Hopfield Net is not good for problems $M > 1000$, so we need a better solution

$$\underline{\underline{C}}_{M\times N} = \begin{pmatrix} - c^{(1)} - \\ - c^{(2)} - \\ \vdots \\ - c^{(M)} - \end{pmatrix} \qquad P_b = \Psi(\underline{\underline{C}}) \;;\; \underline{\underline{C}}_{opt}: \min_{\underline{\underline{C}}} P_b \qquad P_b = g(\underline{\underline{R}})$$

$$x_\ell = \sum_{i=1}^{M} R_{\ell i} y_i + \nu_\ell = R_{\ell\ell} y_\ell + \sum_{\substack{i=1\\ i\neq \ell}}^{M} R_{\ell i} y_i + \nu_\ell = y_\ell + \sum_{\substack{i=1\\ i\neq \ell}}^{M} R_{\ell i} y_i + \nu_\ell$$

$$\to \hat{y}_\ell = \mathrm{sgn}\{x_\ell\}$$

$$P_b = P(\hat{y}_\ell \neq y_\ell) = P(\hat{y}_\ell = 1 \mid y_\ell = -1)\cdot\frac{1}{2} + P(\hat{y}_\ell = -1 \mid y_\ell = 1)\frac{1}{2} =$$

$$= \frac{1}{2}\Big[P\big(\mathrm{sgn}\{-1 + \sum_{\substack{i=1\\ i\neq \ell}}^{M} R_{\ell i} y_i + \nu_\ell\} = 1\big) + P\big(\mathrm{sgn}\{1 + \sum_{\substack{i=1\\ i\neq \ell}}^{M} R_{\ell i} y_i + \nu_\ell\} = -1\big)\Big] =$$

$$= \frac{1}{2}\Big[P\big(-1 + \sum_{\substack{i=1\\ i\neq \ell}}^{M} R_{\ell i} y_i + \nu_\ell > 0\big) + P\big(1 + \sum_{\substack{i=1\\ i\neq \ell}}^{M} R_{\ell i} y_i + \nu_\ell < 0\big)\Big] = \circledast$$

$$\underline{y}' = (y_1 \dots y_{\ell-1}, y_{\ell+1}, \dots y_M)$$
$$P(\underline{y}' = \underline{z}) = \frac{1}{2^{M-1}} \quad : \underline{z} \in \{-1,1\}^{M-1}$$

$$\circledast = \frac{1}{2}\frac{1}{2^{M-1}} \sum_{\underline{z}\in\{-1,1\}^{M-1}} \Big\{P\big(-1 + \sum_{\substack{i=1\\ i\neq \ell}}^{M} R_{\ell i} z_i + \nu_\ell > 0\big) + P\big(1 + \sum_{\substack{i=1\\ i\neq \ell}}^{M} R_{\ell i} z_i + \nu_\ell < 0\big)\Big\} =$$

$$= \frac{1}{2^M} \sum_{\underline{z}\in\{-1,1\}} \Big\{P\big(\nu_\ell > 1 - \sum_{\substack{i=1\\ i\neq \ell}}^{M} R_{\ell i} z_i\big) + P\big(\nu_\ell < -1 - \sum_{\substack{i=1\\ i\neq \ell}}^{M} R_{\ell i} z_i\big)\Big\} = \frac{1}{2^M} \sum_{\underline{z}\in\{-1,1\}^M} \Big\{ \Phi\Big(\frac{-1 + \sum_{\substack{i=1\\ i\neq \ell}}^{M} R_{\ell i} z_i}{\sqrt{N_0}}\Big) + \Phi\Big(\frac{-1 - \sum_{\substack{i=1\\ i\neq \ell}}^{M} R_{\ell i} z_i}{\sqrt{N_0}}\Big) \le \circledast$$

$$P_b = \frac{1}{2^M} \sum_{\underline{z}\in\{-1,1\}^{M-1}} \Big\{\Phi\Big(\frac{-1 + \sum_{\substack{i=1\\ i\neq \ell}}^{M} \underline{c}^{(\ell)T}\underline{c}^{(i)} z_i}{\sqrt{N_0}}\Big) + \Phi\Big(\frac{-1 - \sum_{\substack{i=1\\ i\neq \ell}}^{M} \underline{c}^{(\ell)T}\underline{c}^{(i)} z_i}{\sqrt{N_0}}\Big) \Longrightarrow \underline{\underline{C}}_{opt}: \min_{\underline{\underline{C}}} p_b$$

