

1	2	3	4	5	6	7	8	9	10	Overall	Grade
10	5	5	10	5	10	15	15	10	15	100	

**Each problem must be worked out on separate sheet, except for Problem 5!
A detailed reasoning must accompany each numerical result !!!**

1) Given a code set with lengths $l_1 = 1; l_2 = 2; l_3 = 2; l_4 = 3$

Can this code be uniquely decodable (justify your answer) !

Solution: Not, because it does not fulfill the Kraft inequality.

2) Given a digital data transmission over a bandlimited channel

a) State the relationship between the transmitted and receiver filter for the sake of optimal noise transformation.

b) Define the condition for ISI free transmission

Solution: a) $H_R(f) = H_T^*(f)$

b)
$$\sum_m \left| H \left(f + \frac{m}{T} \right) \right| = 1; \quad |f| \leq \frac{1}{2T}$$

3) a) Define the conditional entropy $H(Y|X)$ of two random variables X and Y .

b) define the channel capacity by using entropies

c) What is the joint entropy of K independent identically distributed random variables (where the marginal entropy is denoted by $H(X)$)

Solution

a)
$$H(Y|X) = \sum_x \sum_y p(x, y) \log \left(\frac{1}{p(y|x)} \right)$$

b)
$$C := \max_{p(x)} I(X, Y) = \max_{p(x)} H(X) - H(Y|X)$$

c)
$$H(X_1, \dots, X_K) = KH(X)$$

4) There is a source with quadrary alphabet with and a digital channel with state transition matrix

$$\mathbf{P} = \begin{pmatrix} 0.125 & 0.125 & 0.25 & 0.5 \\ 0.125 & 0.5 & 0.125 & 0.25 \\ 0.25 & 0.125 & 0.5 & 0.125 \\ 0.5 & 0.25 & 0.125 & 0.125 \end{pmatrix}$$
 into how long codewords should we extend 10 symbol long message sequences in order to guarantee reliable communication ?

Solution:

Channel capacity: $C = \log(4) - 0.125 * 6 - 0.25 * 2 - 0.5 * 1 = 0.25 \rightarrow n = k / C = 40$

5) Indicate the true statements

- The dataspeed is proportional with the bandwidth over which the digital communication system is implemented (TRUE)
- The I-divergence between two distribution $D(p||q)$ is always positive (TRUE)
- The channel capacity is independent of the input distribution
- For a symmetric channel the state transition matrix and its transpose are identical

6) Given a generator polynom of a cyclic code as follows $g(x) = x^9 + y^3x^8 + y^{11}x^7 + y^5x^6 + y^6x^5 + y^3x^4 + yx^3 + y^{14}x^2 + yx + y^9$, the length of the message vector is 6. What are the parameters of the code ?

Solution: $k=6, \deg(g(x)) = 9 = n - k \rightarrow n = 15$

7) Construct a binary Hamming code with parameter $n=15$.

- a) Determine the parameters of the code
- b) Give the generator and the parity check matrix
- c) How many vectors form a group belonging to a given syndrome vector \mathbf{s}
- d) Give the codeword which belongs to message vector $\mathbf{u} = (01000000000)$

Solution:

a) $C(15,11)$

b) The parity check matrix $\mathbf{H} = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 \end{pmatrix}$

The generator matrix is $\mathbf{G} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$

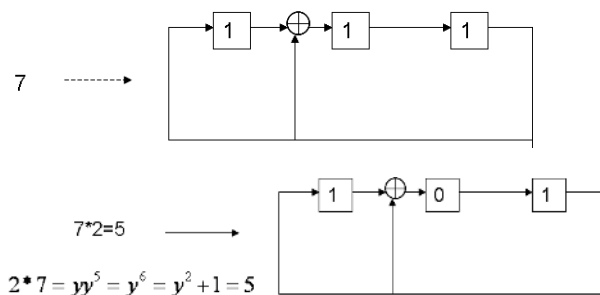
c) The size of the group is $|E_s| = 2^k$

d) The second row of the generator matrix $\mathbf{c} = (0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 1)$

8) Implement the multiplication of $(2*7)$ over GF(8) in a shift register. The representation of 7 must be downloaded as an initial state into the shift register. Give the connection pattern and indicate the final result in the shift register.

