Ismétés

- abszolutt inthatő $\rightarrow \int_{-\infty}^{\infty} |x(t)| dt < \infty$
- véges energiájú $\rightarrow \int_{-\infty}^{\infty} x^2(t)\, dt < \infty$
- véges átlagenergiájú $\rightarrow \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} x^2(t)\, dt < \infty$
- Fourier sor

$\hookrightarrow x(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk2\pi f_0 t}$; $f_0 = \frac{1}{T}$; $c_k = \frac{1}{T} \int_{-T/2}^{T/2} x(t) e^{-jk2\pi f_0 t} dt$

- Fourier trafó

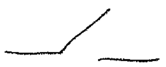
$$X(f) = \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi f t}\, dt$$

- Inverz Fourier trafó

$$x(t) = \int_{-\infty}^{\infty} X(f)\, e^{j2\pi f t}\, df$$

- Laplace trafó $\rightarrow F(s) = \int_0^{\infty} e^{-st} f(t)\, dt$
- Konvolúció $\rightarrow a(n) * b(n) = \sum_{k=-\infty}^{\infty} a(n-k)\, b(k) = \sum_k a(k)\, b(n-k)$

II Mintavételezés

$x(t)$  $x(t_0 + kt) = x_k$

↓ kezdőfázis ; ↳ mintav. period. ideje

$$x_m(f) = \sum_{-\infty}^{\infty} c_k e^{-2\pi j k f t}$$

- $x(t)$ sávkorl. B-n $\Rightarrow x(t) = \int_{-B}^{B} x(f)\, e^{j2\pi f t}\, df$

$\Rightarrow x_k = \int_{-B}^{B} x_m(f)\, e^{j2\pi f k t}\, df$; $\frac{1}{T} > 2B$ mintavételezési frekv. > mint a sávszélesség kétszerese

$$c_k = T \int_{-\frac{1}{2T}}^{\frac{1}{2T}} x_m(f)\, e^{jk2\pi f T}\, df = T \int_{-B}^{B} x_m(f)\, e^{jk2\pi f T}\, df = T x_k$$

$$x(t) = \int_{-B}^{B} x(f)\, e^{j2\pi f t}\, df = \int_{-B}^{B} \overbrace{T \sum x_k}^{c_k}\, e^{-j2\pi k f T} e^{j2\pi f t}\, dt =$$

$$= T \sum_{k=-\infty}^{\infty} x_k \int_{-B}^{B} e^{j2\pi f(t-kT)}\, df$$

Mintavételezési törvény

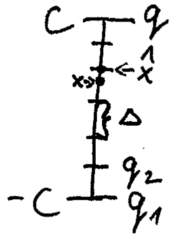
$$\boxed{\begin{aligned} x(t) &= \sum_{k=-\infty}^{\infty} x_k\, h(t-kT) \\ h(t) &= T\cdot\frac{\sin 2\pi B t}{2\pi B t} \\ X_m(f) &= \sum_{l=-\infty}^{\infty} X\left(f-\frac{l}{T}\right) \end{aligned}}$$

$$f_m = \frac{1}{T} > 2B$$

- mintavett jelből aluláteresztő szűrővel lehet rekonstruálni az eredeti jelet

III Kvantálás

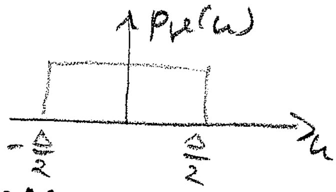
① egyenletes lépésközű



$$\Rightarrow N = \frac{2c}{\Delta} = 2^n \quad (\Rightarrow n \text{ bites kvantáló})$$

