Development of Complex Curricula for Molecular Bionics and Infobionics Programs within a consortial* framework**

Consortium leader

PETER PAZMANY CATHOLIC UNIVERSITY

Consortium members

SEMMELWEIS UNIVERSITY, DIALOG CAMPUS PUBLISHER

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**Molekuláris bionika és Infobionika Szakok tananyagának komplex fejlesztése konzorciumi keretben

***A projekt az Európai Unió támogatásával, az Európai Szociális Alap társfinanszírozásával valósul meg.

Digital- and Neural Based Signal Processing & Kiloprocessor Arrays

Digitális- neurális-, és kiloprocesszoros architektúrákon alapuló jelfeldolgozás

Introduction and Analog to Digital conversion

Bevezetés és az analóg-digitális átalakítás

dr. Oláh András

Outline

- Course Information
- Introduction and focus of the course
- What is signal processing? (objectives: algorithms, architectures and applications)
- First lecture: A/D conversion
  - The sampling theorem
  - Uniform quantization
  - Non uniform quantization
- AD converters and main performances
- Available AD converters on the market
  - Successive Approximation Register ADC
  - Delta-Sigma ADC

Course Information

- **Class Mailing List**: digjel@lists.ppke.hu
- Small tests in classes on all topics;
- One major test on Digital Signal Processing scheduled in the middle of the semester;
- Exam (major questions on Neural Processing, small questions on Digital Signal Processing);
- **Grading**:
  - Final grade=0.33*av. on STs+0.33*DSP+ 0.33*NSP

Course Information (Cont'd)

- **Suggested literature and references:**
  - Lecture notes (essential for the tests and exams)
  - J.G. Proakis, D.G. Manolakis: „Digital Signal Processing", Prentice Hall, 1996, ISBN 0-13394338-9
  - S. Haykin „Adaptive filters" ‚Prentice Hall, 1996 (recommended)
  - Haykin, S.: Neural networks - a comprehensive foundation, MacMillan, 2004
  - Hassoun, M.: Fundamentals of artificial neural networks, MIT Press, 1995
  - Chua, L.O., Roska T. and Venetianer, P.L.: "The CNN is as Universal as the Turing Machine", IEEE Trans. on Circuits and Systems, Vol. 40., March, 1993
  - J.G. Proakis: Digital communications, McGraw Hill, 1996.

Course Syllabus and Scheduling

1. Introduction and Analog to Digital conversion.
2. Description digital signals and systems in time domain.
3. Description digital signals and systems in transform (Z, DFT) domain.
4. Efficient computation of the transform domain (FFT) and filter design.
5. Adaptive signal processing.

   Midterm exam

6. Introduction to neural processing (inspiration, history and approaches).
7. Signal processing by a single neuron (linear set separation).
8. Hopfield network, Hopfield net as associative memory and combinatorial optimizer.
9. Cellular Neural Network.
10. Feed forward Neural Networks (generalization, representation, learning, appl.).
11. Principal Component Analysis.
12. Virtual machines: signal processing with multicore systems.

    Final exam

Objective of Digital Signal Processing

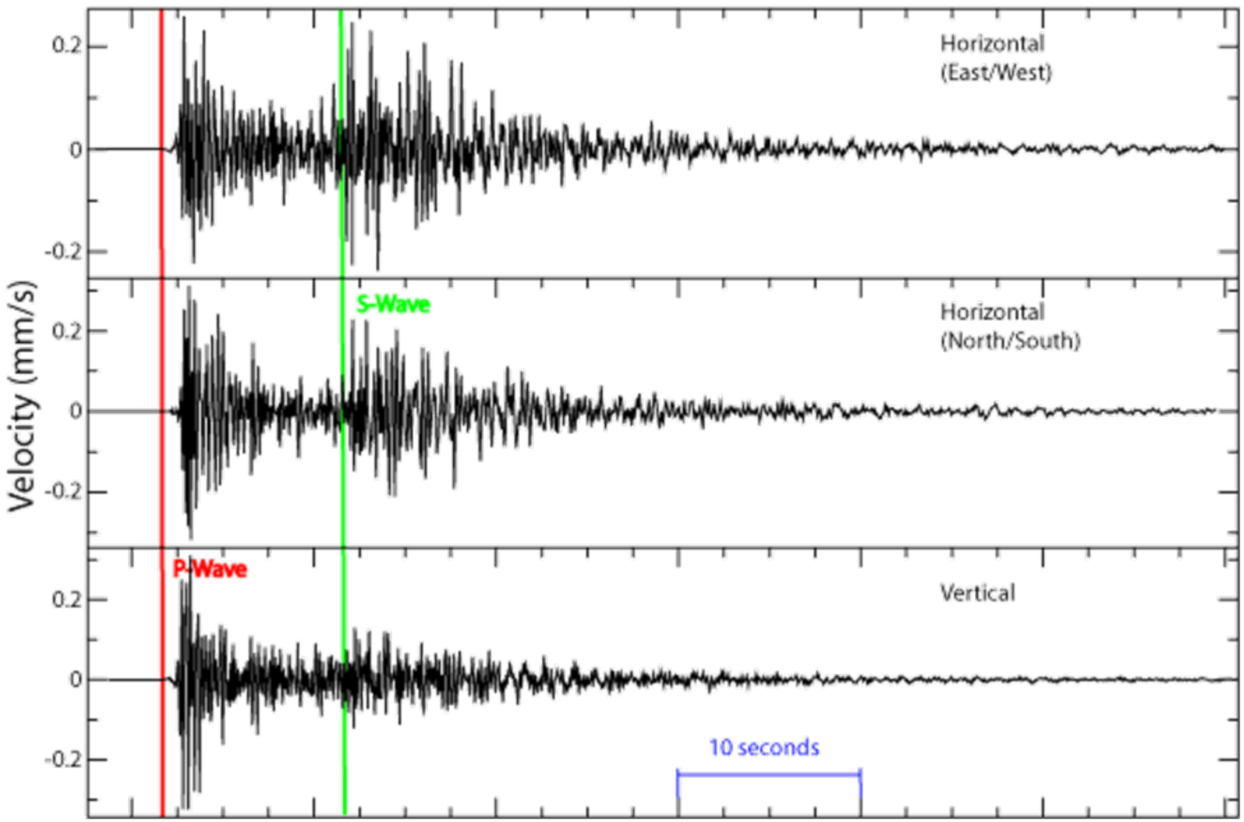
Observed physical process → A/D conversion + computer + SP algorithms → Important feature

Medical signals,
Seismic signals,
Vibro analysis,
Speech
Video
Multimedia
etc.

linear and nonlinear algorithms

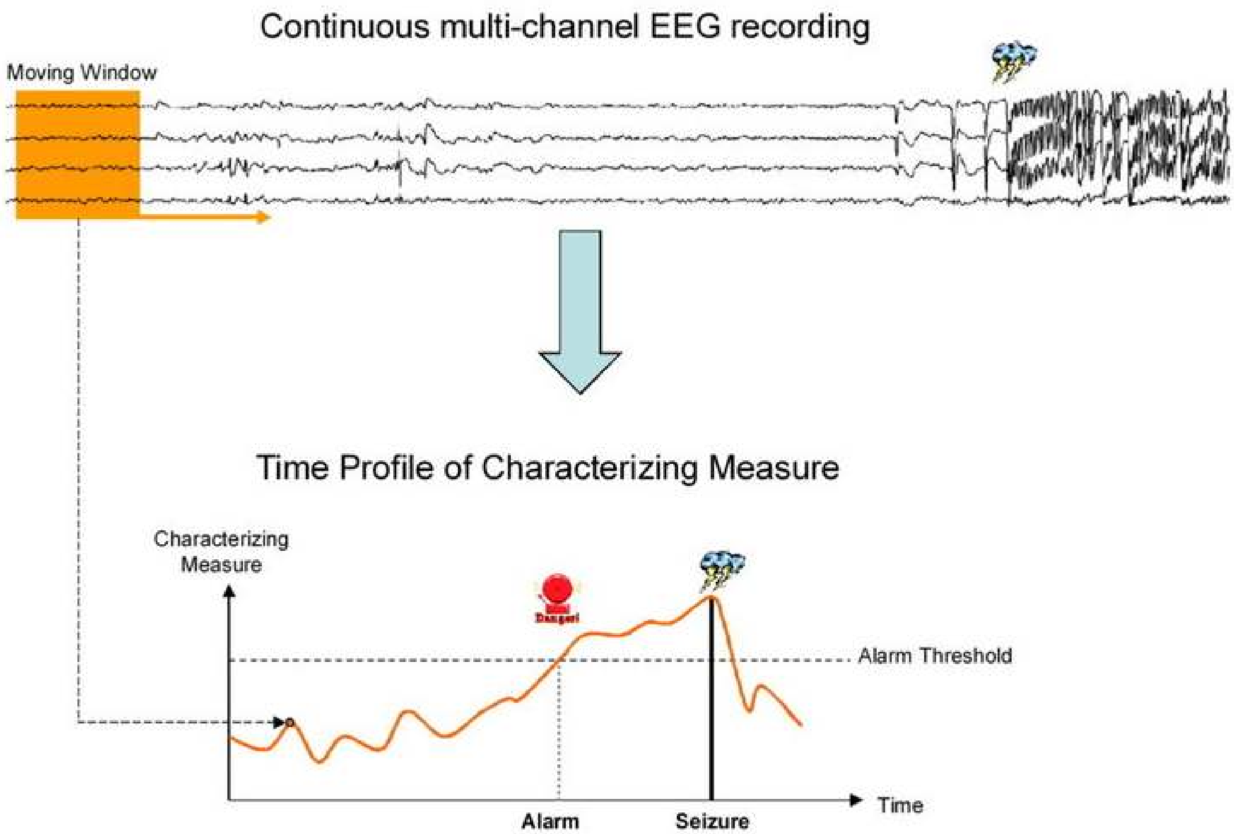
Predicting epileptic seizure,
Predicting earthquake
Testing bearings
Data compression for transmitting multimedia information

Seismic signal analysis



http://en.wikipedia.org/wiki/File:Seismogram.gif

Epileptic seizure prediction



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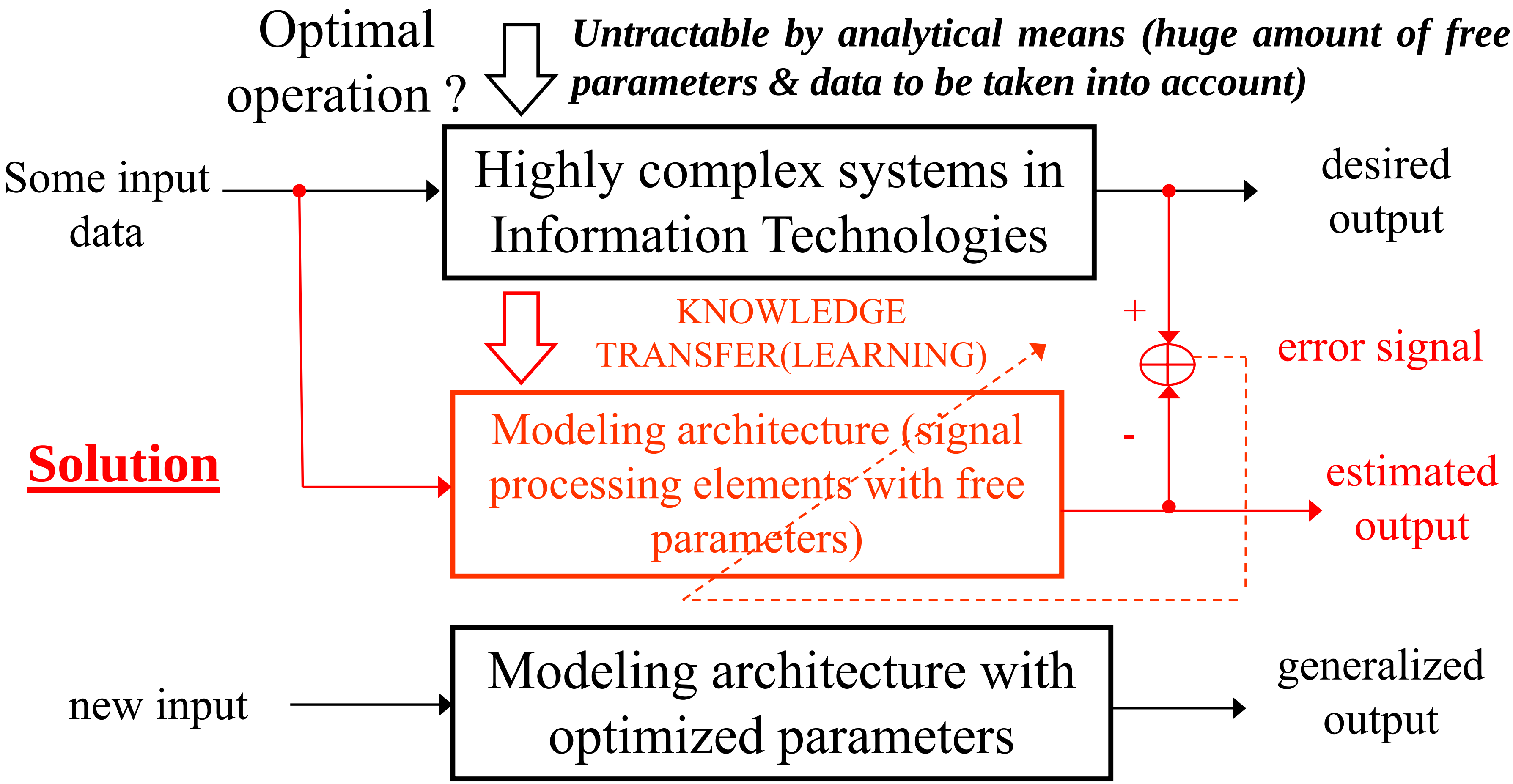
Signal processing in IT



Fundamental issues

- Representation capabilities (is the architecture complex enough to model the system) ?
- Learning (how to adjust the free parameters to capture the hidden characteristics of the modeled system) ?
- Generalization (once the knowledge transfer has taken place, how to trust the output given to an input being not part of the training set) ?

