9. ea   2011. 04. 15.

$$\mathcal{Z}\{x(n)\} = \sum_{n=-\infty}^{\infty} x(n) z^{-n}, \quad z \in ROC$$

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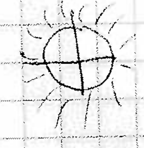
$\mathcal{Z}\{\delta(n)\} = 1$ ROC: z plane , $\mathcal{Z}\{u(n)\} = \frac{z}{z-1}$ ROC: $|z| > 1$



Z transzformáció tulajdonságai

1) lineáris $\mathcal{Z}\{c_1 x_1(n) + c_2 x_2(n)\} = c_1 \mathcal{Z}\{x_1(n)\}_{ROC} + c_2 \mathcal{Z}\{x_2(n)\}$

$$\sum_{n=-\infty}^{\infty} \left(c_1 x_1(n) + c_2 x_2(n)\right) z^{-n} = \sum_{n=-\infty}^{\infty} \left(c_1 x_1(n) z^{-n} + c_2 x_2(n) z^{-n}\right) =$$

$$= c_1 \sum x_1(n) z^{-n} + c_2 \sum x_2(n) z^{-n} = c_1 X_1(z) + c_2 X_2(z)$$

$\mathcal{Z}\{x(n)\} = X(z)$

2) Időeltolás $\mathcal{Z}\{x(n-k)\} = z^{-k} X(z)$; $\sum_{n=-\infty}^{\infty} x(n-k) z^{-n} = \sum_{m=-\infty}^{\infty} x(m) z^{-m} z^{-k}$

$m := n-k \Rightarrow n = m+k$

$$= z^{-k} \underbrace{\sum_{m=-\infty}^{\infty} x(m) z^{-m}}_{X(z)}$$

3) Skálázás : $\mathcal{Z}\{\alpha^n x(n)\} = X(\alpha^{-1} z)$; $\sum_{n=-\infty}^{\infty} \alpha^n x(n) z^{-n} = \sum_{n=-\infty}^{\infty} x(n) \underbrace{(\alpha^{-1} z)^{-n}}_{n}$

$= X(\alpha^{-1} z)$

4) Időbeli fordítás : $\mathcal{Z}\{x(-n)\} = \sum_{n=-\infty}^{\infty} x(-n) z^{-n} = \sum_{m=-\infty}^{\infty} x(m) z^{m} =$

$$= \sum_{m=-\infty}^{\infty} x(m) (z^{-1})^{-m} = X(z^{-1})$$

5) z szerinti deriválás: $\mathcal{Z}\{n\, x(n)\} = -z \frac{dX(z)}{dz}$

$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$; $\frac{dX(z)}{dz} = \sum_{n=-\infty}^{\infty} x(n)(-n) z^{-n-1} = -\left(\sum_{n=-\infty}^{\infty} n \cdot x(n) z^{-n}\right) z^{-1}$

$$\mathcal{Z}\{n\, u(n)\, \alpha^n\} = -z \frac{d}{dz}\left(\frac{z}{z-\alpha}\right) = -z \frac{z-\alpha-z}{(z-\alpha)^2} = \alpha \frac{z}{(z-\alpha)^2}$$

6) Konvolúció : $\mathcal{Z}\left\{\sum_{k=-\infty}^{\infty} x(k)\, g(n-k)\right\} = X(z)\, Y(z)$;

$$\sum_{n=-\infty}^{\infty} \left\{\sum_{k=-\infty}^{\infty} x(k)\, g(n-k)\right\} z^{-n} = \sum_{k=-\infty}^{\infty} x(k) \underbrace{\sum_{n=-\infty}^{\infty} g(n-k) z^{-n}}_{z^{-k} Y(z)} = \underbrace{\sum_{k=-\infty}^{\infty} x(k) z^{-k}}_{X(z)} Y(z)$$

Racionális törtfv alakú $X(z)$ tulajdonságai

$$X(z) = \frac{\sum_{j=0}^{M} b_j z^{-j}}{\sum_{i=0}^{N} a_i z^{-i}} = \cancel{z^{N-M}} \frac{z^M \left(\sum_{j=0}^{M} b_j z^{-j}\right)}{z^N \left(\sum_{i=0}^{N} a_i z^{-i}\right)} = z^{N-M} \frac{\sum_{j=0}^{M} b_j z^{M-j}}{\sum_{i=0}^{N} a_i z^{N-i}} =$$

$$= z^{N-M} \cdot c \cdot \frac{\prod_{j=1}^{M} (z - z_j)}{\prod_{i=1}^{N} (z - p_i)}$$