Solution:

$$y(a_0 + a_1y + a_2y^2) = a_0y + a_1y^2 + a_2y^3 = a_0y + a_1y^2 + a_2(y+1) = a_2 + (a_0 + a_2)y + a_1y^2$$



9. Given a $C\left(\frac{2}{4}\right); L = 2; G = \{15, 9, 9, 5\}$ convolutional encoder

a) Give the number of states

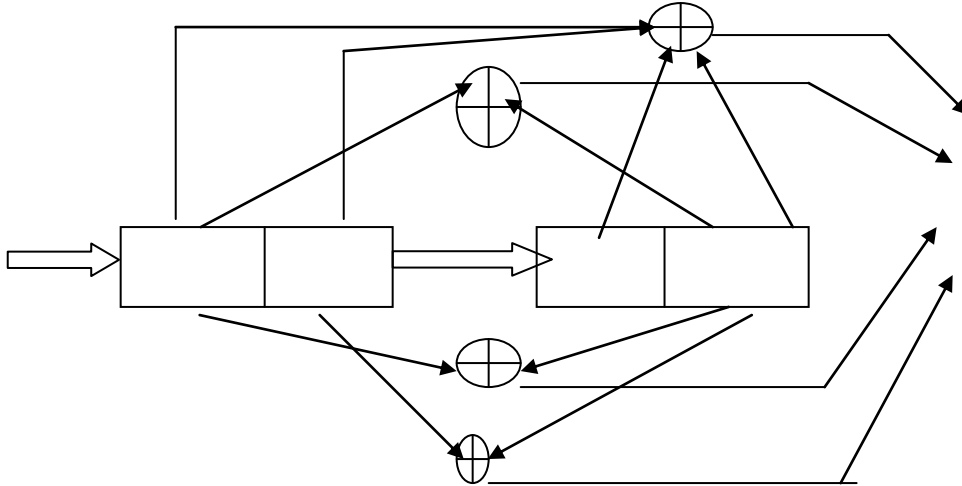
b) What is the complexity of the Viterbi algorithm in the case of receiving the sequence (1110101)

c) Depict the architecture

Megoldás:

a) Az állapotszám: $2^{(L-1)k} = 2^2 = 4$

b) $O(V2^{kL}) = O(3 * 2^4) = O(3 * 16) = O(48)$

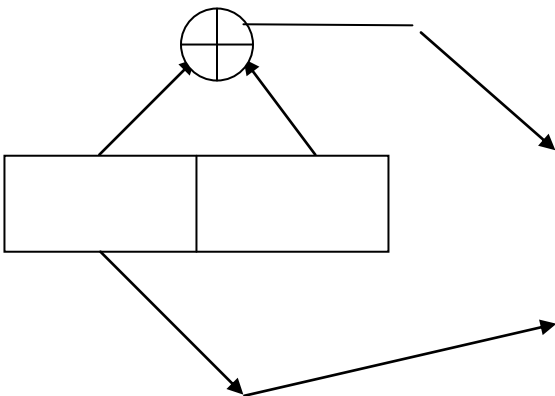


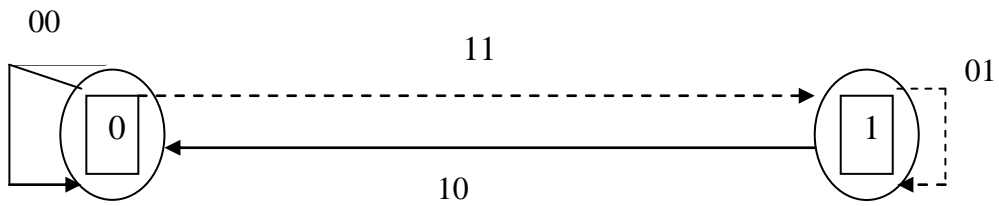
c)

10) Given a $C\left(\frac{1}{2}\right)_{L=2} G = \{3, 2\}$ t convolutional encoder

- a. Give the architecture
- b. Give the state-graph
- c. Give the transfer function
- d. How many codesequence has weight 12

Solution:





$$X_b = D^2 X_a + D X_b \quad T(D) = \frac{D^3}{1-D} = D^3 + D^4 + D^5 + D^6 + \dots$$

$$X_{a'} = D X_b$$

1 db. 12 súlyú útvonal van.