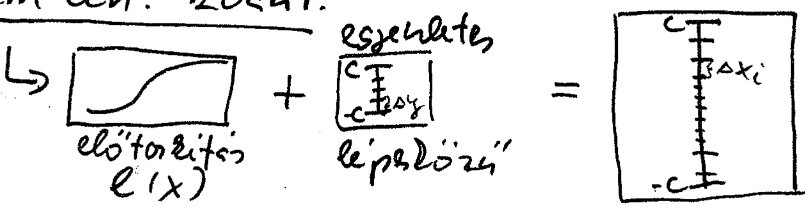
$$SNR_{kvant} = \frac{\text{jelen.}}{\text{kv. zajen.}} = \frac{c^2/2}{\mathbb{E}(\varepsilon^2)} \quad ; \varepsilon = x - \hat{x}$$

$$\mathbb{E}(\varepsilon^2) = \int_{-\Delta/2}^{\Delta/2} p_\varepsilon(u)\, u^2\, du = \int_{-\Delta/2}^{\Delta/2} \frac{1}{\Delta}\cdot u^2\, du = \frac{\Delta^2}{12}$$



$$\Rightarrow \boxed{SNR_{kvant}} = \frac{c^2/2}{\frac{\Delta^2}{12}} = \frac{3}{2}\,\frac{4c^2}{\Delta^2} = \frac{3}{2}\,N^2 = \underline{\frac{3}{2}\,2^{2n}}$$

② nem lin. kvant.



$$e'(x_i) = \frac{\Delta y}{\Delta x_i}$$

$$\Rightarrow \Delta x_i = \frac{\overbrace{2c}^{\Delta y}}{N e'(x_i)}$$

jelenergia

$$\mathbb{E}(\text{minta}^2) = \int_{-c}^{c} x^2\, p(x)\, dx$$

$$\boxed{SNR = \frac{\int_{-c}^{c} x^2\, p(x)\, dx}{A\int_{-c}^{c} \frac{1}{e'^2(x)}\, p(x)\, dx}}$$

$$\boxed{e_{opt}(x) : \max_{e(x)} SNR}$$

zajenergia

$$\sum_i \mathbb{E}(\text{zajen} \mid \text{jel} \in \Delta x_i)\, p(x)\, \Delta x_i = \sum_i \frac{\Delta x_i^2}{12}\cdot p(x)\,\Delta x_i =$$

$$= \sum_i \overbrace{\underbrace{\left(\frac{2c}{N}\right)^2 \cdot \frac{1}{12}}_{A} \cdot \frac{1}{e'^2(x)}}^{\Delta x_i^2}\, p(x)\,\Delta x_i = A\int_{-c}^{c} \frac{1}{e'^2(x)}\, p(x)\, dx$$

③ Univerzális kvantáló - sfv ftlen.

$$x^2 \sim \frac{1}{e'^2(x)} \Rightarrow e'(x) = \frac{1}{x} \Rightarrow \boxed{e(x) = \ln x}$$

IV LTI rendszerek

① $y(n) = \sum_k h(n-k)\,x(k) = h(n) * x(n)$

$x(n) = \sum x(k)\, S^k \delta(n)$

$y(n) = \mathcal{T}\, x(n) = \mathcal{T} \sum x(k)\, S^k \delta(n) = \sum x(k)\, \mathcal{T} S^k \delta(n) =$

$= \sum x(k)\, S^k h(n) = \underline{\sum x(k)\, h(n-k)}$

② Kauzalitás

$y(n) = \sum_{k=0}^{n} h(n-k)\,x(k)$ "jövő nem hat.-hatja meg a múltat!"

③ BIBO stabilitás

"korlátos bemenő jelre kimenet korlátos ⇒ BIBO stabil"

$|x(n)| \le M_x$, $|y(n)| \le M_y$ ∀-re ⇒ BIBO

$|y(n)| = |\sum h(k)\,x(n-k)| \le \sum |h(k)\,x(n-k)| \le M_x \sum |h(k)| \le M_y$