Examples

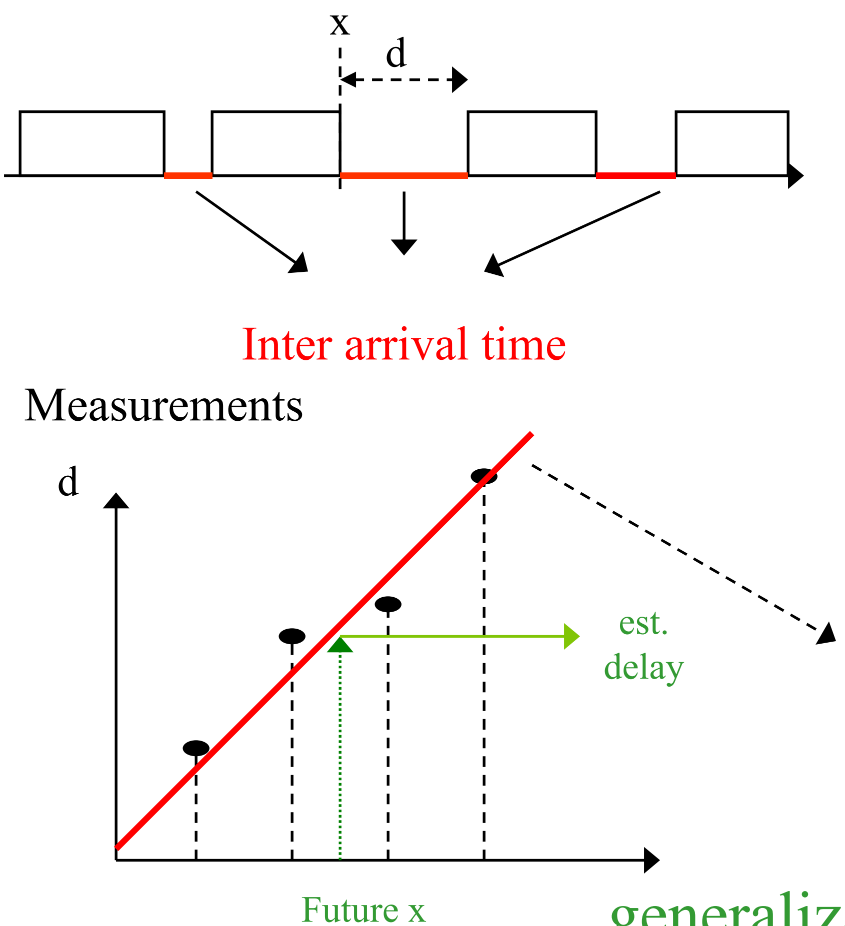
Input traffic volume (voice, video, multimedia) → IP network (with routers, switches, buffers …etc.) → QoS parameters (average packet loss rate, average packet delay)

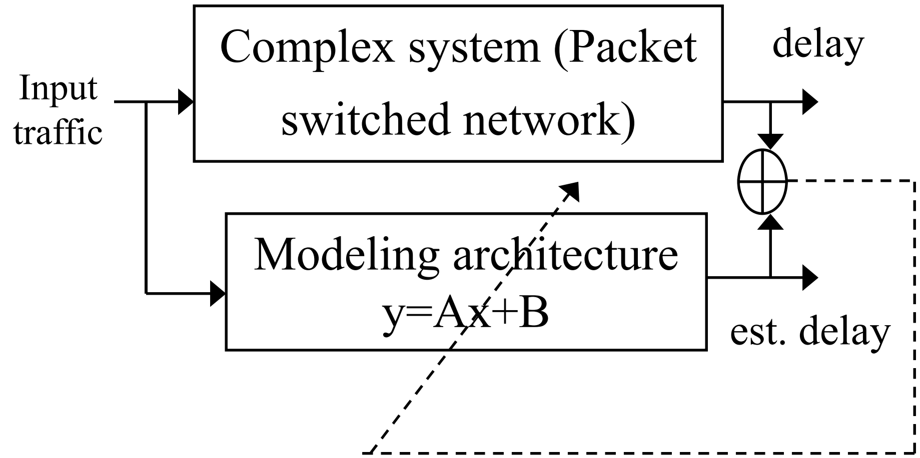
Number of users → GSM or UMTS, b3G systems → Multiuser interference

Stock prices, currency exchange rates (financial data series) → Financial system (stock market, economical factors) → Optimal investment for maximizing the return

Endeavour: HOW TO MODEL AND OPTIMIZE THESE SYSTEMS ?

A simple example – packet delay estimation



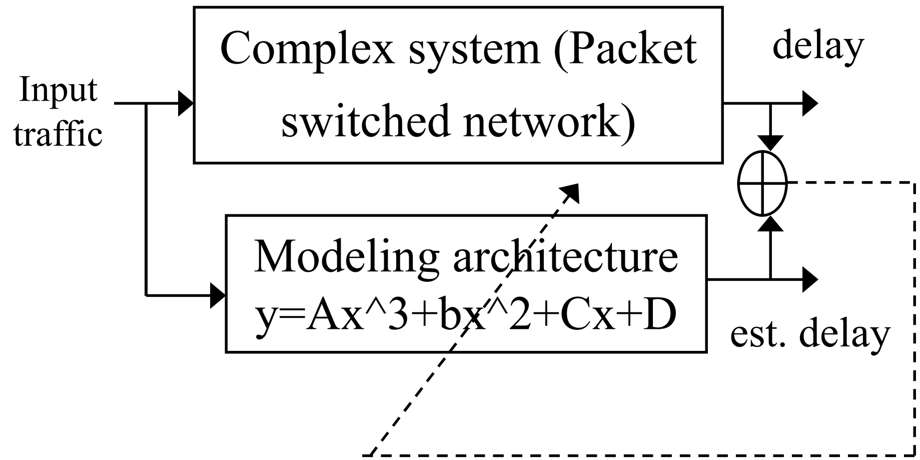


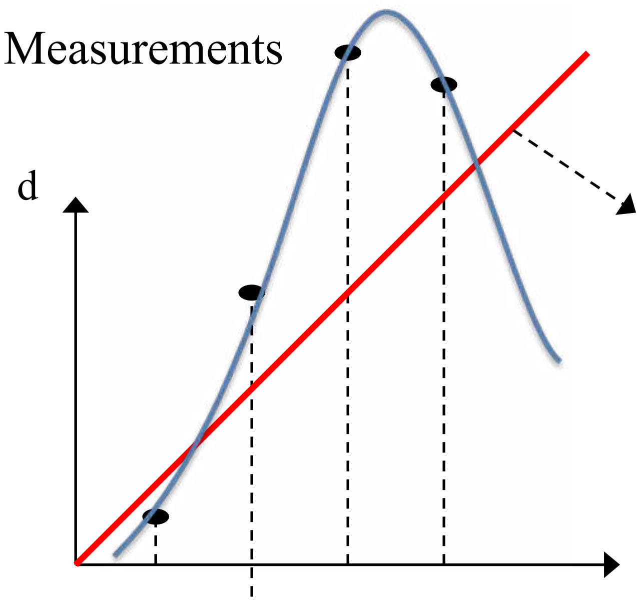
$$A_{opt}, B_{opt} : \min_{A,B} \frac{1}{K} \sum_{k=1}^{K} \left(d_k - Ax_k - B\right)^2$$

learning

generalization

A simple example – packet delay estimation (cont’)



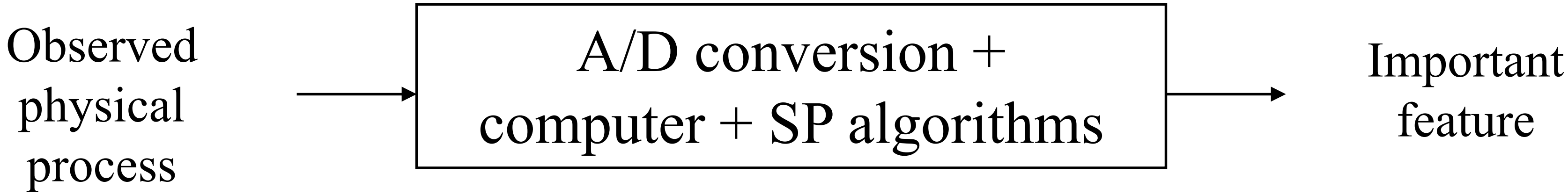


Linear approximation is not good

$$A_{opt}, B_{opt} : \min_{A,B} \frac{1}{K} \sum_{k=1}^{K} \left(d_k - Ax_k^3 - Bx_k^2 - Cx_k - D \right)^2$$

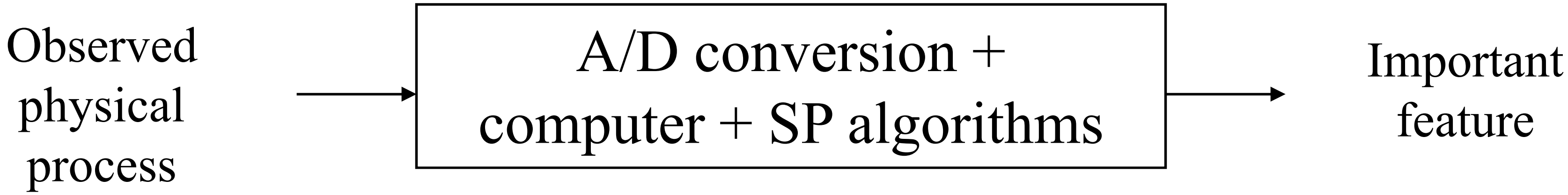
Challenges

- Linear or nonlinear modeling (most of the real-life problems are of highly nonlinear nature) ?
- How to develop fast learning algorithms ?
- How to develop exact measures expressing the quality of generalization ?



Directions

- Fast and real-time linear processing with designated HW architecture (DSP)
- Biologically inspired nonlinear processing
- Emergence of novel computational paradigms by using kilo-processor arrays



Biological inspirations

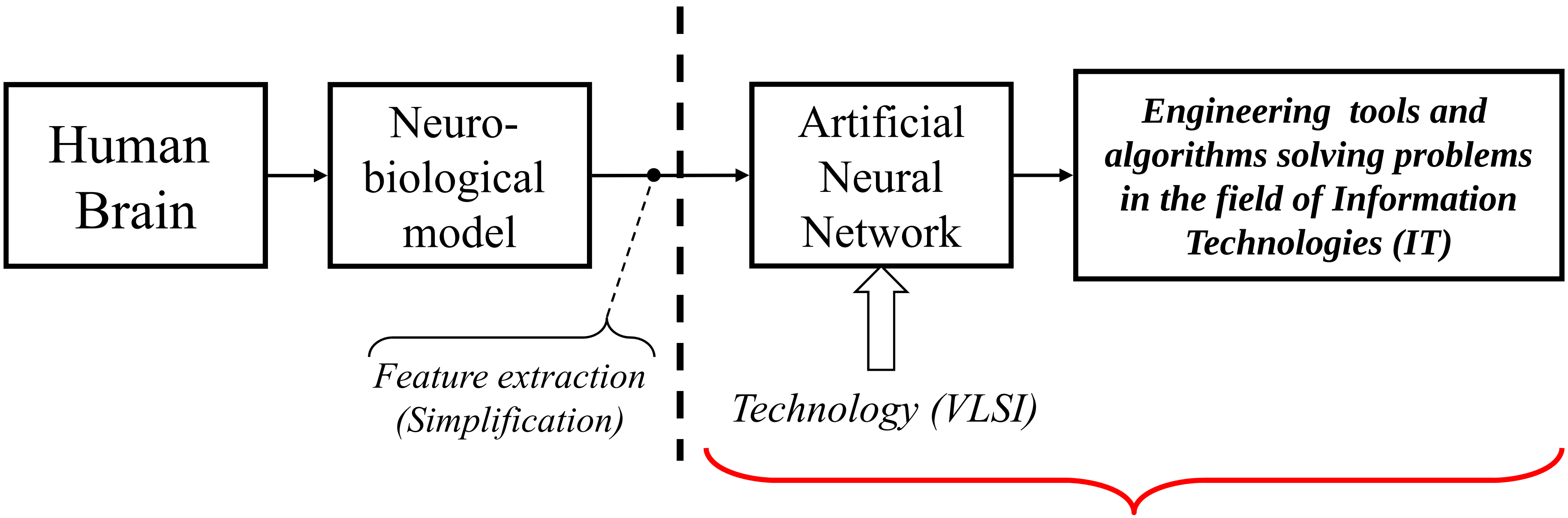
Modeling architecture

Representation Learning Generalization Robustness Modularity

Solution provided by evolution and biology: MAMMAL BRAIN

- high representation capability;
- large scale adaptation;
- far reaching generalizations;
- modular structure (nerve cells, neurons);
- very robust

Copying the Brain?

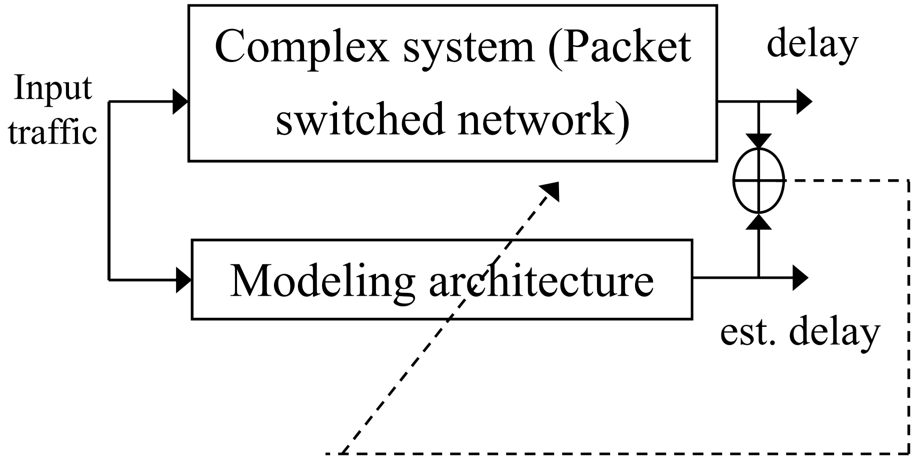


Focus of this course:

Signal Processing algorithms

Signal processing introduction - summary

- Collection of algorithms to solve highly complex problems in real-time (in the field of IT) by using classical methods and novel computational paradigms routed in biology.



Historical notes

- Linear analog filters, 20’s
- Artificial neuron model, 40’s (McCulloch-Pitts, J. von Neumann);
- Hebbian learning rule, 50’s (Hebb)
- Perceptron learning rule, 50’s (Rosenblatt);
- Fast Fourier Transformation, 50’s
- Nonlinear adaptive filter, 50’s (Gabor)
- ADALINE, 60’s (Widrow)
- Critical review , 70’s (Minsky)
- Adaptive linear signal processing (RM, KW algorithms) , 70’s

Historical notes (cont')

- DSPs and digital filters, 80's
- Feed forward neural nets, 80's (Cybenko, Hornik, Stinchcombe)
- Back propagation learning, 80's (Sejnowsky, Grossberg)
- Hopfield net, 80's (Hopfield, Grossberg);
- Self organizing feature map, 70's - 80's (Kohonen)
- CNN, 80's-90's (Roska, Chua)
- PCA networks, 90's (Oja)
- Applications in IT, 90's - 00's
- Kiloprocessor arrays, 2005

Analog-to-Digital Conversion

Signal analysis and processing is engaged with studying the different phenomena of nature and drawing conclusions about how the observed quantities are changing in time. All applications have one thing in common, signals are studied as a function of time and the analysis is carried out by a computer. However, computers can only process digital sequences, thus the analog signal must first be converted into a binary sequence.