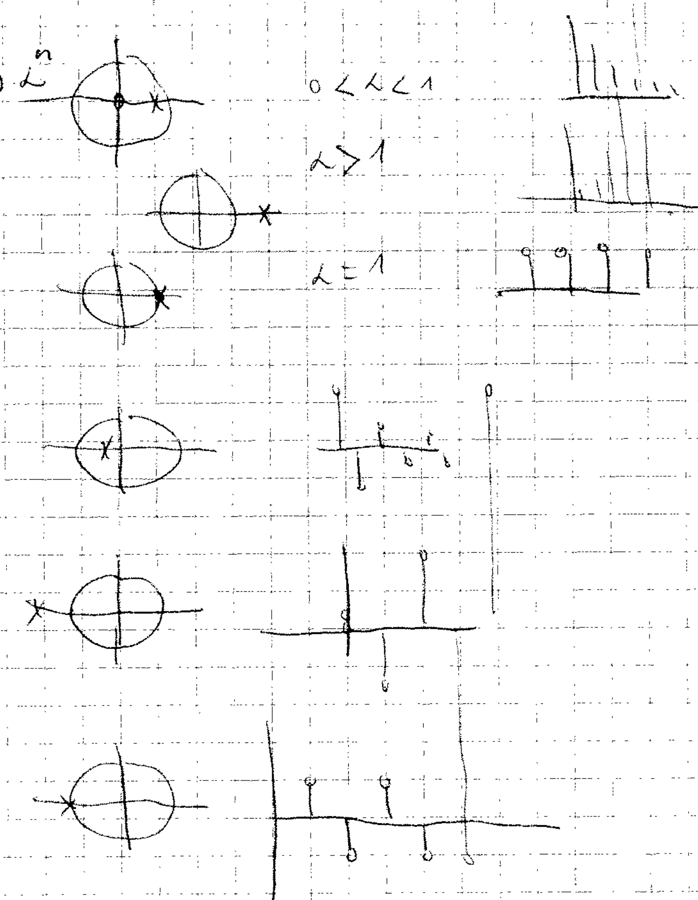
z_j, $j = 1, \dots, M$ => zérusok p_i, $i = 1, \dots, N$ => pólusok

pólus-zérus kép teljesen meghatározza a hálózatot. Ez alapján felírható.

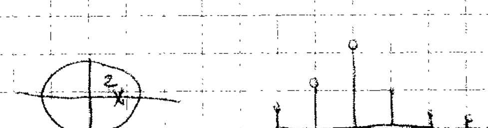
rac törtfv-nek $\forall$ ugyanannyi pólusa és zérusa van

Tfh $N < M$ ($M < N$) -> z^{N-M} tag megnöveli a zérusokat (pólusokat)

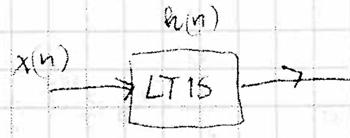
$X(z) = \frac{z}{z - \alpha} = u(n)\, \alpha^n$



