$$H(z) = \frac{B(z)}{A(z)} = \sum_k c_k \frac{z}{z - z_k} \xrightarrow{Z^{-1}} \sum_k c_k z_k^n u(n)$$

pl $$H(z) = \frac{1}{1 - 1{,}5 z^{-1} + 0{,}5 z^{-2}} = \frac{z^2}{z^2 - 1{,}5z + 0{,}5} = z \frac{z}{z^2 - 1{,}5z + 0{,}5} = \text{(szorzás)}$$

$$z \cdot \frac{z}{(z-1)(z-0{,}5)}$$

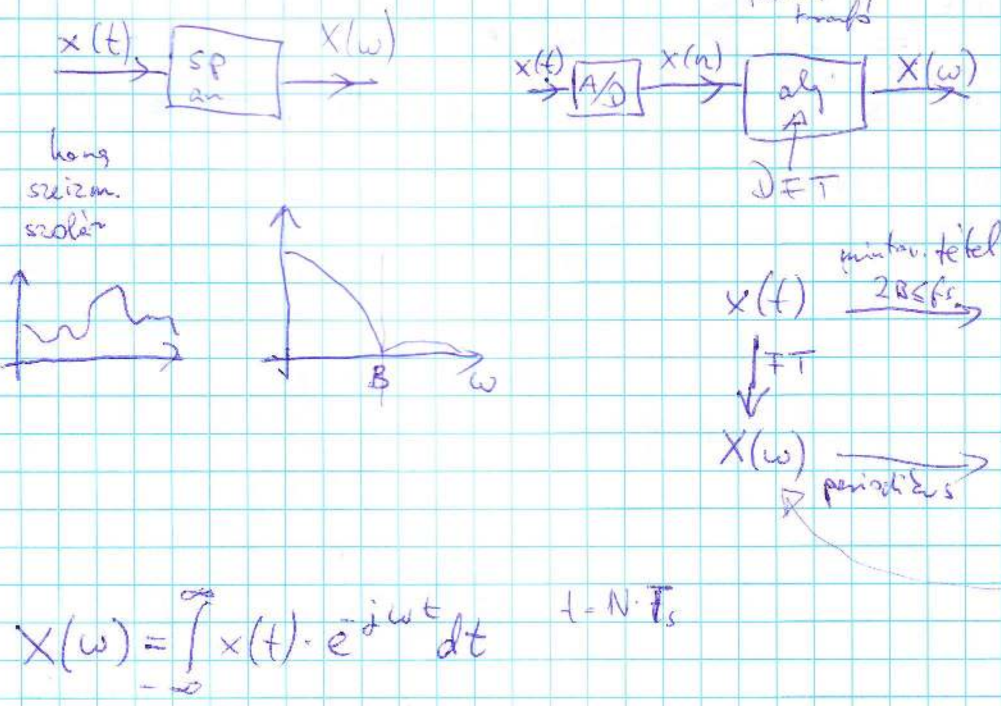
$$z_{1,2} = \frac{1{,}5 \pm \sqrt{2{,}25 - 2}}{2} = \frac{1{,}5 \pm 0{,}5}{2} \begin{cases} 1 \\ +0{,}5 \end{cases}$$

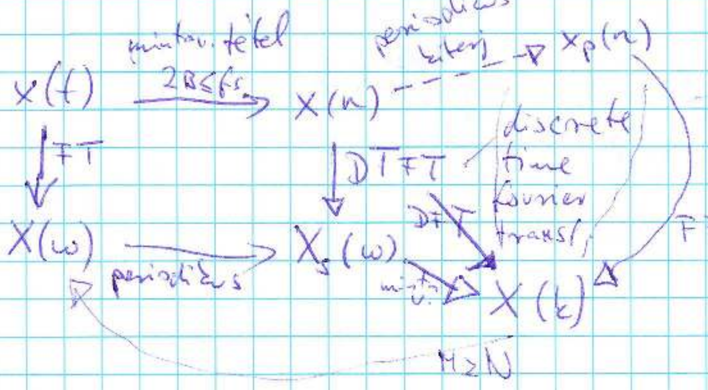
$$= z \left\{ c_1 \frac{1}{z-1} + c_2 \frac{1}{z - 0{,}5} \right\} = z \cdot \left\{ \frac{2}{z - 1} - \frac{1}{z - 0{,}5} \right\}$$

$$H(z) = 2 \frac{z}{z-1} - 1 \frac{z}{z - 0{,}5} \xrightarrow{Z^{-1}} 2 u(n) - 1 \cdot (0{,}5^n) u(n) \quad !!$$