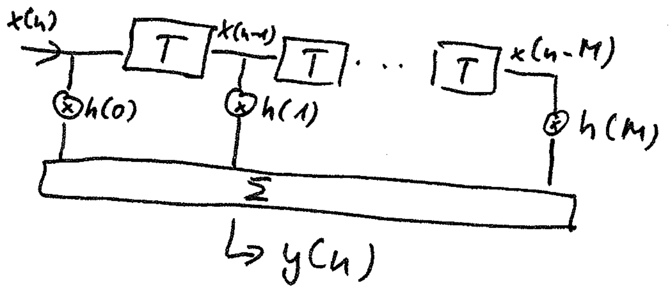
$\Rightarrow \sum_{k=0}^{\infty} |h(k)| < \infty$, akkor igaz

↳ $= c_1 \lambda_1^n + c_2 \lambda_2^n$ ⇒ stabil, ha λ-k egységkörön belül

④ Megvalósítások

a) FIR - "Finite Impulse Response"

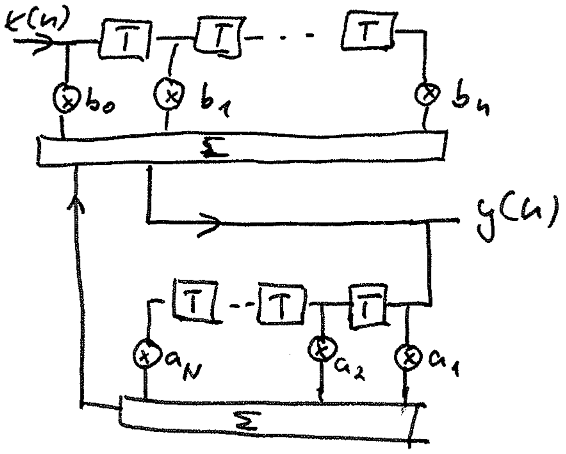
$$\sum_{k=0}^{M} h(k)\,x(n-k)$$



b.) ~ IIR - "recursive Infinite Impulse Response

$$y(n)=\sum_{j=0}^{M} b_j x(n-j) - \sum_{i=1}^{N} a_i y(n-i)$$

$$\Rightarrow \sum_{i=0}^{N} a_i y(n-i) = \sum_{j=0}^{M} b_j x(n-j)$$



⑤ Optimalizálás

$\sum a_i y(n-i) \sum b_j x(n-j)$

$\mathfrak{I}_1 : v(n) = \sum b_j x(n-j)$ $\qquad \tilde{\mathfrak{I}}_2 : \sum a_i x(n-i) = w(n)$

$\mathfrak{I}_2 : u(n) = \sum a_j y(n-i)$ $\qquad \mathfrak{I}_1 : y(n) = \sum b_j w(n-j)$

V A Z-transzformált

$$\boxed{Z\{x(n)\} = \sum_{n=-\infty}^{\infty} x(n) z^{-n}} \qquad z = re^{j\Theta}$$

- konvergenciatart - ROC
  - mindig egy gyűrű

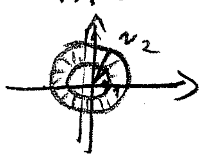
$$Z\{x(n)\} = \sum_{n=-\infty}^{\infty} x(n) z^{-n} = \sum_{n=-\infty}^{-1} x(n) z^{-n} + \sum_{0}^{\infty} x(n) z^{-n}$$

véges, ha

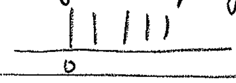
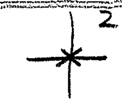
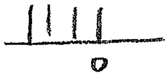
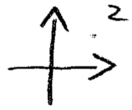
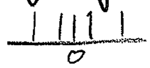
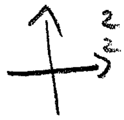
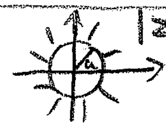
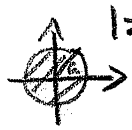
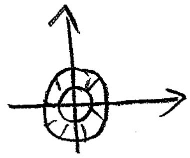
$$|X(z)| = \left|\sum_{-\infty}^{\infty} x(n) z^{-n}\right| \le \sum |x(n)| r^{-n} = \sum_{-\infty}^{-1} |x(n)| r^{-n} + \sum_{0}^{\infty} |x(n)| r^{-n} =$$

$$= \underbrace{\sum_{1}^{\infty} |x(n)| r^{n}}_{r<r_2} + \underbrace{\sum_{0}^{\infty} |x(n)| r^{-n}}_{r>r_1}$$