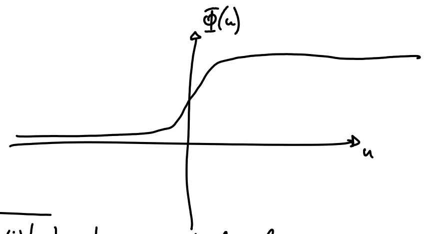
Problem:

$$\underline{\underline{C}} \to O(2^{M-N}\cdot 2^{M-1}) \sim O(2^{M(M-1)(N)})$$

Lemma: $0 < a < b < c$

$$\Phi\Big(\frac{-c+b}{\sigma}\Big) + \Phi\Big(\frac{-c-b}{\sigma}\Big) > \Phi\Big(\frac{-c+a}{\sigma}\Big) + \Phi\Big(\frac{-c-a}{\sigma}\Big)$$

$$\Phi\Big(\frac{-c+b}{\sigma}\Big) - \Phi\Big(\frac{-c+a}{\sigma}\Big) > \Phi(??) + \Phi(??)$$



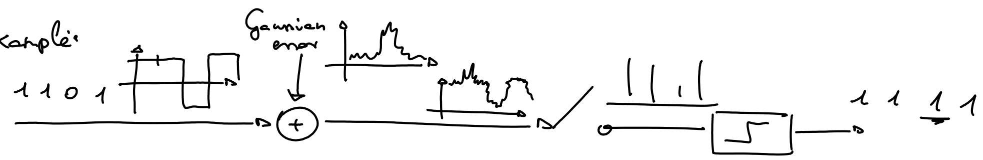
$$\circledast \le \frac{1}{2}\Big[\Phi\Big(\frac{-1 + \sum_{\substack{i=1\\ i\neq \ell}}^{M} |R_{\ell i}|}{\sqrt{N_0}}\Big) + \Phi\Big(\frac{-1 - \sum_{\substack{i=1\\ i\neq \ell}}^{M} |R_{\ell i}|}{\sqrt{N_0}}\Big)$$

$$\boxed{P_b \le \frac{1}{2}\Big[\Phi\Big(\frac{-1 + \sum_i' |\underline{c}^{(\ell)T}\underline{c}^{(i)}|}{\sqrt{N_0}}\Big) + \Phi\Big(\frac{-1 - \sum_i' |\underline{c}^{(\ell)T}\underline{c}^{(i)}|}{\sqrt{N_0}}\Big)}$$

it can truly be optimized but we minimize an upper bound of P_b

Resources — bandwidth, transmission power $\longleftrightarrow$ **QoS** — Dataspeed, P_b $\;\to\;$ $QoS \overset{??}{=} \Psi(\text{resources})$

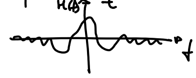
Example:



Problem with that model:

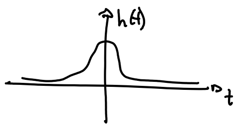
we produce the signal as $y(t) = \sum_i y_i h(t - iT)$ where $h(t)$ looks like:

$\to$ but that $h(t)$ has finite support, so its spectrum is infinite $\to$

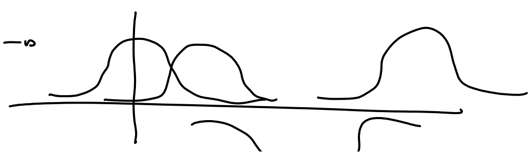


$\to$ we do not have infinite bandwidth

$\Rightarrow$ then we can use $h(t)$ that has infinite support and finite spectrum



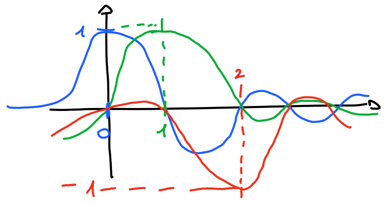
$\to$ but these signals interfere with each other. $\to$



$$x_k = \sum_i y_i h_{k-i} + \nu_k = h_0 y_k + \cancel{\sum h_{k-i}} y_i + \nu_k$$

InterSymbol Interference
ISI

Solution: Use sinusoid signals that do not interfere:



⇒ sum of these signals is accurate at the given point but arbitrary everywhere else

→ Problem: needs very well synchronized system clocks

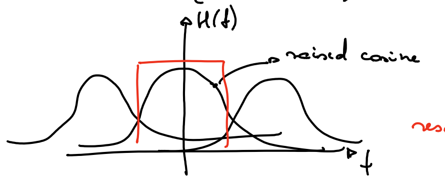
→ if we fail to do that we get samples from bad places

↙ Solution

$$h_\ell = \delta_{0\ell} = \begin{cases} 1 : \ell = 0 \\ 0 : \ell \neq 0 \end{cases}$$

DFT: $H(e^{i\omega T}) = \sum_\ell h_\ell e^{-i\ell\omega T} = 1$

$$H(e^{i\omega T}) = \sum_\ell H\left(f + \frac{\ell}{T}\right)$$



$\boxed{B \sim \frac{1}{T}}$

resource QoS