power

$\frac{1}{N_0}$: SNR

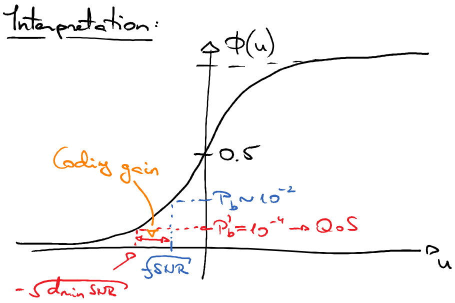
$$\boxed{\begin{array}{c} P_b = \Phi\left(-\sqrt{SNR}\right) \\ \hline B \sim \frac{1}{T} \end{array}}$$

→ Modem technology





Interpretation:



Design of a practical communication system:

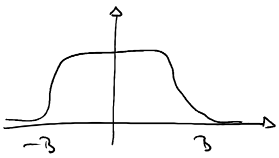
Given QoS parameters: R (dataspeed), $P'_b \leq 10^{-\gamma}$

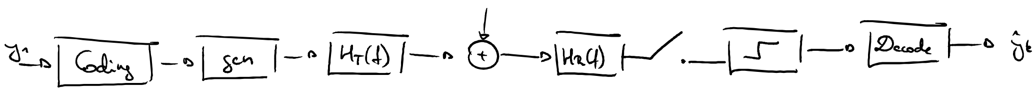
$SNR \to P_b = \Phi(-\sqrt{SNR})$

1) $C(n,k) \to d_{min}$: $\Phi(-\sqrt{d_{min}\, SNR}) \leq 10^{-\gamma}$

loss in dataspeed $\frac{k}{n} R$

$R' = \frac{n}{k} R \to B \sim \frac{n}{k} R$





$R' =$