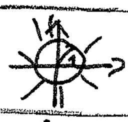
$\Rightarrow r_1 < r < r_2$



- ROC - összefoglalás

x(n)	ROC	
véges tartójú belépőfv  0	$z \neq 0$ 	$\sum_{0}^{M} x(n) z^{-n}$
véges tartójú antikauzális  0	$z \neq \infty$ 	$\sum_{-M}^{-1} x(n) z^{-n}$
végestartójú jel  0	$z \neq 0$, $z \neq \infty$ 	$\sum_{-M_1}^{M_2} x(n) z^{-n}$
végtelen tartójú belépőfv $u(n)\,a^n$	 $\lvert z\rvert > a$	$\sum_{0}^{\infty} x(n) z^{-n}$
végtelen tartójú antikauzális jel	 $\lvert z\rvert < a$	
végtelen tartójú jel kauzális		

- Gyakori fv-ek z-transzformáltja

x(n)	X(Z)	ROC	Biz
$\delta(n)$	1	teljes z	$\sum_{-\infty}^{\infty} \delta(n) z^{-n} = 1$
$u(n)$	$\frac{z}{z-1}$		$\sum_{0}^{\infty} 1\, z^{-n} = \frac{1}{1-z^{-1}} = \frac{z}{z-1}$
$a^n u(n)$	$\frac{z}{z-a}$		
$u(n) \cos n\omega_0$	$\frac{1}{2}\left\{\frac{z}{z-e^{j\omega_0}} + \frac{z}{z-e^{j\omega_0}}\right\}$		$\sum_{0}^{\infty} \cos n\omega_0\, z^{-n} = \sum \frac{1}{2} e^{jn\omega_0} z^{-n} + \sum \frac{1}{2} e^{-jn\omega_0} z^{-n}$

– Z-trafó tulajdonságai

① Linearitás

$$\mathcal{Z}\{a x_1(n) + b x_2(n)\} = a X_1(z) + b X_2(z)$$

biz $\sum_{-\infty}^{\infty} (a \cdot x_1(n) + b x_2(n)) z^{-n} = a \sum x_1(n) z^{-n} + b \sum x_2(n) z^{-n}$

② Eltolás

$$\mathcal{Z}\{S^k x(n)\} = \mathcal{Z}\{x(n-k)\} = z^{-k} X(z)$$

biz $\sum_n x(n-k) z^{-n} = \sum_m x(m) z^{-(m+k)} = z^{-k} \sum x(m) z^{-m} = z^{-k} X(z^m)$

③ Skálázás

$$\mathcal{Z}\{u(n)\, a^n x(n)\} = X(a^{-1} z)$$

biz $\sum_{-\infty}^{\infty} a^n x(n) z^{-n} \overset{u(n)}{=} \sum_0^{\infty} x(n) (a^{-1} z)^{-n} = X(a^{-1} z)$

④ "Időmegfordítás"

$$\mathcal{Z}\{x(-n)\} = X(z^{-1})$$

biz
$\sum_{-\infty}^{\infty} x(-n) z^{-n} = \sum_m x(m) z^m = \sum x(m) (z^{-1})^{-m} = X(z^{-1})$

⑤ "Deriválás"

$$\mathcal{Z}\{n x(n)\} = -z \frac{dX(z)}{dz}$$

biz $\frac{dX(z)}{dz} = \frac{d}{dz} \sum x(n) z^{-n} = -\sum n x(n) z^{-(n+1)} = -z^{-1} \sum n x(n) z^{-n} =$