2013.04.26 **Spektrálanalízis (DFT / FF)**

DFT: diszkrét fourier transzf.
FF: fast fourier

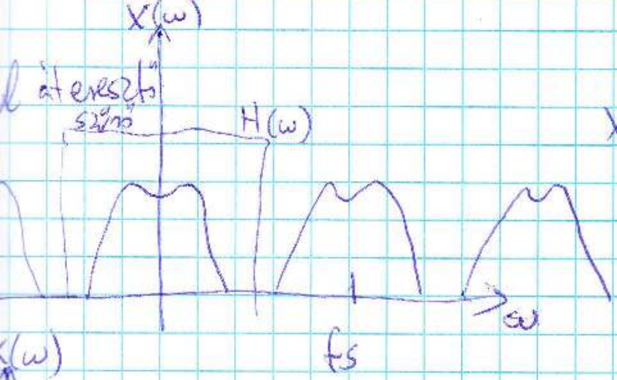




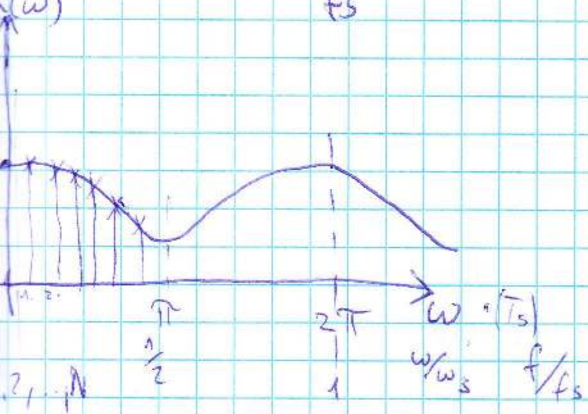
$$X(\omega) = \int_{-\infty}^{\infty} x(t) \cdot e^{-j\omega t} dt \qquad t = N \cdot T_s$$

$$\text{DTFT}: X_s(\omega) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n}$$

$$X_s(\omega) = \sum_{l=-\infty}^{\infty} X(\omega - 2\pi f_s \cdot l)$$



$$X(\omega) = H(\omega)\cdot X_s(\omega)$$



$$X(k) = X_s(\omega)\Big|_{\omega=\frac{2\pi}{N}\ell} = \sum_{n=-\infty}^{\infty} x(n)\cdot e^{-j\frac{2\pi}{N}k\cdot n}$$

$$\ldots + \sum_{n=-N}^{-1} x(n)\cdot e^{-j\frac{2\pi}{N}k\cdot n} + \sum_{n=0}^{N-1} x(n)\cdot e^{-j\frac{2\pi}{N}n\cdot k} + \sum_{n=N}^{2N-1} x(n)\cdot e^{-j\frac{2\pi}{N}n\cdot k} + \ldots =$$

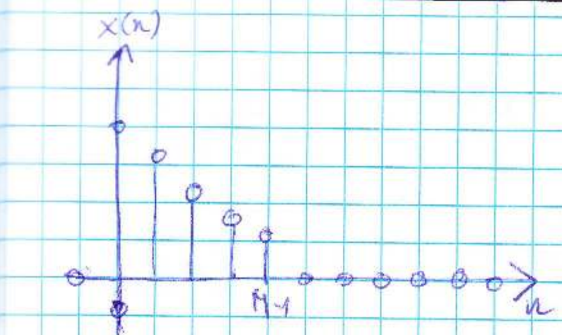
$$= \sum_{\ell=-\infty}^{\infty}\ \sum_{n=\ell\cdot N}^{\ell N+N-1} x(n)\cdot e^{-j\frac{2\pi}{N}\cdot n\cdot k} = \sum_{n=0}^{N-1} \underbrace{\sum_{\ell=-\infty}^{\infty} x(n-\ell\cdot N)}_{x_p(n)} \cdot e^{-j\frac{2\pi}{N}(n-\ell N)k} \Rightarrow$$