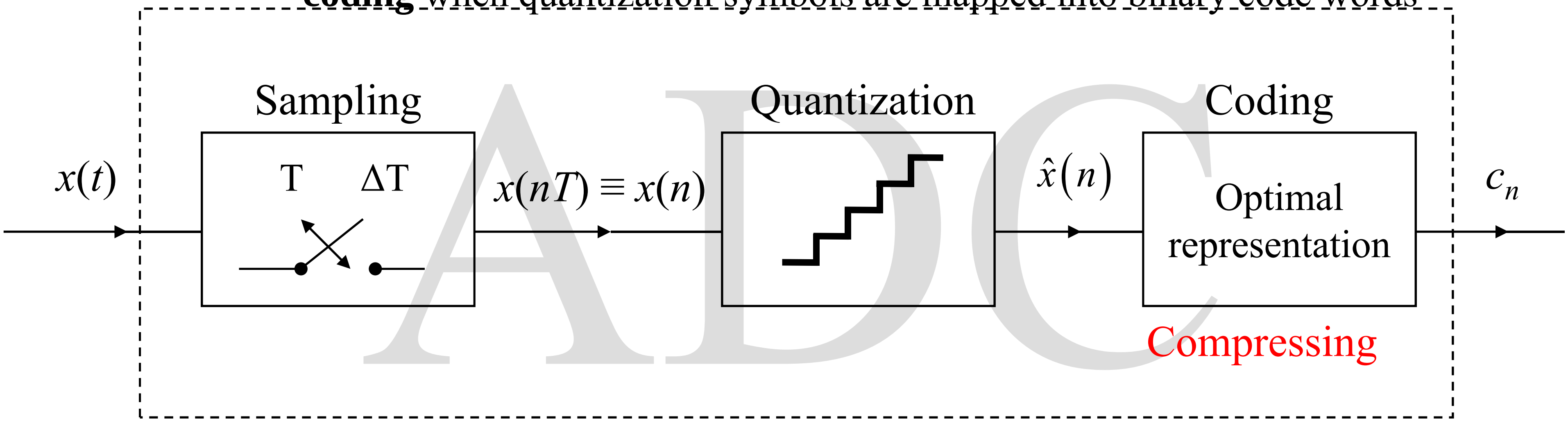
Notations

The underlying notation is summarized by the following table:

Signal		***Time***	***Voltage***
Analog signal	$x(t)$	*Continuous*	*Continuous*
Sampled signal	$x(n)$ or $x(nT)$	*Discrete*	*Continuous*
Quantized signal	$\hat{x}_k$	*Discrete*	*Discrete*
Coded signal	c_n	*Discrete*	*Binary*

Analog-to-Digital Conversion

- ADC has three main steps:
  - **sampling** when sample the value of the signal $x(t)$ at certain discrete time instants obtaining a sequence x_k;
  - **quantization** when the values of the samples x_k are rounded to some allowed discrete levels (referred to as quantization levels) and having a finite set of these levels they can then easily be represented by binary code words.
  - **coding** when quantization symbols are mapped into binary code words

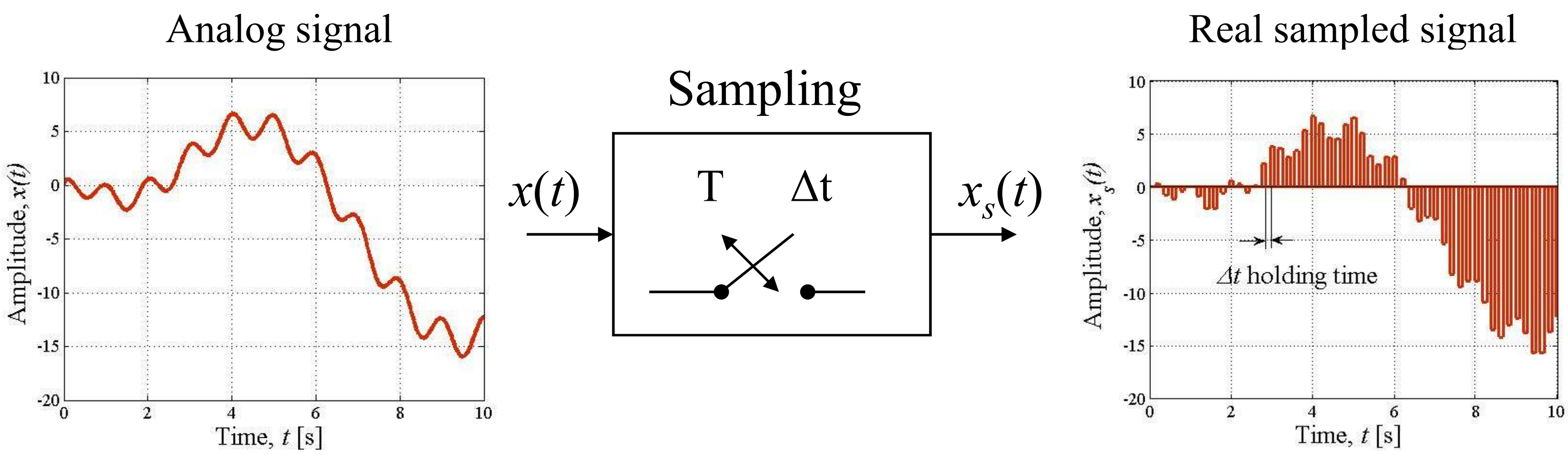


The challenge of ADC

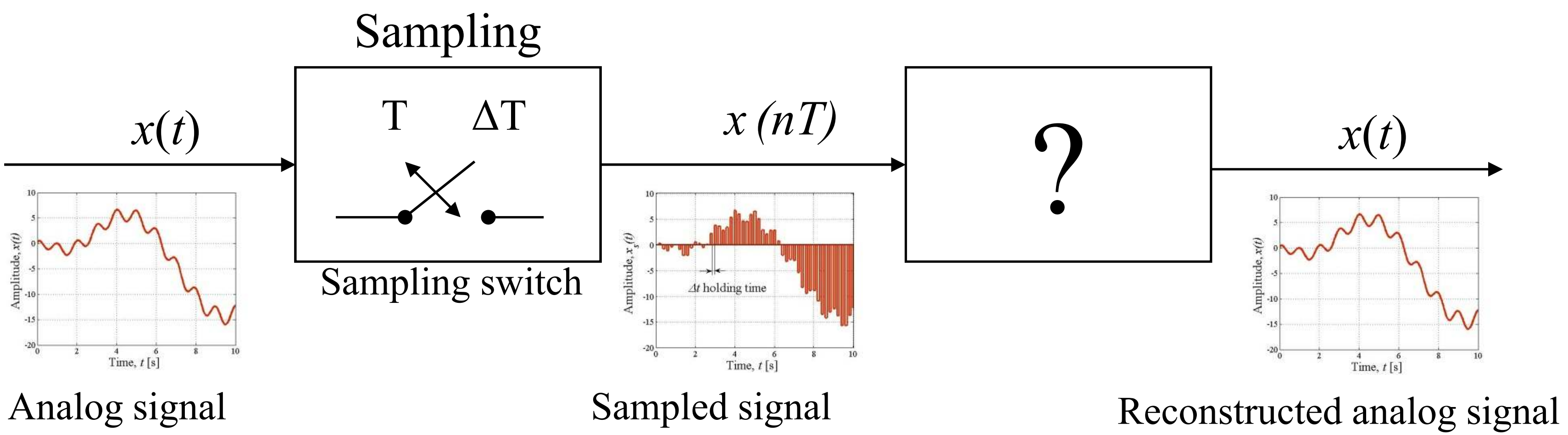
- Question:
  - Is there any loss of information in the course of the conversion?
  - What is the optimal representation of signals by binary sequences (in terms of length …etc.) ?
- Fundamental challenges of sampling and of quantization: choosing proper sampling frequency and quantization levels. ADC is fully characterized by
  - the sampling frequency (denoted by f_s);
  - the number of quantization levels (N),
  - and the rule of quantization.
- Optimizing ADC means that we seek the optimal values of these parameters in order to obtain efficient binary representation of signals with minimum loss of information.

Sampling

Sampling is carried out by a switch and temporary we assume that the switch is ideal (i.e. the holding period is zero).



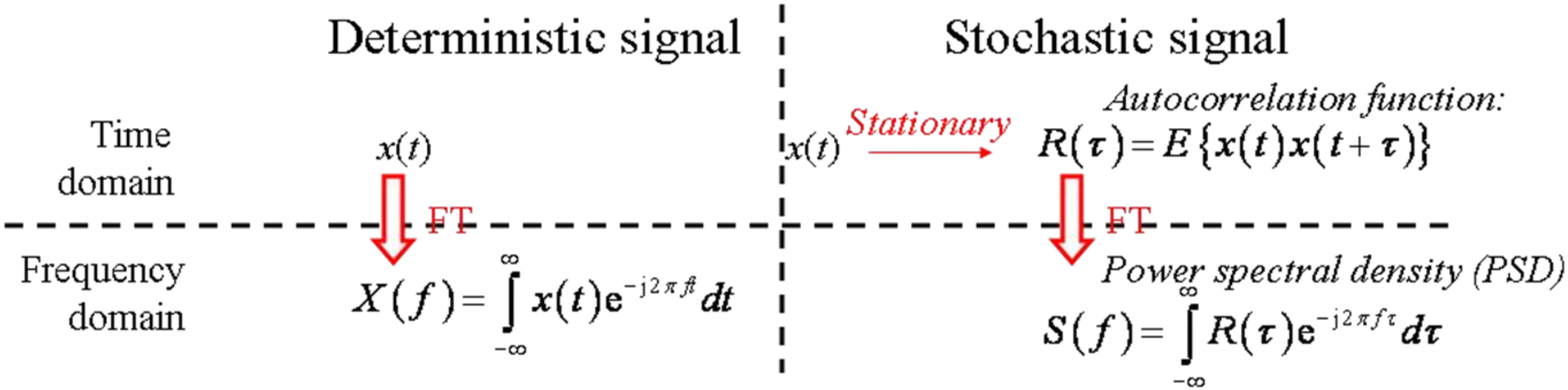
Sampling (cont')



Can analog signals be reconstructed from their samples without any loss?

Bandwidth of a signal: the concept

- It is desirable to classify signals according to their frequency-domain characteristics (their frequency content):
  - **Low-frequency signal**: if a signal has its spectrum concentrated about zero frequency
  - **High-frequency signal**: if the signal spectrum concentrated at high frequencies.
  - **Band pass-signal**: a signal having spectrum concentrated somewhere in the broad frequency range between low frequencies and high frequencies.



Bandwidth of a signal: the concept (cont’)

- The quantative measure of the range over which the spectrum is concentrated is called the bandwidth of signal.
- We shall say that a signal is **band limited** if its spectrum is zero outside the frequency range $|f| \geq B$, where B is the *absolute bandwidth*. The absolute bandwidth dilemma:
  - Band limited signals are not realizable!
  - Realizable signals have infinite bandwidth!
  - (No signal can be time-limited and band limited simultaneously.)

Bandwidth of a signal: the concept (cont')

- In the case of a band pass signal ($f_{\min} \le f \le f_{\max}$), the term **narrowband** is used to describe the signal if its bandwidth

$$B = f_{\max} - f_{\min},$$

is much smaller than the median frequency

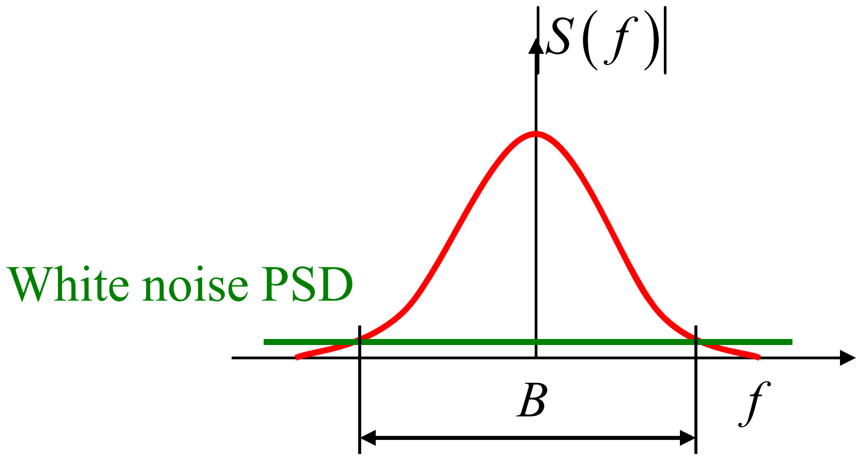
$$(f_{\max} + f_{\min})/2.$$

Otherwise, the signal is called **wideband**.

- There are many bandwidth definitions depending on application:
  - noise equivalent bandwidth
  - 3 dB bandwidth
  - η% energy bandwidth

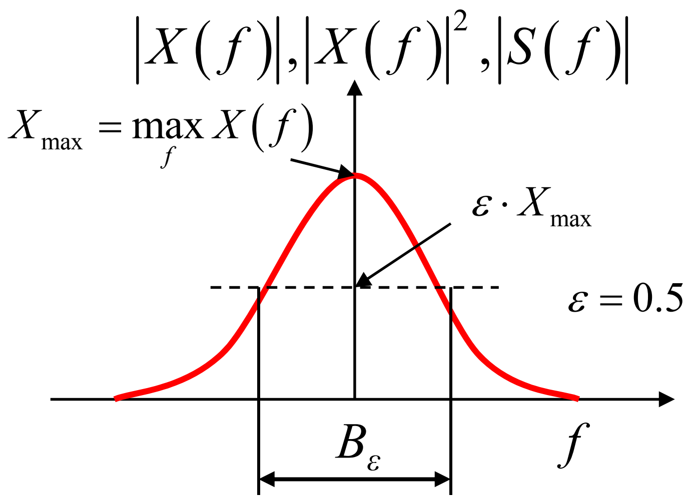
The noise equivalent bandwidth

It is defined as the bandwidth of a system with a rectangular transfer function that receives as much noise as the system under consideration



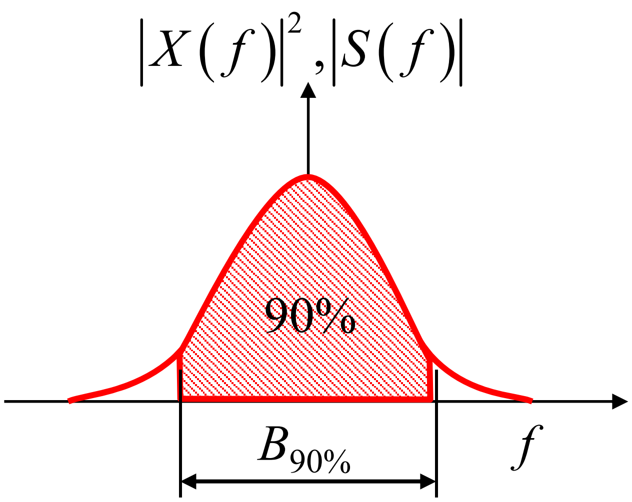
The 3 dB bandwidth

Is the bandwidth at which the absolute value of the spectrum (energy spectrum or PSD) has decreased to a value that is 3 dB below its maximum value.



The η% energy bandwidth

Is the bandwidth that contains η % of total emitted.



Frequency ranges of a some natural signals

	Type of Signal	Frequency Range [Hz]
Biological Signals	Electroretinogram	0 - 20
	Pneumogram	0 - 40
	Electrocardiogram (ECG)	0 -100
	Electroenchephalogram (EEG)	0 - 100
	Electromyogram	10 - 200
	Sphygmomanogram	0 - 200
	Speech	100 - 4000
Seicmic signals	Seismic exploration signals	10 - 100
	Eartquake and nuclear explosion signals	0.01-10
Electromagnetic signals	Radio bradcast	$3x10^4$ - $3x10^6$
	Common-carrier comm.	$3x10^8$ - $3x10^{10}$
	Infrared	$3x10^{11}$ - $3x10^{14}$
	Visible light	$3.7x10^{14}$ - $7.7x10^{14}$

The sampling theorem

(Shannon – Kotelnikov 1949)

If a band limited signal $x(t)$ (the band is limited to B) is sampled with sampling frequency $f_s \geq 2B$ then $x(t)$ can be uniquely reconstructed form its samples as follows:

$$x(t) = \sum_{n=-\infty}^{\infty} x(nT)\,h(t - nT)$$

where

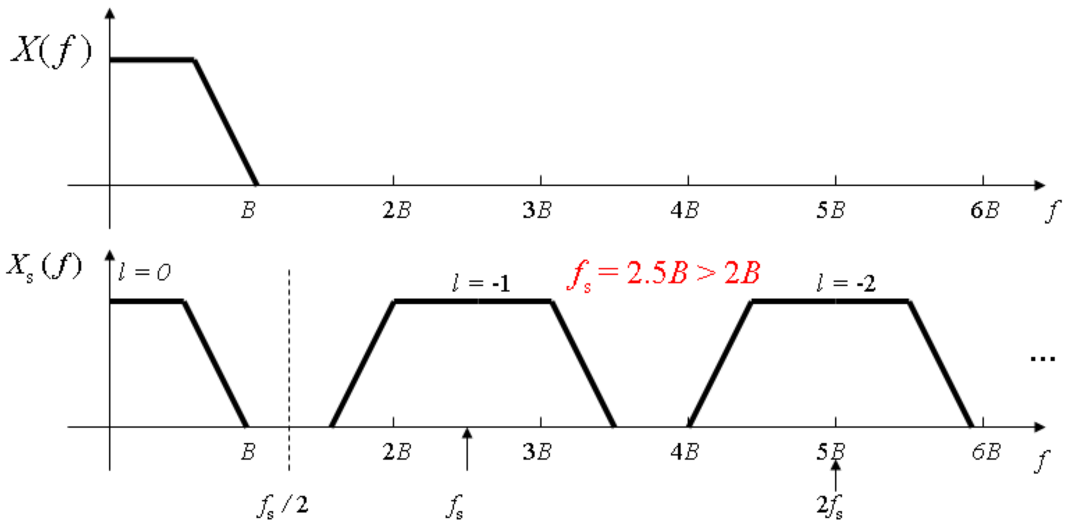
$$h(t) = 2T\frac{\sin(2\pi Bt)}{2\pi Bt}$$