$= -z^{-1} \; \mathcal{Z}\{n \cdot x(n)\}$

⑥ Konvolúció

$$\mathcal{Z}\{\sum a(n-k) b(k)\} = A(z) B(z)$$

biz $\sum \sum a(n-l) b(l) z^{-(n-l)} z^{-l} = \sum b(l) \{\sum a(n-l) z^{-(n-l)}\} z^{-l} =$

$= \sum b(l) \{\sum a(m) z^{-m}\} z^{-l} = A(z) \sum b(l) z^{-l} = A(z) B(z)$

VI Inverz "z" transzformáció

$$\boxed{x(n)=\frac{1}{2\pi j}\oint_G X(z)z^{n-1}dz}$$

bíz

$$x(n)=\frac{1}{2\pi j}\oint_G X(z)z^{n-1}dz=\frac{1}{2\pi j}\oint_G\sum_{l=-\infty}^{\infty}x(l)z^{-l}z^{n-1}dz=$$

$$=\sum x(l)\frac{1}{2\pi j}\oint_G z^{\overset{0\Leftarrow n=l}{n-l}-1}dz=\sum x(l)\delta_{n0}=x(n)$$

- átviteli fv

$$Z\{h(n)\}=H(z)=\frac{Y(z)}{X(z)}\Leftarrow y(n)\sum h(n-l)x(l)\overset{z}{\Rightarrow}Y(z)=H(z)X(z)$$

$$H(z)=\frac{\sum b_j z^{-j}}{\sum a_i z^{-i}}\Leftarrow\sum a_i\overbrace{y(n-i)}^{z^{-i}Y(z)}=\sum b_j\overbrace{x(n-j)}^{z^{-j}X(z)}$$

$$H(z)=z^{N-M}\frac{\sum b_j z^{M-j}}{\sum a_i z^{N-i}}=z^{N-M}\frac{B(z)}{A(z)}$$

BIBO stabil $H(z)$, ha $A(z)$ gyökei egységkörön belül

$$\xrightarrow{z^{-1}}h(n)=\sum c_i(z_i)^n\cdot u(n)$$

VII Frekvencia analízis

$$X\left(k\frac{2\pi}{N}\right)=X(\omega)\Big|_{\omega=k\frac{2\pi}{N}}=X(z)\Big|_{z=e^{j\omega}}\Rightarrow\sum\overbrace{x(n)e^{-jk\frac{2\pi}{N}n}}^{u(n)}=$$

$$=\ldots+\sum_{n=-N}^{-1}u(n)+\sum_{0}^{N-1}u(n)+\sum_{N}^{2N-1}u(n)+\ldots=\sum_{l=-\infty}^{\infty}\sum_{n=lN}^{(l+1)N-1}u(n)=\sum_{n=0}^{N-1}\underbrace{\sum_{l=-\infty}^{\infty}x(n-lN)}_{x_p(n)}e^{-jnk\frac{2\pi}{N}}$$

DFT

$$\boxed{X\left(k\frac{2\pi}{N}\right)=\sum_{n=0}^{N-1}x_p(n)e^{-jnk\frac{2\pi}{N}}}$$

$$x_p(n)=\sum_{k=0}^{N-1}c_k e^{jnk\frac{2\pi}{N}}$$

$$\hookrightarrow X(k)=\sum x(n)W_N^{kn};\ W_N=e^{-j\frac{2\pi}{N}}$$

$$c_k=\frac{1}{N}\sum x_p(n)e^{-jnk\frac{2\pi}{N}}$$

$$X(\omega)=\sum_{k=0}^{N-1}X\left(k\frac{2\pi}{N}\right)P\left(\omega-k\frac{2\pi}{N}\right)\Leftarrow\sum x_p(n)e^{-jn\omega}=\sum_0^{N-1}\frac{1}{N}\sum_0^{N-1}X\left(k\frac{2\pi}{N}\right)e^{jnk\frac{2\pi}{N}}e^{-jn\omega}=$$

$$P(\omega)=\frac{1}{N}\frac{1-e^{-j\omega N}}{1-e^{-j\omega}}\qquad =\sum X\left(k\frac{2\pi}{N}\right)\frac{1}{N}\sum e^{-jn\left(\omega-k\frac{2\pi}{N}\right)}$$