① $\Sigma \leftrightarrow \Sigma$

② $n = n - \ell\cdot N$

$$e^{-j\frac{2\pi}{N}\cdot n\cdot k}\cdot \underbrace{e^{j\frac{2\pi}{N}\cdot\ell\cdot k}}_{1}$$

$$= \sum_{n=0}^{N-1} x_p(n)\cdot e^{-j\frac{2\pi}{N}\cdot n\cdot k}$$



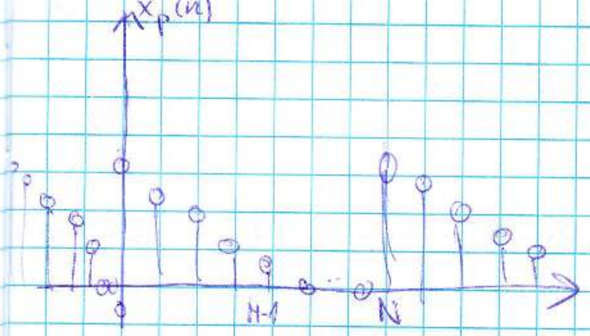
Fourier-sorba fejtés.

$x_p(n)$ periodikus $\Rightarrow$ FS

$$x_p(n)=\sum_{k=0}^{N-1} c_k \cdot e^{j\frac{2\pi}{N}k\cdot n}$$

ahol $c_k=\frac{1}{N}\sum_{n=0}^{N-1} x_p(n)\, e^{-j\frac{2\pi}{N}k\cdot n}$

$$c_k=\frac{1}{N}X(k)$$



$M<N$

$$X(\omega)=\sum_{n=-\infty}^{\infty} x(n)\cdot e^{-j\omega n} = \sum_{n=0}^{N-1} x_p(n)\cdot e^{-j\omega n} =$$

ha $\boxed{M\leq N}$ $x(n)=x_p(n)$

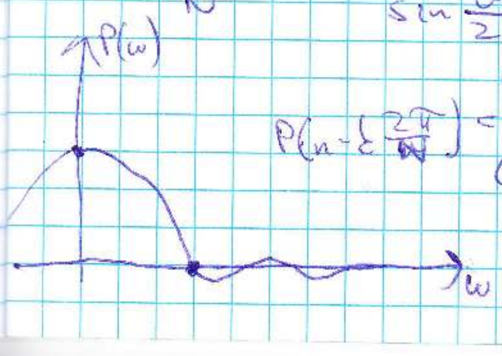
eredeti spektrum!

$$=\sum_{n=0}^{N-1}\sum_{k=0}^{N-1}\frac{1}{N}X(k)\cdot e^{j\frac{2\pi}{N}n\cdot k}\cdot e^{-j\omega n}=\sum_{k=0}^{N-1}X(k)\cdot\sum_{n=0}^{N-1}\frac{1}{N}\cdot e^{j\frac{2\pi}{N}n\cdot k}\cdot e^{-j\omega n}=$$

$$=\sum_{k=0}^{N-1}X(k)\sum_{n=0}^{N-1}\frac{e^{-jn\left(\frac{2\pi}{N}k+\omega\right)}}{N}=\sum_{k=0}^{N-1}\underbrace{X(k)\cdot P\left(\omega-\frac{2\pi}{N}k\right)}_{\text{interpolációs fv}}, \text{ ahol}$$