Proof of sampling theorem

Since $X(f)$ is band limited it can be extended to form a periodic signal

$$X_s(f) = \sum_l X\left(f + \frac{l}{T_s}\right)$$

if $1/T_s > 2B$ as indicated by the next figure:



Proof of sampling theorem (cont')

One may notice, that the condition $f_s = 1/T_s > 2B$ guarantee that there is no overlapping in $X_s(f)$ and as a result:

$$X_s(f) = X(f) \quad -1/2T_s \le f \le 1/2T_s$$

(furthermore since $f_s = 1/T_s > 2B$ this statement is also true

$$X_s(f) = X(f) \quad -B \le f \le B$$

Let us also note that $X_s(f)$ is a periodic signal, i.e.

$$X_s(f) = X_s\left(f + \frac{l}{T_s}\right)$$

Proof of sampling theorem (cont')

Let us now express a sample $x(n)$ by the means of inverse Fourier transform

$$x(n)=\int_{-B}^{B} x(t)e^{j2\pi fnT}df$$

On the other hand, $X_s(f)$ being a periodic signal it can be expanded into Fourier series as follows:

$$X_s(f)=\sum_n c_n e^{-j2\pi nfT_s}$$

where

$$c_n=\frac{1}{T_s}\int_{-T_s/2}^{T_s/2} X_s(f)e^{j2\pi nT_s}df=\frac{1}{T_s}\int_{-B}^{B} X_s(f)e^{j2\pi nT_s}df$$

Proof of sampling theorem (cont')

From the Fourier series of $X_s(f)$ follows that

$$Tx(n) = c_n \qquad \text{and} \qquad X_s(f) = T\sum_n x(n)e^{-j2\pi nfT_s}$$

Taking into account that $X_s(f) = X(f) \quad -B \le f \le B$ we can write

$$x(t) = \int_{-B}^{B} X(f)e^{j2\pi ft}dt = \int_{-B}^{B} X_s(f)e^{j2\pi ft}dt$$

and substituting

$$X_s(f) = T_s\sum_n x(n)e^{-j2\pi nfT_s}$$

into the integral, we obtain

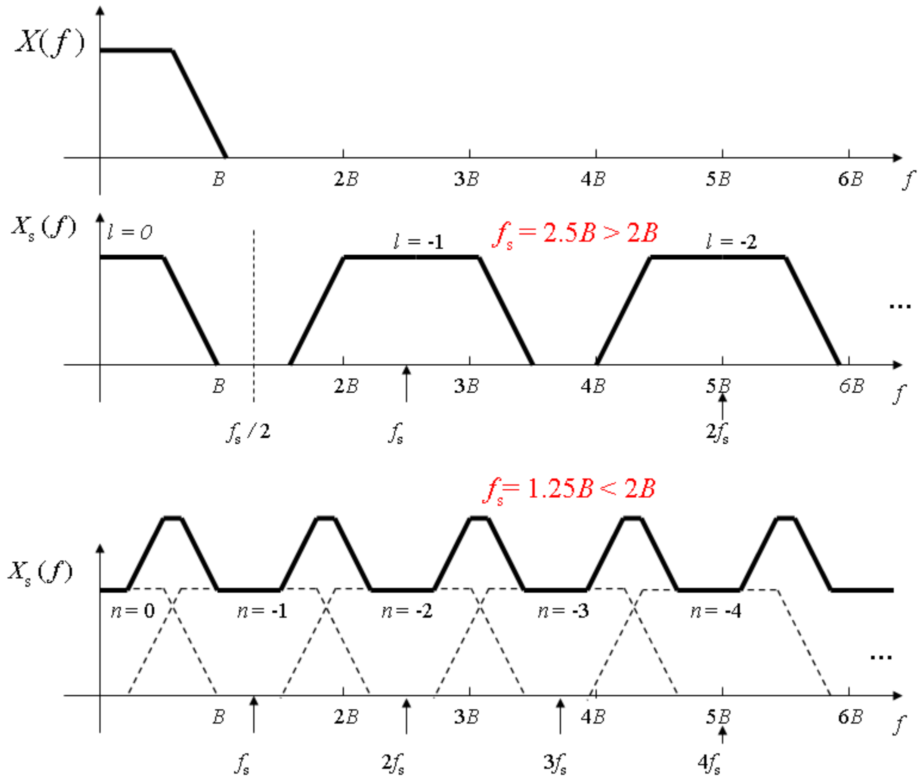
$$x(t) = \int_{-B}^{B}\left(T\sum_n x(n)e^{-j2\pi nT_s f}\right)e^{j2\pi ft}dt = T\sum_n x(n)\int_{-B}^{B} e^{j2\pi f(t-nT_s)}df =$$

$$= T_s\sum_n x(n)\frac{\sin(2\pi B(t-nT_s))}{2\pi B(t-nT_s)} = T_s\sum_n x(n)h(t-nT_s),$$

which proves the theorem.

Phenomena of aliasing

If the sample frequency is not chosen to be high enough (i.e. frequency $f_s \geq 2B$), then $X_s(f)$ then there is an overlap in the spectrum, which implies that $X(f)$ cannot be regained from $X_s(f)$.



Aliasing

Problem 1:

We sample the functions $x_1(t)=u(t)e^{-t}$ and $x_2(t)=u(t)te^{-t}$, respectively.

- Which function has larger bandwidth ? (To determine the bandwidth use parameter ε=0.01)
- What is the minimum sampling frequency to uniquely restore the signals form their samples ?

Problem 2:

Given a the frequency response of a system as follows:

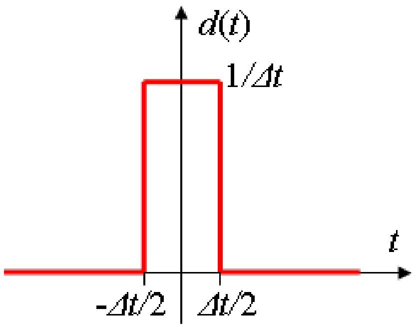
$$H(\omega)=\frac{1}{1+j\omega\cdot 1,59\cdot 10^{-4}}$$

- Determine the bandwidth of the system with parameter ε=0.1!
- What is the impact on the bandwidth if we set ε=0.01?
- What type of filtering does this system implement ?

Sampling in practice

- In practice the sampling is carried out by a switch which has a finite (non-zero) holding time.
- If the holding time Δt is small enough then $x_s(t)$ can be perceived as

$$x_s(t) = \sum_n x(nT) d(t - nT)$$

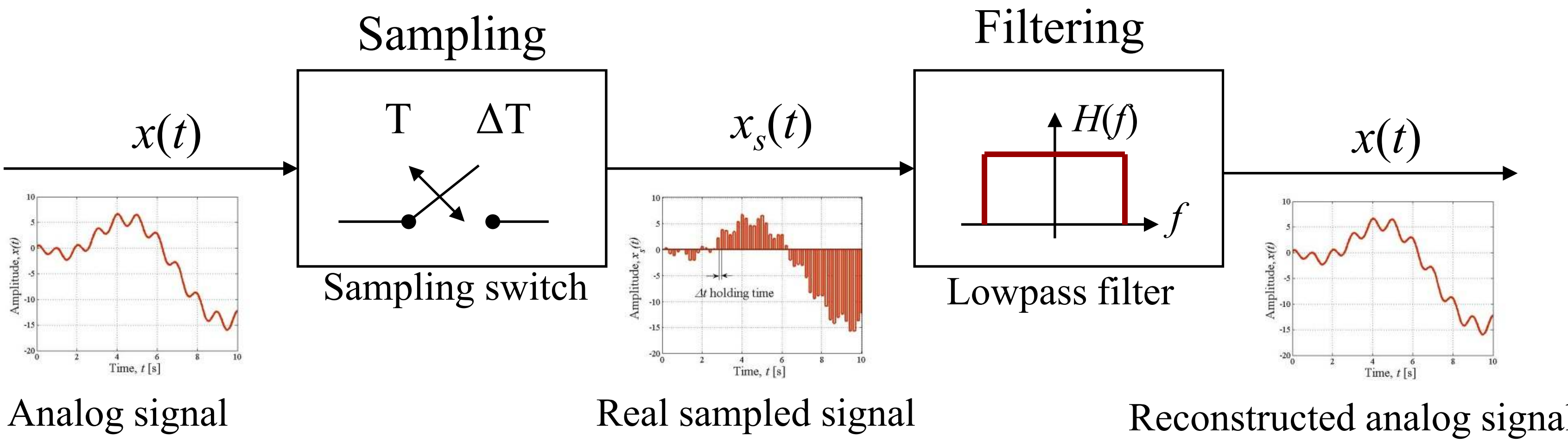


$$\lim_{\Delta t \to 0} d(t) = \delta(t)$$

- The signal $x_s(t)$ is often called real sampled signal, as $x_s(t)$ can be obtained from $x(t)$ by a proper electronic circuitry.
- Since the $d(t) \to \delta(t)$ when $\Delta t \to 0$, thus if we construct a low-pass filter with impulse response function $h(t)$ then the output of this filter to the input $d(t)$ is approximately $h(t)$ as well.

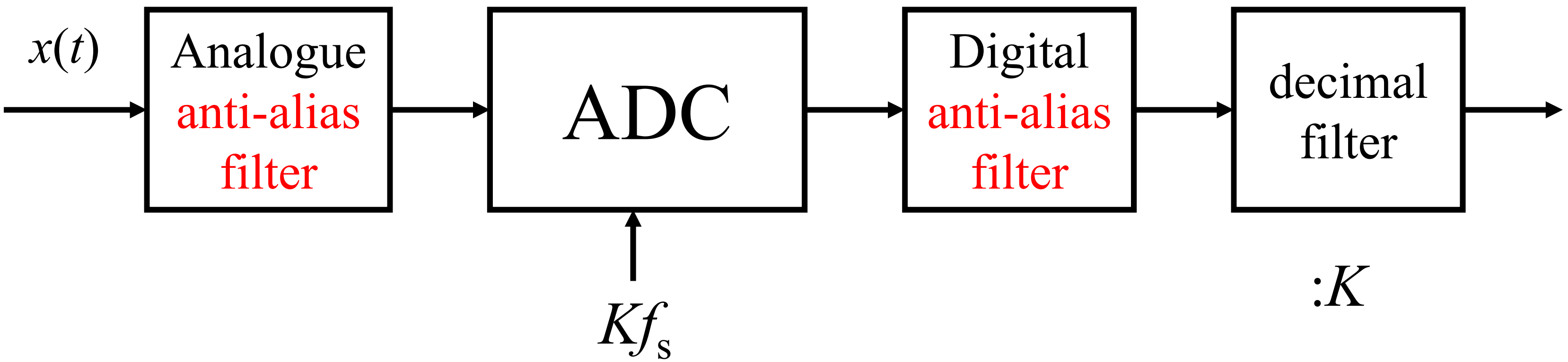
Summarizing of sampling

In the case of practical sampling first we obtain $x_s(t)$ from $x(t)$ and then from $x_s(t)$ the original signal $x(t)$ can be regained by letting $x_s(t)$ pass through a low pass filter.

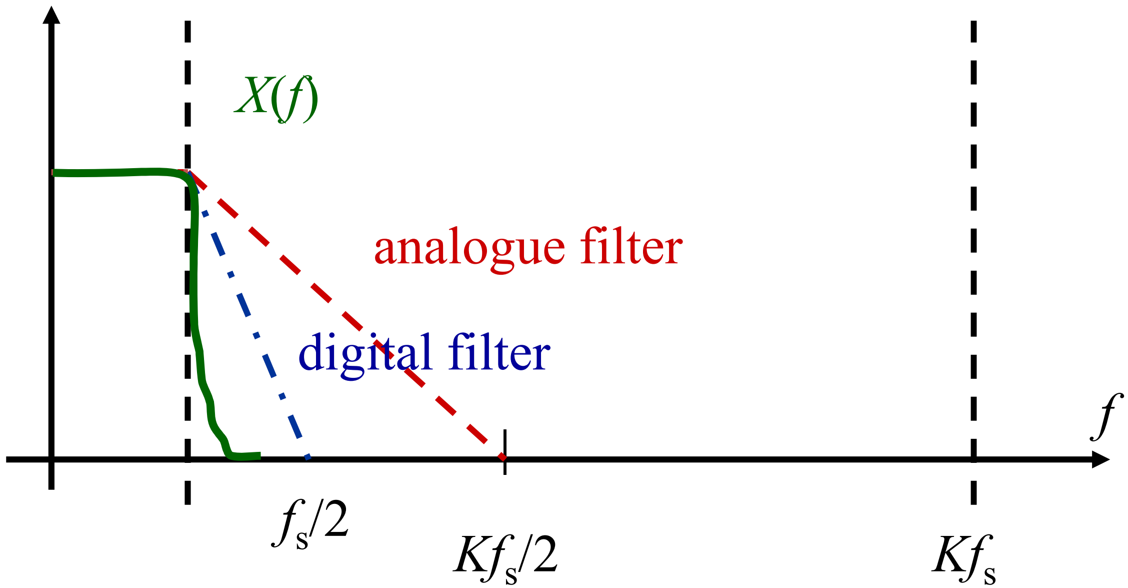


Oversampling technique

According to the Shannon theorem the sampling frequency f_s should be two times larger than the signal bandwidth B. Such a choice of sampling frequency creates a risk that the signals of frequency $f_H > B$ can generate the signals f_H-f_s in the bandwidth after sampling. For that reason it is safer to set the sampling frequency f_s two times larger than the frequency when the anti-alias filter sufficiently attenuates the signals.



Oversampling technique (cont')



Oversampling technique (cont')

Higher sampling frequency means less critical requirements of the filter performances. The profit related to oversampling:

- cheaper and less complicated anti-alias filter
- noise reduction increases the quantization SNR (see later)

This method is currently applied in high quality sound processing:

- in SACD system introduced by Sony (*SACD – Super Audio Compact Disc*) the sampling frequency is *2.82 MHz* which means the oversampling factor $K = 64$.
- in *DVD Audio* system introduced by Technics the sampling frequency is *192 kHz* and the oversampling factor is $K = 4$.