IDFT

$$\boxed{x(n)=\frac{1}{N}\sum_{k=0}^{N-1}X\left(k\frac{2\pi}{N}\right)e^{jnk\frac{2\pi}{N}}}\xrightarrow{W_N=e^{-j\frac{2\pi}{N}}}x(n)=\frac{1}{N}\sum X(k)W_N^{-nk}$$

FFT

$$\boxed{X(p,q)=\sum_{l=0}^{L-1}\left\{W_N^{ql}\left(\sum_{m=0}^{M-1}x(l,m)\,W_M^{q\cdot m}\right)\right\}W_L^{pl}}$$

$e^{-jmq\frac{2\pi}{M}}$

$N=M\cdot L$

$$X(p,q)=\sum_l\sum_m x(l,m)\,W_N^{(pM+q)(mL+l)}=\sum\sum x(l,m)\,\underbrace{W_N^{pmML}}_{1}\,\underbrace{W_N^{qmL}}_{W_M^{q\cdot m}}\,\underbrace{W_N^{plM}}_{W_L^{pl}}\,W_N^{qL}$$

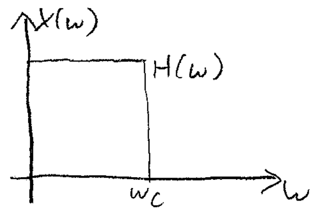
Ért

	Kompl. szorzás	összeadás
① L db M hossz. DFT; $F(l,q):=\sum_m x(l,m)W_M^{qm}$	LM^2	$L\cdot M(M-1)$
② M·L db szorzás; $G(l,q)=F(l,q)\cdot W_N^{qL}$	ML	0
③ M db L hossz. DFT $X(p,q)=\sum_l G(l,q)\,W_L^{pl}$	ML^2	$ML(L-1)$

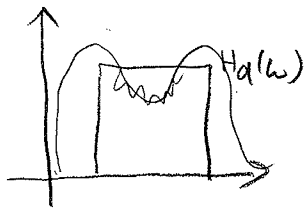
VIII A jelek spektrális formálása – szűrés

$$H(\omega)=H(z)\big|_{z=e^{j\omega}}=\sum_n h(n)e^{-jn\omega}$$

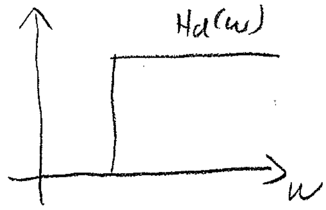
Aluláteresztő



Sávszűrő



Felüláteresztő

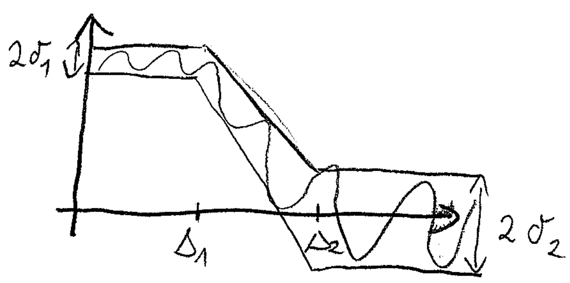


$$\hookrightarrow H_d(\omega)\to H_d(n)=\frac{1}{2\pi}\int_{-\pi}^{\pi}H(\omega)e^{-jn\omega}d\omega=\frac{\omega_c}{\pi}\,\frac{\sin(n\omega_c)}{n\,\omega_c}$$