$$P(\omega)=\frac{1}{N}\sum_{n=0}^{N-1}e^{-jn(\omega)}=\frac{1}{N}\,\frac{1-e^{-jN\omega}}{1-e^{-j\omega}}=\frac{1}{N}\,\frac{e^{-j\frac{N}{2}\omega}}{e^{-j\frac{\omega}{2}}}\,\frac{\frac{e^{j\frac{N}{2}\omega}-e^{-j\frac{N}{2}\omega}}{2j}}{\frac{e^{j\frac{\omega}{2}}-e^{-j\frac{\omega}{2}}}{2j}}=$$

$$=\frac{e^{-j\omega\left(\frac{N}{2}+\frac{1}{2}\right)}}{N}\,\frac{\sin\left(\frac{N}{2}\omega\right)}{\sin\frac{\omega}{2}}$$



$$P\left(n-k\frac{2\pi}{N}\right)=\begin{cases}1 & k=0\\ 0 & k=1\end{cases}$$

DFT: $X(k)=\sum_{n=0}^{N-1} x(n)\cdot e^{-j\frac{2\pi}{N}\cdot n\cdot k}$ $\quad k=0,1,\dots,N-1$

$$W_N = e^{-j\frac{2\pi}{N}}$$

$$X(k)=\sum_{n=0}^{N-1} x(n)\cdot W_N^{n\cdot k}$$

Inverz: $\boxed{\text{IDFT:} \quad x(n)=\frac{1}{N}\sum_{k=0}^{N-1} X(k)\, W_N^{-nk}}$ $\quad n=0,1,\dots,N-1$

$$\begin{bmatrix} X(0) \\ X(1) \\ \vdots \\ X(N-1) \end{bmatrix} = \begin{array}{c|c} & 0 \quad \xrightarrow{k} \quad N-1 \\ \hline \begin{matrix} 0 \\ \\ n \downarrow \\ \\ N-1 \end{matrix} & \begin{bmatrix} W_N^0 & W_N^0 & \dots & W_N^0 \\ W_N^0 & W_N^1 & W_N^2 & W_N^{N-1} \\ \vdots & W_N^2 & W_N^4 & W_N^{(N-1)\cdot 2} \\ \vdots & & & \vdots \\ W_N^0 & W_N^{N-1} & \dots & W_N^{(N-1)^2} \end{bmatrix} \end{array} \begin{bmatrix} x(0) \\ x(1) \\ \vdots \\ x(N-1) \end{bmatrix}$$

komplexitás: szorzás: N^2 $\rightarrow$ FF

összeadás: $N(N-1)$

N-pontos DFT mátrix

Vandermonde-típusú $\det(\underline{DFT}) \neq 0$ $\exists$ inv $\quad$ transzf^{-1} = eredeti

Tul-k

1. Vandermonde $\rightarrow \det(\underline{\underline{W}}) \neq 0 \rightarrow \underline{\underline{W}}^{-1} = \frac{1}{N}\cdot \underline{\underline{W}}^{*T}$

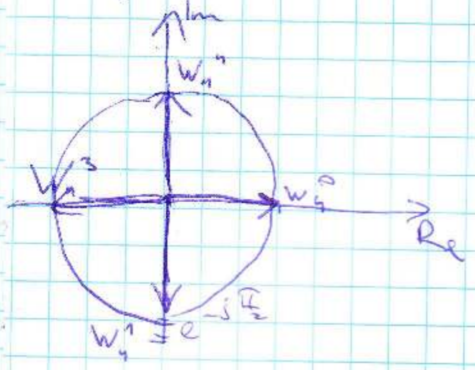
2. $W_N^{k+\frac{N}{2}} = -W_N^k$

3. $W_N^{k\cdot L} = W_{\frac{N}{L}}^k$

4. $W_N^{k+N} = W_N^k$

példa 4 pontos DFT $\underline{W} = ?$

$W_4 = e^{-j\frac{2\pi}{4}} = e^{-j\frac{\pi}{2}}$



$$\underline{W_4} = \begin{array}{c|c} & k: \; 0 \quad 1 \quad 2 \quad 3 \\ \hline \begin{matrix} n: 0 \\ 1 \\ 2 \\ 3 \end{matrix} & \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \end{array}$$

Alk:

→ konvolúció helyett

$$x(n) \longrightarrow \boxed{h(n)} \longrightarrow y(n) = \sum_{k=-\infty}^{\infty} h(k)\, x(n-k)$$

$$X(k) \qquad H(k) \qquad Y(k) = H(k) \cdot X(k)$$

→ spektrálanalízis, korr. analízis
(spektogram

→ DCT (mp3, jpeg)
diszkrét cos
transzf

→ OFDM (wifi)

→ ...