Under sampling technique

Let us consider another case when we process the signal in the bandwidth 30MHz – 55MHz. Applying the sampling frequency 110 MHz (according to the Shannon theorem) seems to be extravagant. In such a case we can modify the Shannon rule (aliasing free sampling):

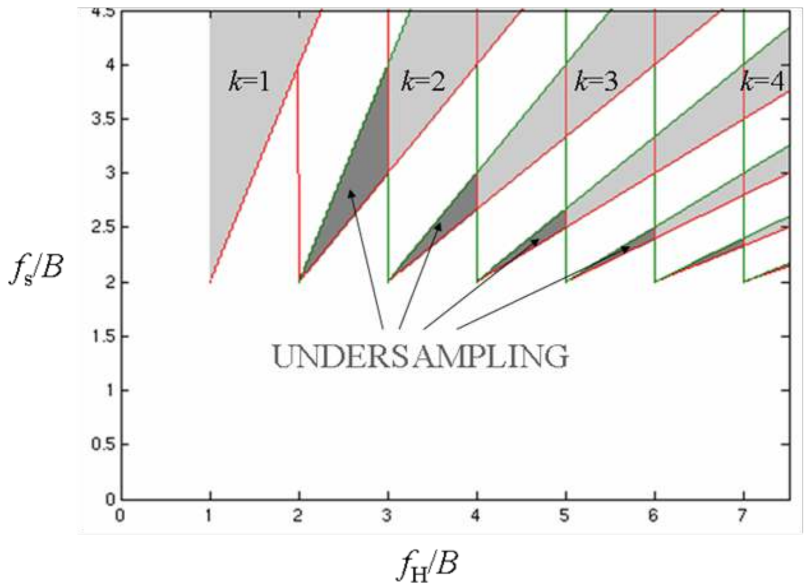
$$\frac{2}{k}\cdot\left(\frac{f_{\mathrm{H}}}{B}\right)<\left(\frac{f_{\mathrm{s}}}{B}\right)<\frac{2}{k-1}\cdot\left(\frac{f_{\mathrm{H}}}{B}-1\right)$$

where

$$1\le k\le \mathrm{trunc}\left(\frac{f_{\mathrm{H}}}{B}\right)$$

Note: k=1 is returned the original Shannon sampling rule

Under sampling technique (cont')

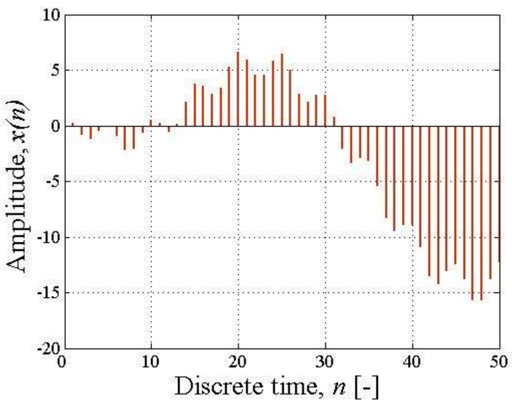


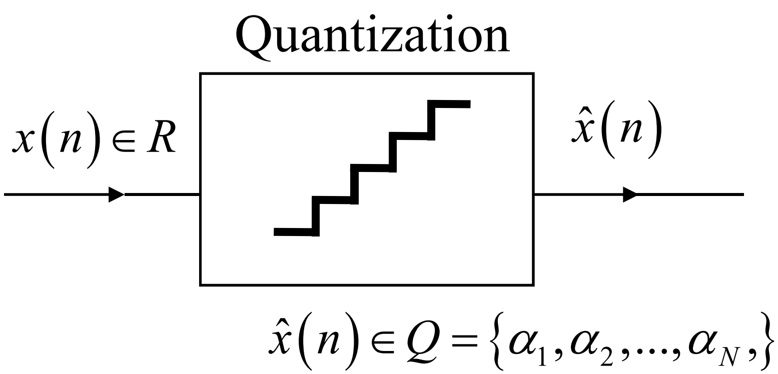
In our case of the signals in bandwidth 30 MHz – 55 MHz it is sufficient to use sampling frequency 55MHz$\leq f_s \leq$60MHz instead of 110 MHz. Of course, by using the under sampling technique we apply a band-pass anti-alias filter instead of a low-pass filter.

Quantization

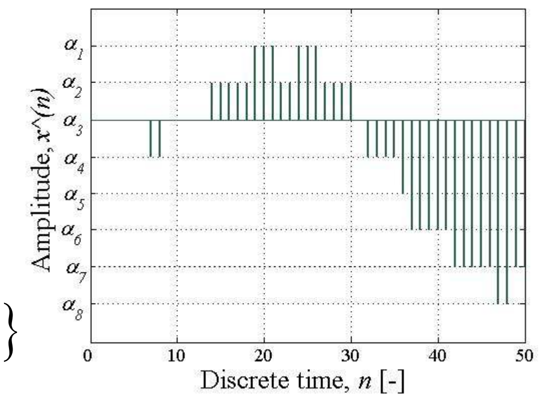
We assume that the signal is already sampled and we deal with samples $x(n)$. Since each sample has continuous amplitude, quantization is concerned to mapping $x(n)$ into $\hat{x}(n)$ which may have only a finite number of values.

Sampled signal





Quantified signal



Quantization (cont')

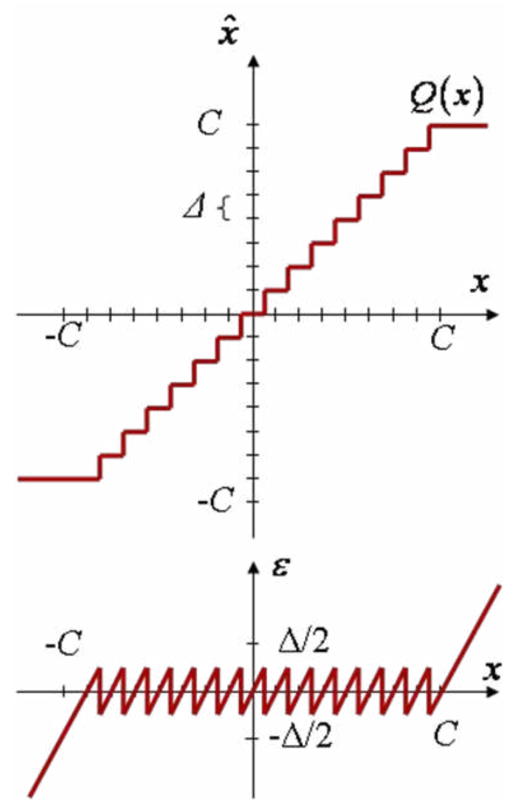
- Quantization always entails loss of information due to the rounding process.
- The design of a quantizer is concerned with two parameters:
  - number of quantization levels;
  - location of quantization levels (uniform or non-uniform);
- The quality of quantization is described by a Signal-to-Quantization Noise Ratio (SQNR) where the average signal power is compared to the noise power resulting from the quantization error:

$$SQNR := \frac{\text{average signal power}}{\text{average noise power due to quantization}}$$

$$(SQNR^{[dB]} := 10 \log SQNR)$$

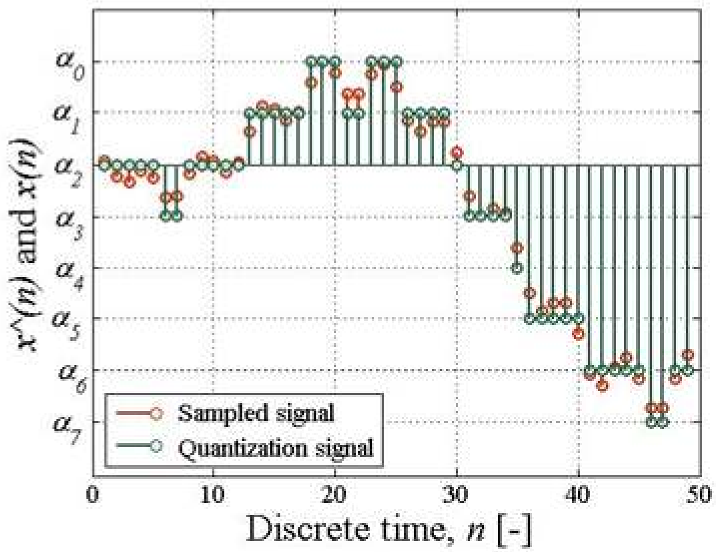
Uniform quantization

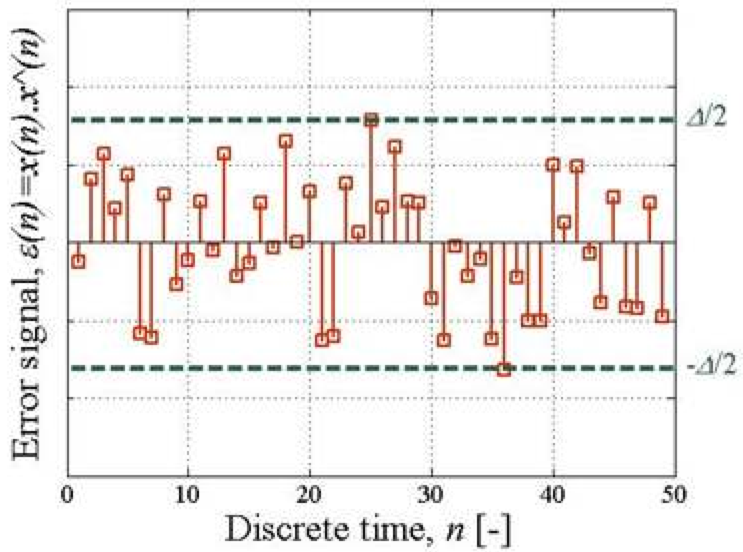
- Signal value is rounded off to predefined thresholds called as quantization values which are equidistantly placed.
- Notations:
  - the sample range is [-C,C]
  - the distance between the thresholds is Δ,
  - the number of quantization level is $N = 2C/\Delta = 2^n$, where n represents the number of bits by which the quantized signal can be represented.
  - the error signal is $\varepsilon := x - \hat{x}$ and -Δ/2≤ ε ≤ -Δ/2.



The quantization characteristics and the quantization error function

Uniform quantization (cont')

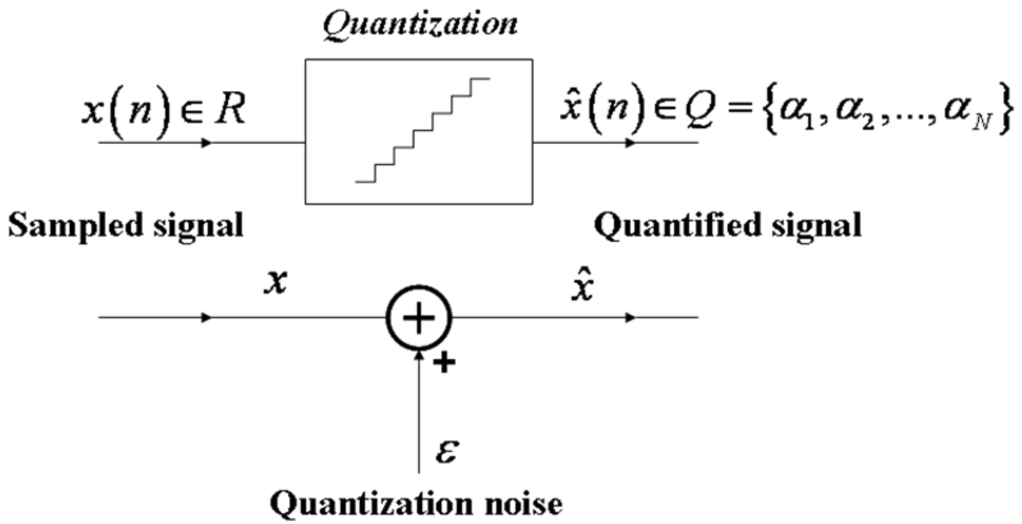


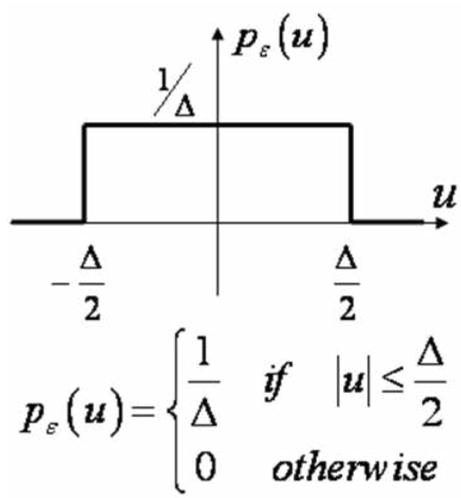


Modeling the quantization noise

Since the nature of errors are random the specific value of ε depends on the value of the current sample, thus ε is regarded as a random variable subject to uniform probability density function, and the average noise power is

$$E\left(\varepsilon^2\right)=\int_{-\Delta/2}^{\Delta/2} u^2 p_\varepsilon(u)du=\int_{-\Delta/2}^{\Delta/2} u^2\frac{1}{\Delta}du=\frac{\Delta^2}{12}$$





SQNR of the uniform quantization

- In the case of full-scale sine wave (with amplitude C):

$$SQNR := \frac{C^2/2}{\Delta^2/12} = \frac{3}{2}\frac{4C^2}{\Delta^2} = \frac{3}{2}N^2 = \frac{3}{2}2^{2n} \qquad (SQNR^{[dB]} := 6.02n + 1.78)$$

- In the case of random input variable subject to uniform probability density function over the interval [-C,C]:

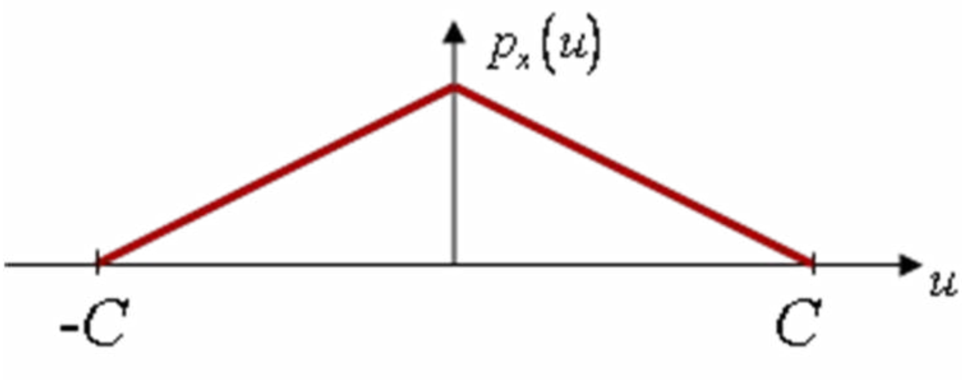
$$SQNR := \frac{(2C)^2/12}{\Delta^2/12} = \frac{4C^2}{\Delta^2} = N^2 = 2^{2n} \qquad (SQNR^{[dB]} := 6.02n)$$

- In the case of sine wave with amplitude A (in normal operation i.e. $A<C$)

$$SQNR^{[dB]} := 6.02n + 1.78 - 20\log(C/A)$$

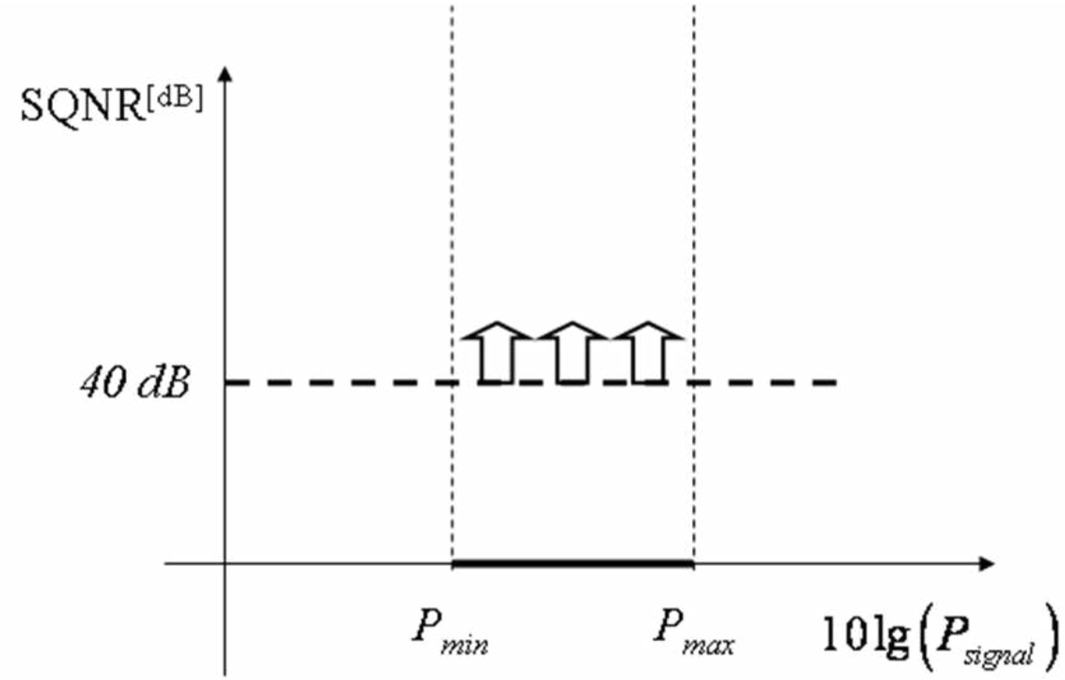
Problems for uniform quantization

Given a random signal the samples of which follow the p.d.f. indicated bellow. What is the quantization signal-to-noise ratio if we use an n=5 bit quantizer? (The quantizer is matched to the amplitude C.) What happens if the system is overdriven, what is its impact in the signal-to-noise ratio ?