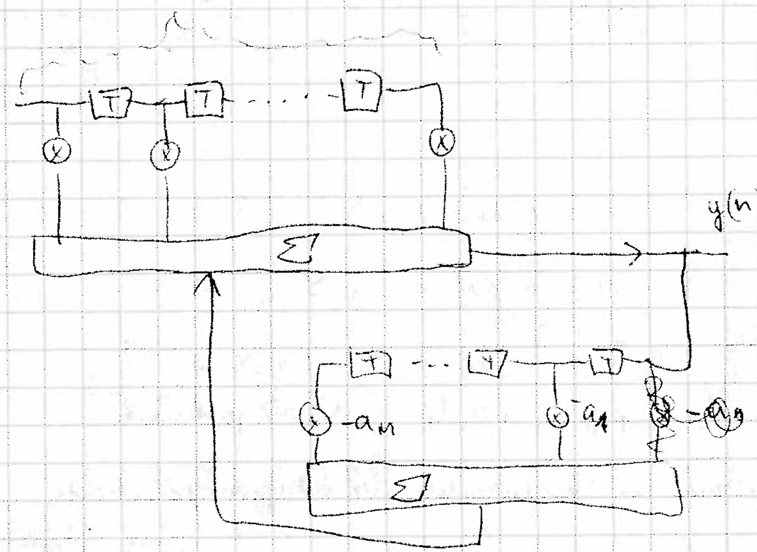
2 multiplicitás



lineárisan nő, majd exp. csökken



$$y(n) = \sum_{\ell=-\infty}^{\infty} h(n-\ell)\, x(\ell)$$



$$y(n) = \sum_{i=1}^{N} a_i\, y(n-i) + \sum_{j=0}^{M} b_j\, x(n-j)$$

$$Y(z) = H(z)\, X(z) \Rightarrow H(z) = \frac{Y(z)}{X(z)} \qquad H(z) = \sum_{n=-\infty}^{\infty} h(n)\, z^{-n}$$

$$\sum_{i=0}^{N} a_i\, y(n-i) = \sum_{j=0}^{M} b_j\, x(n-j) \underset{Z}{\Rightarrow} \sum_{i=0}^{N} a_i z^{-i}\, Y(z) = \left(\sum_{j=0}^{M} b_j z^{-j}\right) X(z)$$

Rendszeranalízis célja $H(z)$ megtalálása

$$\frac{Y(z)}{X(z)} = H(z) = \frac{\sum_{j=0}^{M} b_j z^{-j}}{\sum_{i=0}^{N} a_i z^{-i}} = z^{N-M} \cdot c \cdot \frac{\prod_{j=1}^{M} (z - z_j)}{\prod_{i=1}^{N} (z - p_i)}$$

$|p_i| < 1$

ez a karakterisztikus polinom
összes gyöke az egységkörön belül van

$$Z\{x(n)\} = X(z) = \sum_{n=-\infty}^{\infty} x(n)\, z^{-n}$$

$$Z^{-1}\{X(z)\} = x(n)$$

$$Z^{-1}\{X(z)\} = x(n) = \frac{1}{2\pi j} \oint_G X(z)\, z^{n-1}\, dz$$

Cauchy $$\frac{1}{2\pi j} \oint_G z^{n-1} = \delta_{n0} = \begin{cases} 1 & \text{ha } n = 0 \\ 0 & \text{ha } n \neq 0 \end{cases}$$

$$\frac{1}{2\pi j}\oint_G \Big(\underbrace{\sum_{l=-\infty}^{\infty} x(l)\, z^{-l}}_{\text{nem függ z-től kiemelhető}}\Big) z^{n-1}\, dz = \sum_{l=-\infty}^{\infty} x(l) \underbrace{\left(\frac{1}{2\pi j}\oint_G z^{n-l-1}\, dz\right)}_{\delta_{n,l} \; \{ 0 \text{ ha } n \neq l} =$$

$$= \sum_{l=-\infty}^{\infty} x(l)\, \delta_{n,l} = x(n)$$

$$Z^{-1}\left\{ \frac{\sum_{j=0}^{M} b_j z^{-j}}{\sum_{i=0}^{N} a_i z^{-i}} \right\}$$

parciális törtekre bontunk

$$Z^{-1}\left\{ \sum_i c_i \frac{z}{z-p_i} \right\} = \sum_i c_i\, u(n)\, p_i^{\,n}$$

$$c_i = X(z) \cdot \frac{(z-p_i)}{z}\bigg|_{z=p_i}$$

$$X(z) = \frac{1}{1-1{,}5z^{-1}+0{,}5z^{-2}} = \frac{z^2}{z^2-1{,}5z+0{,}5} \Rightarrow \frac{X(z)}{z} = \frac{z}{z^2-1{,}5z+0{,}5} = \frac{z}{(z-1)(z-0{,}5)}$$

$$p_{1,2} = \frac{1{,}5 \pm \sqrt{2{,}25-2}}{2} = \frac{1{,}5 \pm 0{,}5}{2} = \begin{cases} p_1 = 1 \\ p_2 = 0{,}5 \end{cases}$$

$$c_1 = \frac{z}{z-0{,}5}\bigg|_{z=1} = \frac{1}{1-0{,}5} = 2$$

$$c_2 = \frac{z}{z-1}\bigg|_{z=0{,}5} = \frac{0{,}5}{0{,}5-1} = -1$$

$$x(n) = 2 \cdot u(n) - 1 \cdot u(n)\, 0{,}5^{\,n}$$