STFT
short time fourier transf.

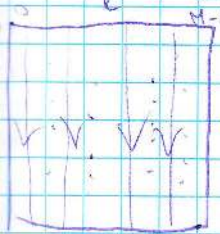
$$X(n,m) = \sum_{n=0}^{N-1} x(n) \underbrace{\boxed{N(n-m)}}_{\text{ablak-fv}} \cdot e^{-j\frac{2\pi}{N}kn}$$

Fast Fourier Transf.

'divide et impera'

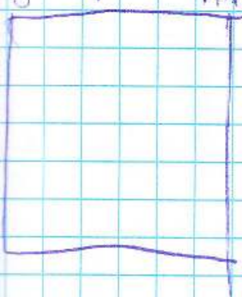
$$N = M \cdot L$$

l oszlopfolytonos



$$x(l,m) = \cancel{x(l\cdot M+m)} = x(m\cdot L+l)$$

sorfolytonos



$$X(p,q) = X(p\cdot M+q)$$

$$X(k) = \sum_{n=0}^{N-1} x(n)\,\underbrace{e^{-j\frac{2\pi}{N}n\cdot k}}_{W_N^{nk}}$$

$$X(p,q) = \sum_{l=0}^{L-1}\sum_{m=0}^{M-1} x(l,m)\cdot W_N^{(m\cdot L+l)(pM+q)} = $$

$$= \sum_{l=0}^{L-1}\sum_{m=0}^{M-1} x(l,m)\cdot \underbrace{W_N^{mpLM}}_{=1}\cdot \underbrace{W_N^{lpM}}_{=W_{N/M}^{lp}=W_L^{lp}}\cdot \underbrace{W_N^{qmL}}_{=W_{N/L}^{q\cdot m}=W_M^{qm}}\cdot W_N^{lq} =$$

$$= \sum_{l=0}^{L-1} \left\{ W_L^{lp} \left[\sum_{m=0}^{M-1} x(l,m) \cdot w_M^{qm} \right] \cdot W_N^{lq} \right\}$$

1, M pontos DFT : L db

2, $G(l,q) = w_L^{lp} F(l,q)$

3, M db L pontos DFT

$$X(p,q) = \sum_{l=0}^{L-1} G(l,q) \cdot w_N^{lq}$$

	szorzás	összeadás
1)	$L \cdot M^2$	$LM(M-1)$
2)	L	$\emptyset$
3)	$M \cdot L^2$	$ML(L+1)$
	$M^2 L + L + ML^2$	$ML(M+L-2)$
	N^2	$N(N-1)$