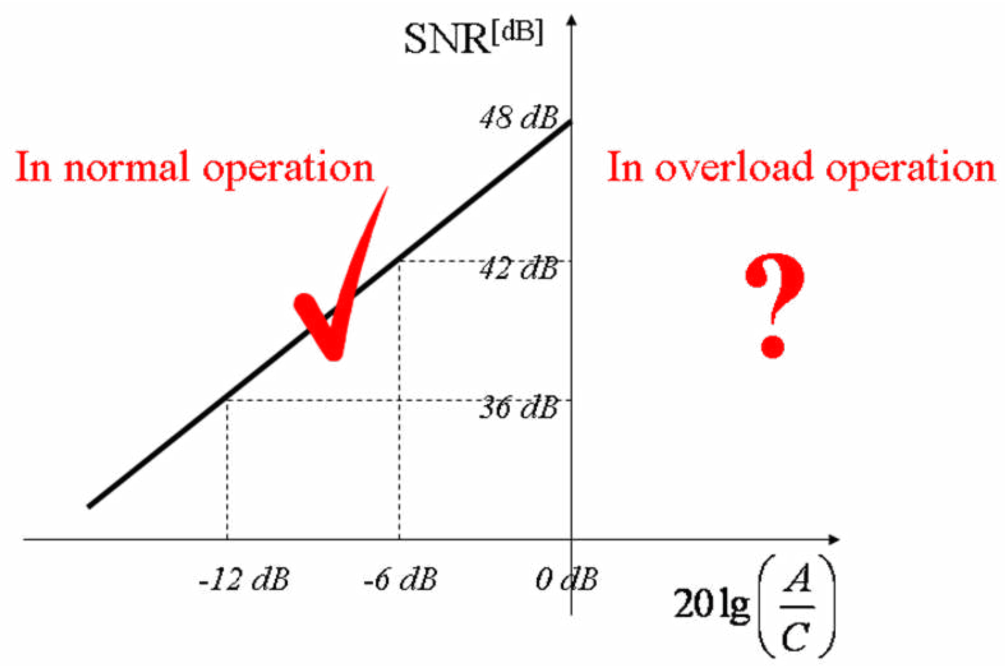
Problems for uniform quantization

How many bit is required for the quantizer to achieve at least 40 dB signal-to-noise ratio over 40 dB dynamics?



Problems for uniform quantization

What is the SQNR of an n=8 bit quantizer in the case 10dB overdrive ? (Under the assumption that the input signal follows uniform distribution.)



Oversampling SQNR

The relation of SQNR in the case of sine wave is valid only if the noise is determined in bandwidth $f_s/2$. If the signal bandwidth B is less than $f_s/2$ then the expression should be corrected to the form

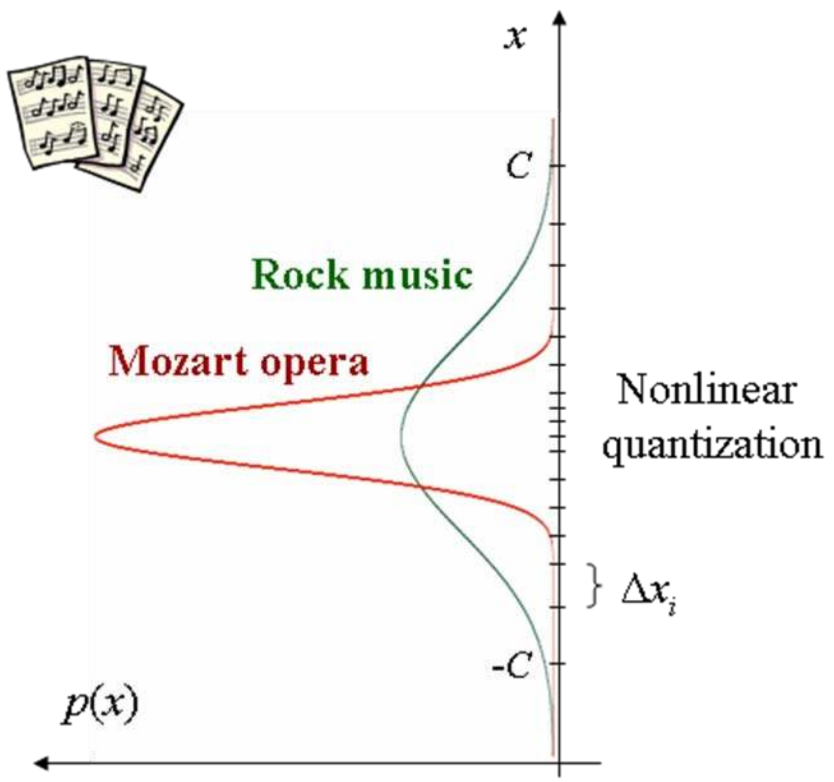
$$SQNR^{[dB]} := 6.02n + 1.78 + 10\log\left(f_s / 2B\right)$$

This expression reflects the effect of noise reduction due to oversampling – for given signal bandwidth doubling of sampling frequency increases the SQNR ratio by 3dB.

Non-uniform quantization

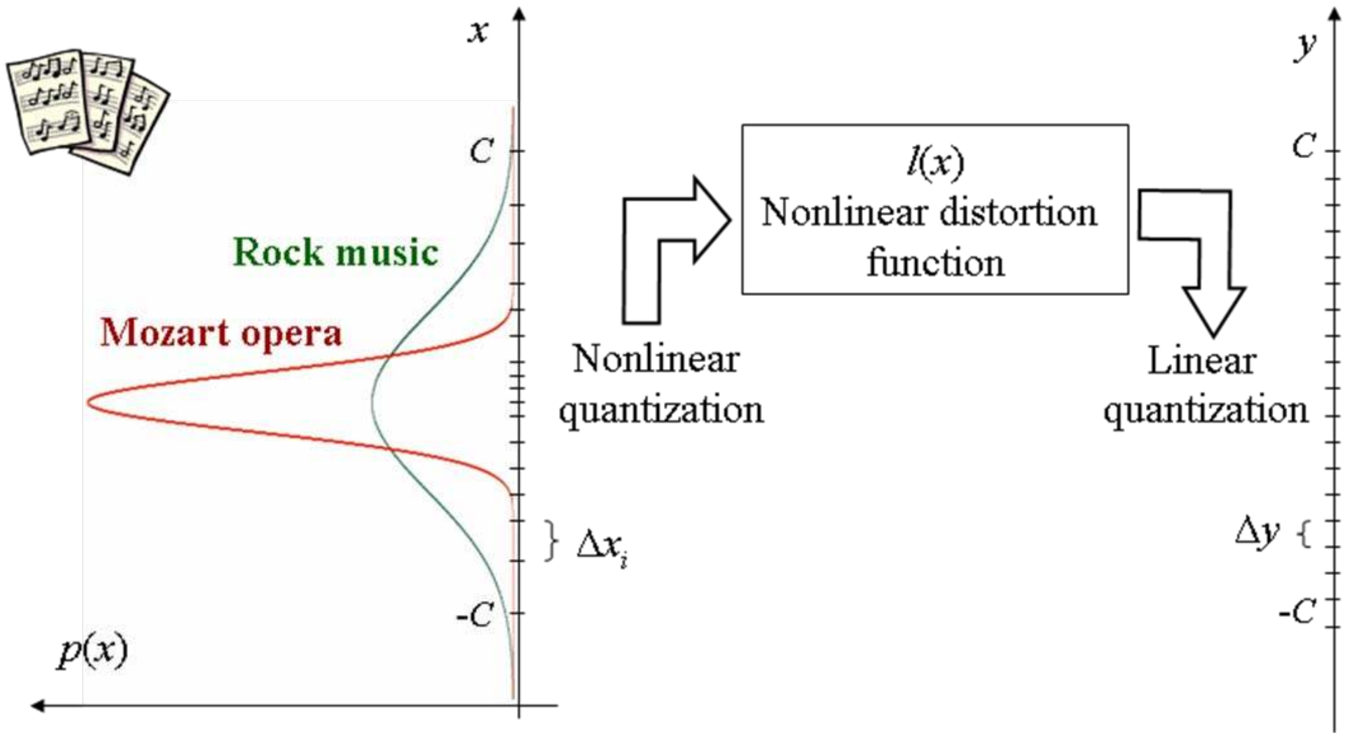
- Uniform quantization suffer from one bottleneck: if the sample to be quantized does not exploit the full range of quantization (i.e. $[-C,C]$ the interval) then SNR can deteriorate severely. As result a user having smaller dynamic range suffers a drop in Quality of Service (QoS).
- Non-uniform quantization is way to compensate this effect: smaller dynamic range there are plenty of quantization levels (to help the users with smaller dynamics) whereas in the case of large dynamic signal there are less quantization levels

Non-uniform quantization (cont')



Probability density function of samples in the case of small and large dynamics

Non-uniform quantization (cont')



The implementation of nonlinear quantization can be reduced to applying an equidistant quantizer preceded by a proper nonlinear distortion function $l(x)$.

SQNR of the non-uniform quantization

- The average noise in an elementary interval: $E\left\{\varepsilon^2 \middle| x \in \Delta x_i\right\} = \dfrac{\Delta x_i^{\ 2}}{12}$
- The average noise:

$$E\left\{\varepsilon^2\right\} \approx \sum_i E\left\{\varepsilon^2 \middle| x \in \Delta x_i\right\} P\left(x \in \Delta x_i\right) \approx \sum_i E\left\{\varepsilon^2 \middle| x \in \Delta x_i\right\} p\left(x_i\right)\Delta x_i$$

  – However, $l'\left(x_i\right) \approx \dfrac{\Delta y}{\Delta x_i}$ thus $\Delta x_i \approx \dfrac{\Delta y}{l'\left(x_i\right)}$

$$E\left\{\varepsilon^2\right\} \approx \sum_i \frac{\Delta x_i^2}{12} p_x\left(x_i\right)\Delta x_i = \frac{1}{12}\sum_i \frac{\Delta y^2}{l'\left(x_i\right)^2} p_x\left(x_i\right)\Delta x_i = \frac{4C^2}{12N^2}\sum_i \frac{1}{l'\left(x_i\right)^2} p_x\left(x_i\right)\Delta x_i \approx$$

$$\approx \frac{4C^2}{12N^2}\int_{-C}^{C} \frac{1}{l'^2(x)} p(x)dx$$

- Therefore, the SQNR is: $SQNR = \dfrac{E\left\{x^2\right\}}{E\left\{\varepsilon^2\right\}} = const \cdot \dfrac{\int_{-C}^{C} u^2 p_x\left(u\right)du}{\int_{-C}^{C} \dfrac{1}{l'\left(x\right)^2} p_x\left(x\right)dx}$

The optimal non-uniform quantization

- The optimal characteristics $l(x)$ can be found by solving the following problem:

$$l_{\text{opt}}(x) : \max_{l(x)} \frac{\int_{-C}^{C} u^2 p_x(u)\,du}{\int_{-C}^{C} \frac{1}{l'(x)^2} p_x(x)\,dx}$$

- This optimization is a hard problem itself (solved in the domain of functional analysis), but it is made more difficult by the fact that real life processes are non stationer (the sample p.d.f. $p(x)$ is changing in time) and as result this problem must be solved again and again in order to adopt to the changing nature of the process.

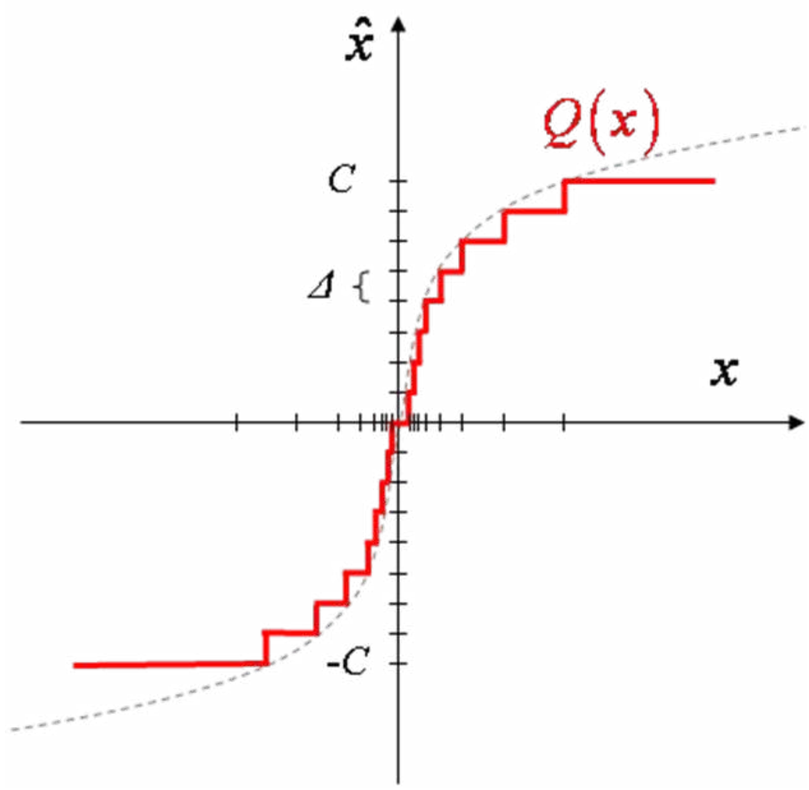
The logarithmic quantization

- To circumvent the difficulties of optimization, we are satisfied by choosing an $l_{\text{opt}}(x)$ subject to a modified objective function which guarantees uniform SQRN:

$$l_{\text{opt}}(x) : \max_{l(x)} \frac{\int_{-C}^{C} u^2 p_x(u)\,du}{\int_{-C}^{C} \frac{1}{l'(x)^2} p_x(x)\,dx} = const.$$

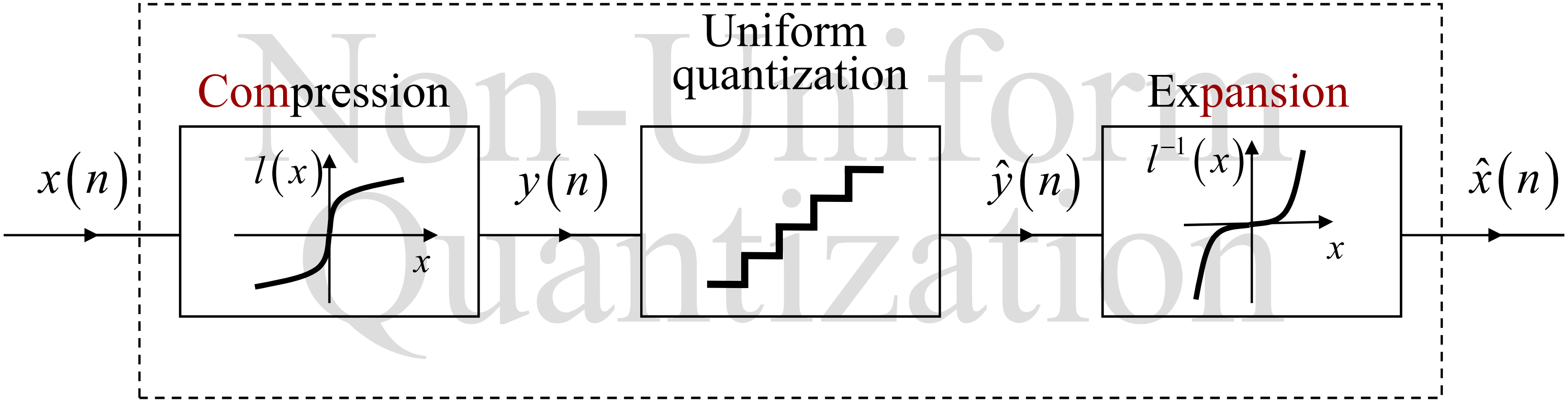
- One can easily see that if $x^2 \sim 1 / l'(x)^2$, then indeed the SNR is constant and independent of $p_x(u)$. Thus $l'(x) \sim 1 / x$, from which $l(x) \sim \log(x)$, which entails logarithmic quantization.