Paley-Wiener tétel

$$\int_{-\pi}^{\pi}\left|\ln|H(\omega)|\right|d\omega<\infty$$

$\Rightarrow$ akkor tudunk szűrni
↳ nem létezik végtelen meredek szűrő
↳ ezért is kell tűrésmezőt definiálni



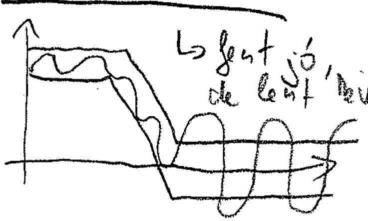
$$H_{opt}(\omega)=\min\int_{-\pi}^{\pi}\left(H_d(\omega)-H(\omega)\right)^2d\omega$$

$$H(\omega)=\sum_0^{M-1}h(n)e^{-jn\omega}\;;\;h(n)=\frac{1}{2\pi}\int_{-\pi}^{\pi}H_d(\omega)e^{jn\omega}d\omega$$

$$\underset{\text{átvf.}}{h(n)}=\underset{\text{ablak}}{w(n)}\,\underset{\text{elvárt}}{h_d(n)}\Rightarrow H(\omega)=\frac{1}{2\pi}\int_{-\pi}^{\pi}H_d(\nu)W(\omega-\nu)d\nu$$

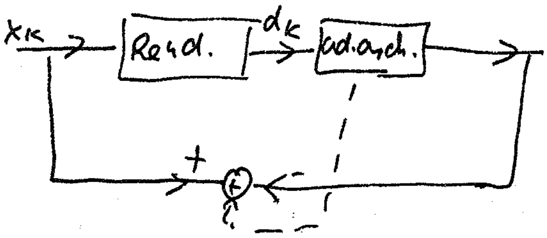
$$W(\omega)=\sum_0^{M-1}1\cdot e^{-jn\omega}=\frac{1-e^{-jM\omega}}{1-e^{-j\omega}}$$

Gibbs oszcilláció



↳ fent jó, de lent billög

Rendszer opt. és inverz identifikáció



cél rend. inverzének meghat.,
⇒ d_k-ból visszakapjuk x_k-t

Hibakritériumok

$e_k = d_k - y_k$

① $E = (d_k - \sum w_j x_{k-j})^2$

② $\frac{1}{K} = \sum_1^K (d_k - \sum_0^J w_j \cdot x_{k-j})^2$

Wiener szűrés

→ legoptimálisabb együttható sorozatot ker. (w_{opt})

$$\boxed{\bar W_{opt} : \min_{\bar w} E(d_k - \sum_0^J w_j X_{k-j})^2}$$

$$Y(\bar w) = E(d_k - \sum w_j \cdot x_{k-j})^2 = E(d_k^2 - 2\sum w_j x_{k-j} d_k + (\sum_j w_j x_{k-j})(\sum_i w_i x_{k-i}))$$

$$= D(0) - 2\sum r(j) w_j + \sum\sum w_j w_i E(x_{k-i} x_{k-j})$$

$$= D(0) - 2\sum \underbrace{r(j)}_{\bar r^T} \underbrace{w_j}_{\bar w} + \sum\sum \underbrace{w_j}_{\bar w} \cdot \underbrace{w_i}_{\bar w^T} \underbrace{R(i-j)}_{\bar{\bar R}}$$

$\bar{\bar R} = \sum E(x_{k-i} x_{k-j})$

$\bar r = \sum x_k x_{k-j}$

↳ keresztkorr.