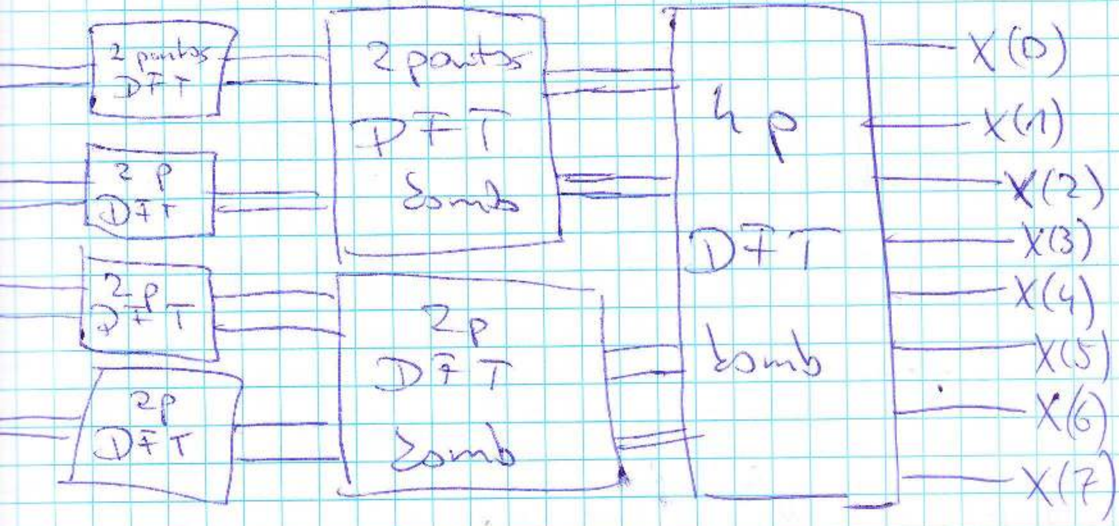
faktorizáció: $N = i_1 \cdot i_2 \cdot \ldots \cdot i_r$

→ Radix-2 $\quad N = 2^r$

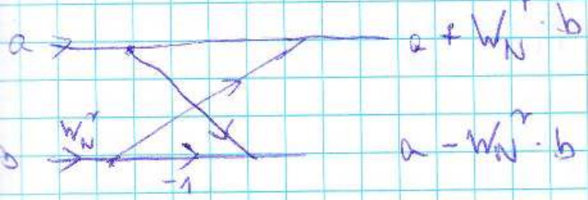
· Radix-4 $\quad N = 4^w$

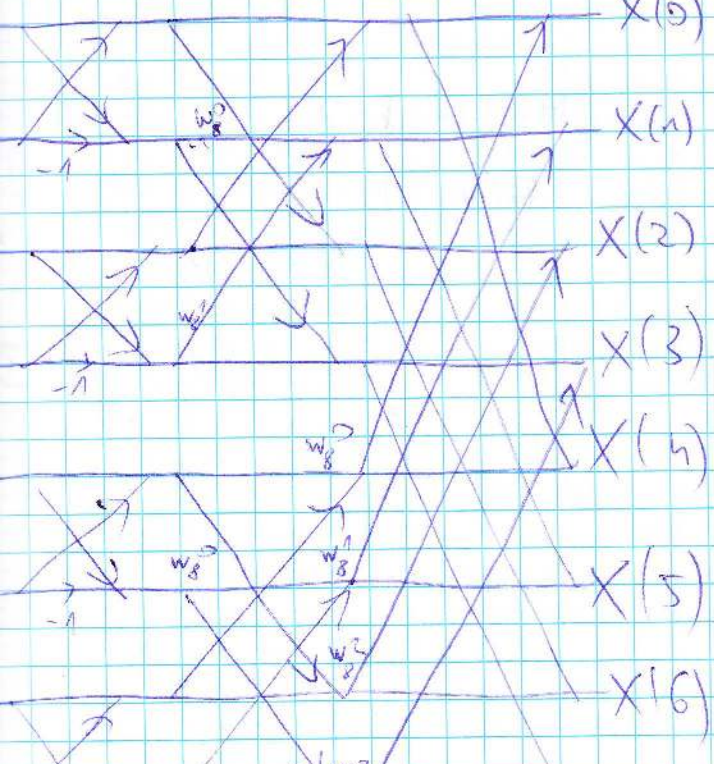
-2

$N = 8 = 2^3$


2 points DFT
2 p DFT
2 p DFT
2 p DFT
2 points DFT comb
2 p DFT comb
4 p DFT comb
X(0)
X(1)
X(2)
X(3)
X(4)
X(5)
X(6)
X(7)

p2e


a
b
W_N^r
-1
$a + W_N^r \cdot b$
$a - W_N^r \cdot b$


X(0)
X(1)
X(2)
X(3)
X(4)
X(5)
X(6)
X(7)
-1
W_8^0
W_8^2
W_8^1
W_8^3