The logarithmic quantization (cont')



Characteristics of logarithmic quantizer

The logarithmic quantization (cont')



The real compressor $l(x)$ is chosen differently in Europe ("A-law") or in the US and Far East ("μ-law").

The logarithmic quantization

- The „A-law”:

$$l(x)=\begin{cases} \dfrac{A|x|}{1+\ln(A)}\operatorname{sgn}(x) & ha \quad 0\le \dfrac{|x|}{x_{\max}}\le \dfrac{1}{A} \\ x_{\max}\dfrac{1+\ln\left(A\dfrac{|x|}{x_{\max}}\right)}{1+\ln(A)}\operatorname{sgn}(x) & ha \quad \dfrac{1}{A}\le \dfrac{|x|}{x_{\max}}\le 1 \end{cases}$$

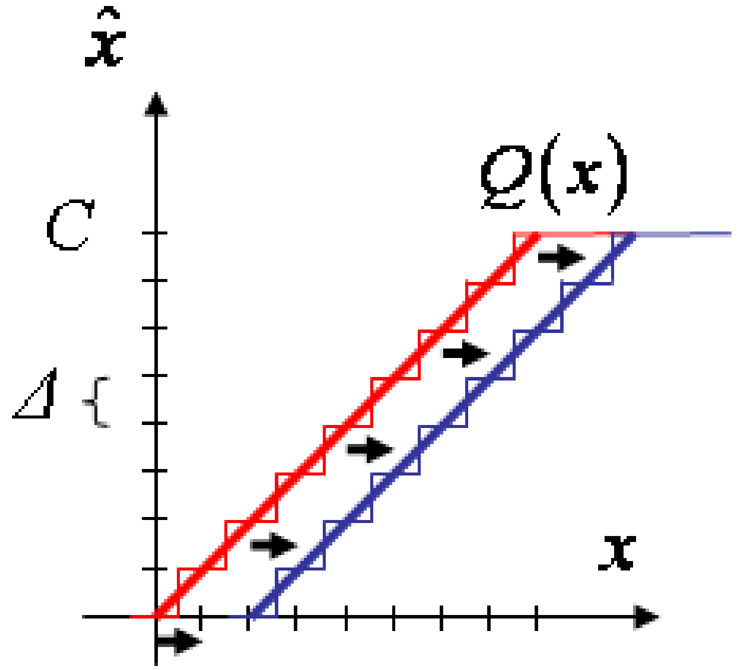
where A=87.56

- The „μ-law”:

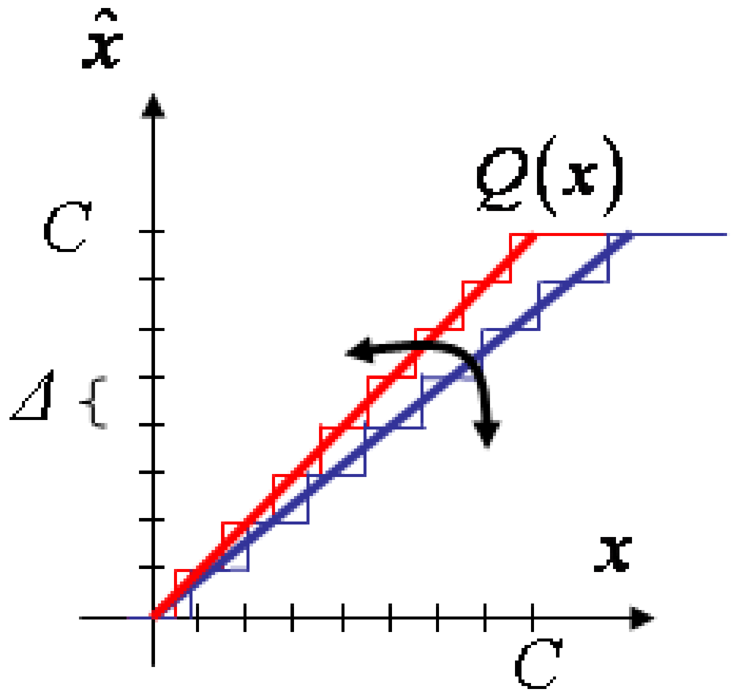
$$l(x)=x_{\max}\frac{\ln\left(1+\mu\dfrac{|x|}{x_{\max}}\right)}{\ln(1+\mu)}\operatorname{sgn}(x)$$

where μ=255

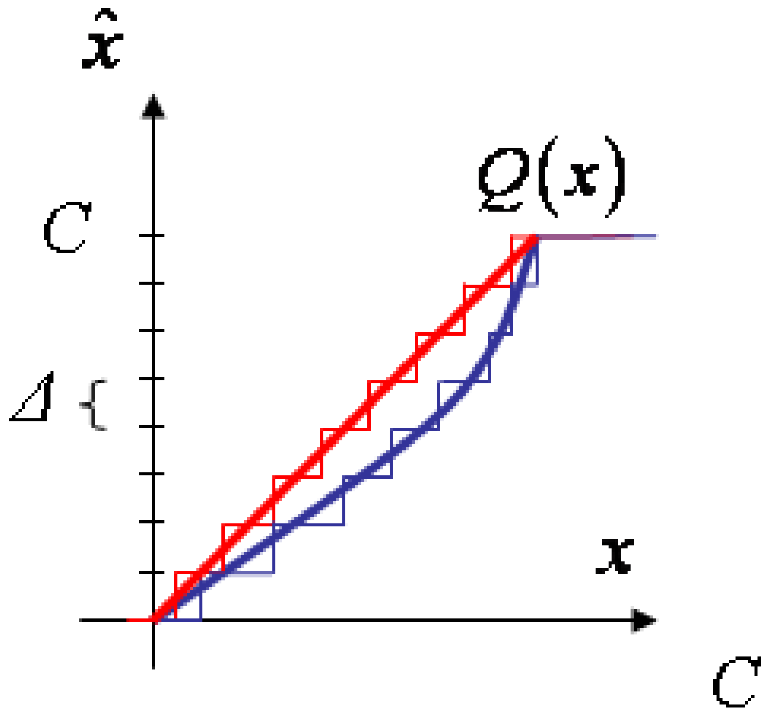
Quantization errors: zero drift



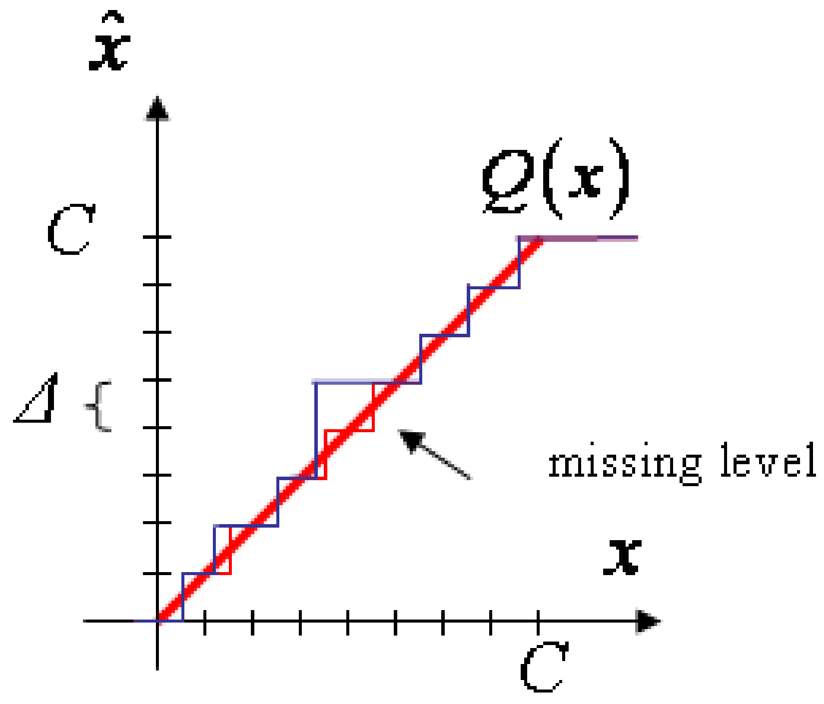
Quantization errors: gain error



Quantization errors: integral nonlinearity

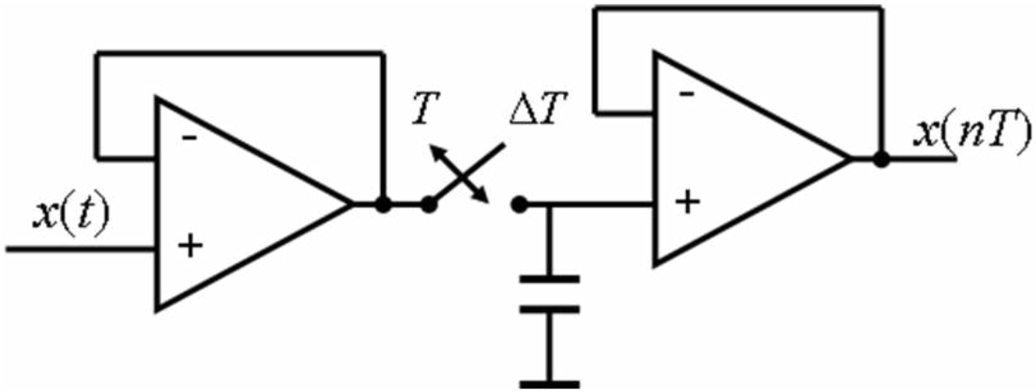


Quantization errors: differential nonlinearity



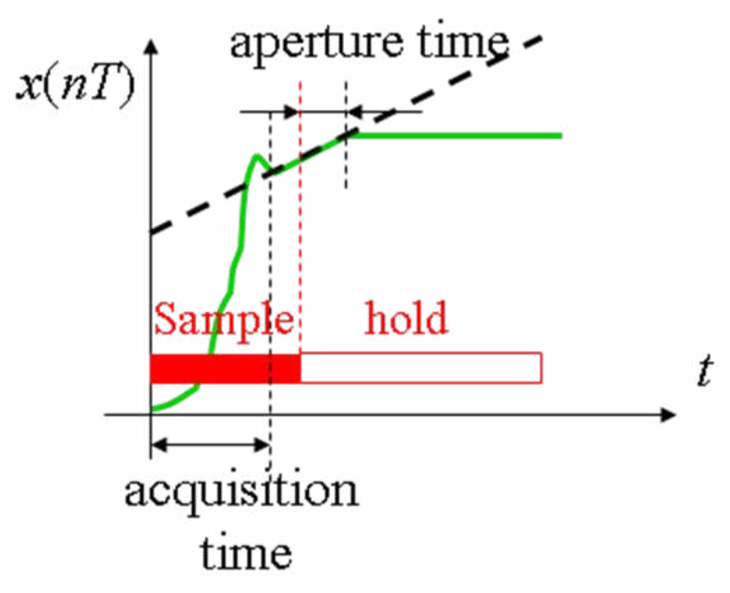
Sample and hold circuit

Although modern analogue-to-digital converters are very fast they need certain time to perform sampling and quantization process. Therefore, the AD converters are usually preceded by a special circuit holding the processed signal for the time necessary for the conversion. These circuits are called SH – sample-and-hold circuits.



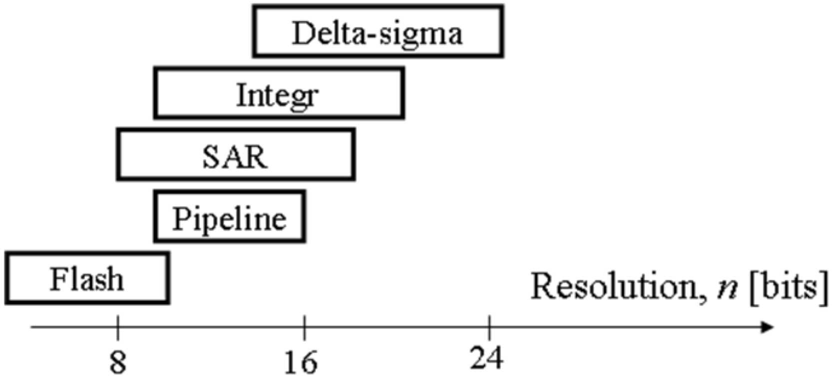
Sample and hold circuit (cont')

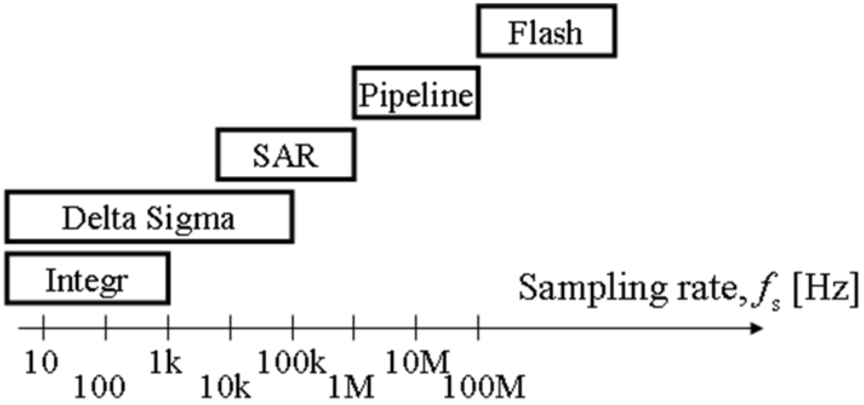
The typical times of sampling are of about 1 μs and the aperture time1 is not larger than several ps. There are also very fast sample-and-hold circuits with sampling time of about 10 ns and aperture time less than 1 ps.



AD converters and main performances

Many various AD converters have been designed and developed. However, currently on the market there are only a few main types of them: successive approximations register SAR, pipeline, delta-sigma, flash and integrating converters.



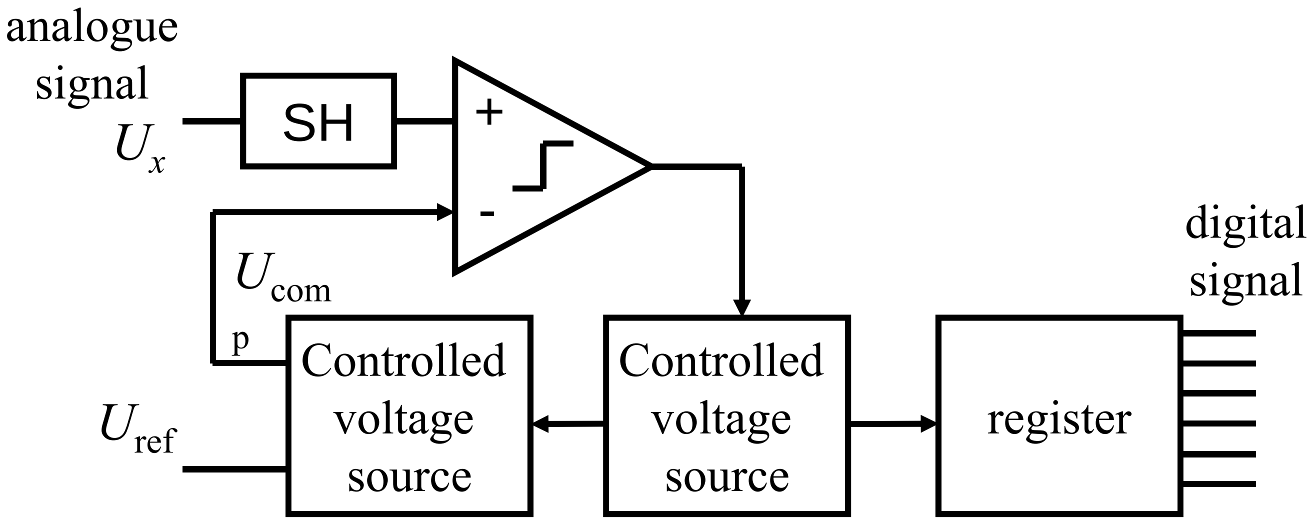


AD converters and main performances (cont')

- We can see that there is no one universal AD converter – the converters of high speed are of the poor resolution and vice versa – accurate (large number of bits) converters are rather slow.
- The most commonly used are the **SAR** (Successive Approximation Register) and Delta-Sigma converters. SAR converters are very accurate, operate with relatively high accuracy (16-bit) and wide range of speed – up to 1 MSPS.
- The **Delta-Sigma** converters (16-bit and 24-bit) are used when high accuracy and resolution are required. Recently, these converters are still in significant progress.

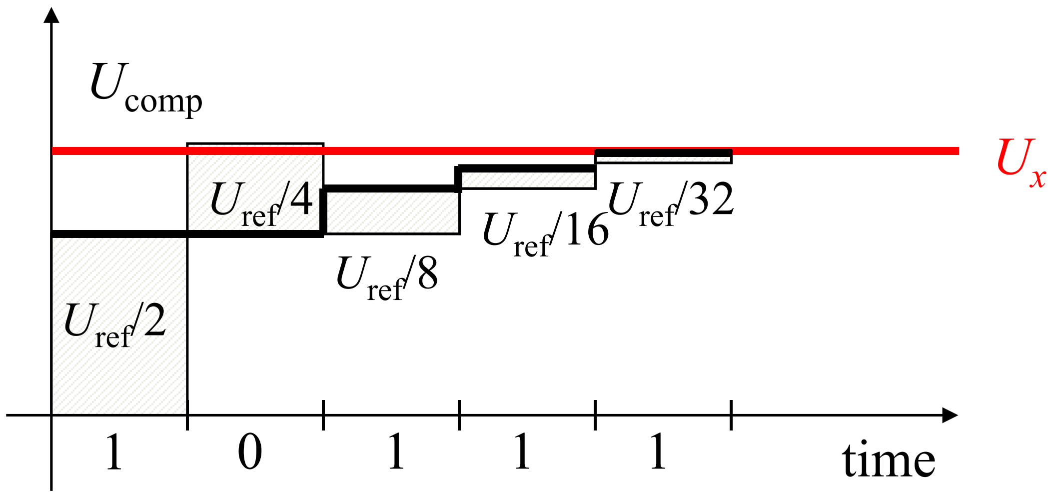
Successive Approximation Register (SAR)

The principle of operation of the SAR device resembles the weighting on the beam scale. Successively the standard voltages in sequence: $U_{ref}/2$, $U_{ref}/4$, $U_{ref}/8$... $U_{ref}/2^n$ are connected to the comparator. These voltages are compared with converted U_x voltage.



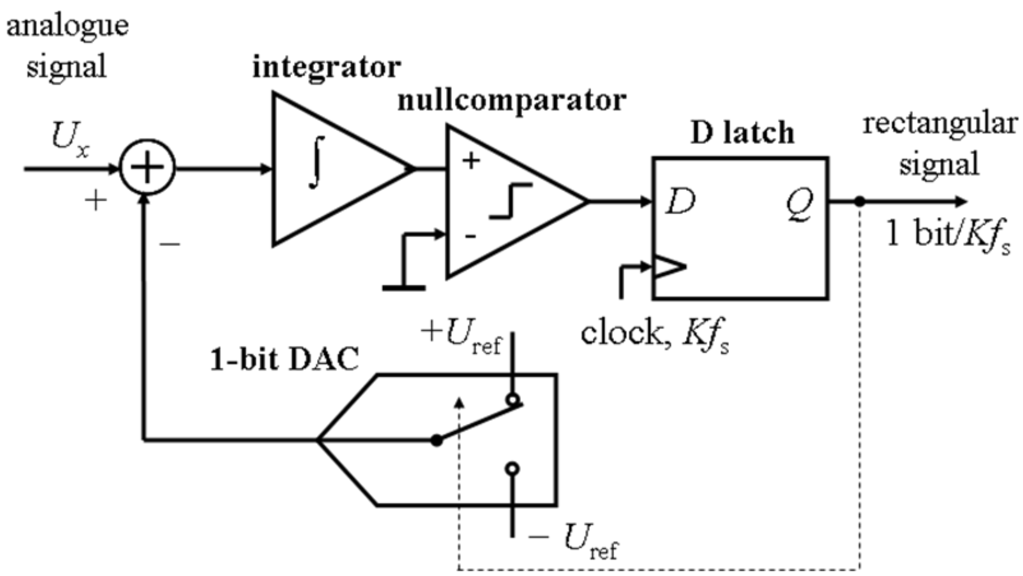
SAR (cont’)

If the connected standard voltage is smaller than the converted voltage in the register this increment is accepted and the register sends to the output 1 signal. If the connected standard voltage exceeds the converted voltage the increment is not accepted and register sends to the output 0 signal.



Delta-Sigma AD converters

The delta-sigma converters utilize the oversampling technique. Due to many advantages (most of all the best resolution – even up to 24-bit) these converters are currently very intensively developed. The principle of operation of such converters is presented in following figure:

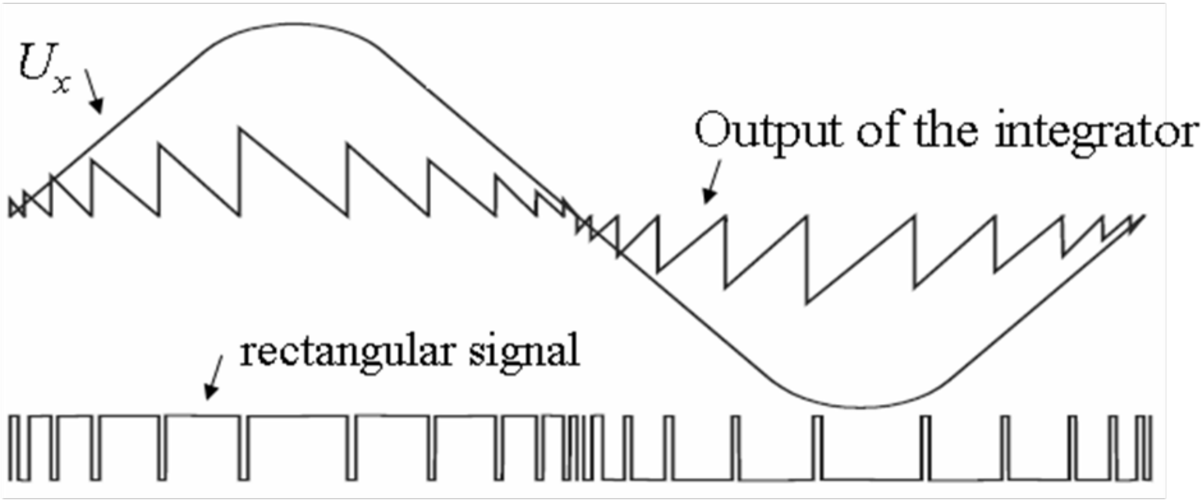


Delta-Sigma AD converters (cont')

In delta-sigma conversion the delta modulation is used (hence the name of this device). In delta modulation the width of the impulse is proportional to the value of converted signal. As the 1-bit ADC quantizer operates the comparator and latch switched with the frequency Kf_s forced by the clock (K is the oversampling factor). The output voltage is converted again to analogue form by 1-bit DAC. The adder in the input compares the input value and the output signal. Due to feedback the average value of output signal should be equal to the value of the input signal. If the input signal increases the integrating circuit need more time to obtain the zero value, the width of the impulse decreases and the average value of the output signal increases.

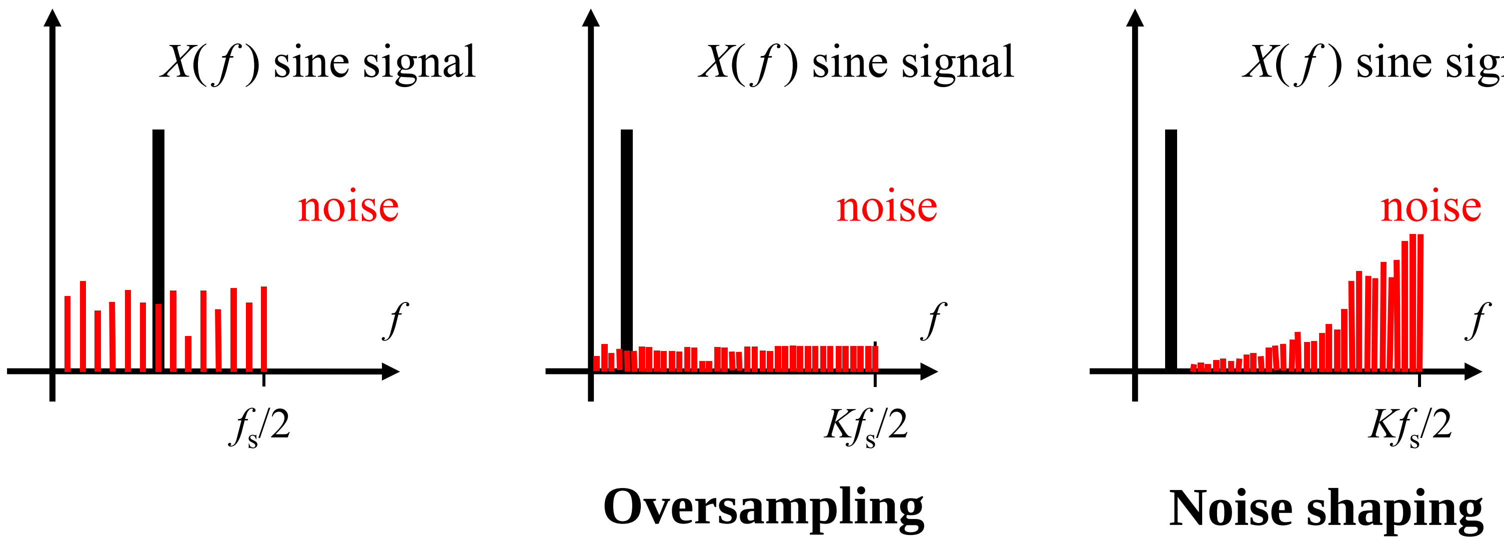
Delta-Sigma AD converters (cont')

The integrator and output signal of the delta-sigma converter as the dependence of the sine input signal.

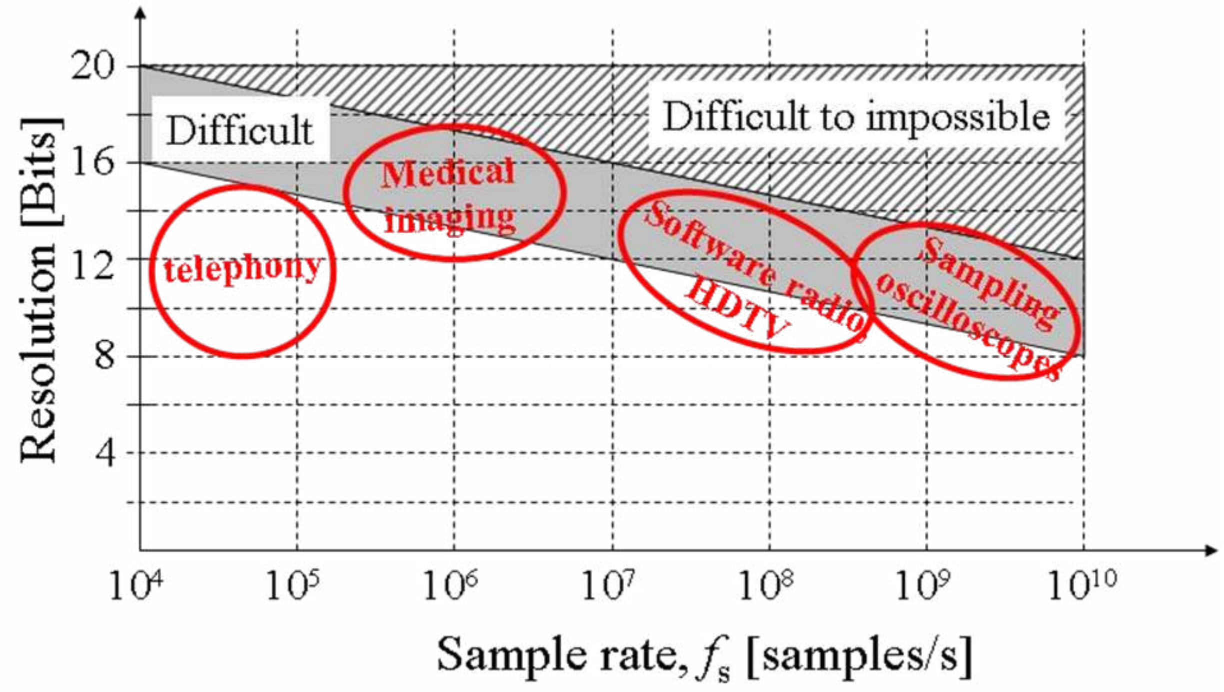


Delta-Sigma AD converters (cont')

The important advantage of the delta-sigma converter is the noise suppression. To obtain a noise suppression of about 40 dB it is necessary to apply a oversampling factor equal to 64.



Performance trade-offs of ADC



In the realization of the ADC converters improving the sample rate and the resolution at the same time are conflicting requirements.

Available ADC on the market

Part	Type	Bits	Sampling rate	Manufacturer	Price, $
ADC180	Integration	26	2048ms	Thaler	210
ADS1256	Delta-sigma	24	300kHz	Texas	9
AD7714	Delta-sigma	24	1kHz	AD	9
AD1556	Delta-sigma	24	16kHz	AD	27
MAX132	Integration	18	63ms	Maxim	8
AD7678	SAR	18	100kHz	AD	27
ADS8412	SAR	16	2MHz	AD	23
MAX1200	Pipeline	15	1MHz	Maxim	20
AD9480	pipeline	8	500MHz	AD	200
MAX105	Flash	6	800MHz	Maxim	36

Characteristics of ADC per application

Application	Architecture	Resolution	Sampling rate
Audio	SAR Delta-sigma	10-16 bits 14-18 bits	85-500 kHz 48-50kHz
Medical	SAR Delta-sigma	8-16 bits 16 bits	50-500 kHz 192 kHz
Automatic control	SAR Delta-sigma	8-16 bits 16 bits	40-500 kHz 250Hz
Wireless comm.	SAR Delta-sigma	8 bits 13 bits	270kHz

Summary

- Fundamental issues: representation capabilities, learning, generalization.
- Collection of algorithms to solve highly complex problems in real-time (in the field of IT) by using classical methods and novel computational paradigms routed in biology.
- ADC has three main steps: sampling, quantization and coding.
- The quantitative measure of the range over which the spectrum is concentrated is called the bandwidth of signal.
- If a band limited signal is sampled with sampling frequency $f_s \geq 2B$ then it can be uniquely reconstructed form its samples.
- Quantization is concerned to mapping sampled signal into rounded signal which may have only a finite number of values.
- In the realization of the ADC converters improving the sample rate and the resolution at the same time are conflicting requirements.

Next lecture: *Description digital signals and systems in time domain.*