↳ autokorr.

$$\boxed{J(\bar w) = D(0) - 2\bar r^T \bar w + \bar w^T \bar{\bar R}\, \bar w}$$

$$\bar W_{opt} : \min_{\bar w} J(\bar w) \Rightarrow \underline{grad(J(\bar w)) = \bar 0} = \frac{\partial J(\bar w)}{w_i} = \sum_{j=0}^J R_{ij} w_j - r_i = \underline{\bar{\bar R}\bar w - \bar r}$$

$$J(w_{opt}) = D(0) - 2\bar r^T \bar w_{opt} + \bar w_{opt}^T \bar{\bar R}\, \bar w_{opt} = D(0) - 2\bar r^T \bar{\bar R}^{-1} \bar r + \bar r^T \bar{\bar R}^{-1T} \bar{\bar R} \bar{\bar R}^{-1} \bar r$$

$$\Rightarrow \boxed{J(\bar W_{opt}) = D(0) - \bar r^T \bar{\bar R}^{-1} \bar r}$$

Elmélet

$\bar{\bar{R}}$ mátrix tulajdonságai

① Toeplitz típusú

② $\bar{\bar{R}} = \bar{\bar{R}}^T$; $R_{ij} = E(x_{k-i}, x_{k-j}) = E(x_{k-j}, x_{k-i}) = R_{ji}$ szimmetrikus

③ poz. szemidefinit $\forall \bar{a} \in R^{J+1}$ $\bar{a}^T \bar{\bar{R}} \bar{a} \geq 0$

$$\sum_i \sum_j a_i a_j E(x_{k-i}, x_{k-j}) = E(\sum\sum a_i a_j x_{k-i} x_{k-j}) = E(\sum a_i x_{k-i})^2 \geq 0$$

④ Hermetikus $\bar{a}^T \bar{\bar{R}} \bar{b} = \bar{b}^T \bar{\bar{R}} \bar{a}$

⑤ SV-ok ortonormáltak

$$\bar{\bar{R}} \bar{s}_i = \lambda_i \bar{s}_i \; ; \; \bar{s}_j^T \bar{\bar{R}} \bar{s}_i = \bar{s}_i^T \bar{\bar{R}} \bar{s}_j \; ; \; \lambda_i (\bar{s}_j^T \bar{s}_i) = \lambda_j (\bar{s}_i^T \bar{s}_j)$$

$\Downarrow$ $\bar{s}_i^T \bar{s}_j = \delta_{ij}$ $\Downarrow$ $(\lambda_i - \lambda_j) \bar{s}_i^T \bar{s}_j = 0$

⑥ SÉ spektrum pozitív

$\lambda_i \geq 0 \;\; \forall i$-re $\Leftarrow$ ③

$$0 \leq \bar{s}_i^T \bar{\bar{R}} \bar{s}_i = \lambda (\bar{s}_i^T \bar{s}_i) = \lambda \|\bar{s}_i\|^2 = \lambda_i$$

prob
- $\bar{\bar{R}}^{-1}$ Diszim
- $\bar{\bar{R}}$ és $\bar{r}$ ismeretlen

IX Rekurzív Wiener szűrő

$$\bar{w}(k+1) = \bar{w}(k) - \Delta \{ \bar{\bar{R}} \bar{w}(k) - \bar{r} \}$$ → Percent 2011.

$\Rightarrow \bar{w}_{opt} = \bar{w}(k+1) = \bar{w}(k)$; $\lim_{k\to\infty} \bar{w}(k) = \bar{w}_{opt}$

Konvergenciaanalízis

$\bar{\bar{R}} \bar{s}_i = \lambda_i \bar{s}_i \quad \det(\bar{\bar{R}} - \lambda \bar{\bar{I}}) = 0$

$\bar{w}(k+1) = \sum_0^J v_j(k+1) \bar{s}_j$; $\bar{w}(k) = \sum v_j(k) \bar{s}_i$ $\quad r = \sum \varrho_i \bar{s}_i$

$$\sum v_i(k+1) \bar{s}_i = \sum v_i(k) \bar{s}_i - \Delta \{ \bar{\bar{R}} \sum v_i(k) \bar{s}_i - \sum \varrho_i \bar{s}_i \}$$

$$\sum v_i(k+1) \bar{s}_i = \sum (1 - \Delta \lambda_i) v_i(k) + \Delta \varrho_i \} \bar{s}_i$$

$$\Rightarrow \boxed{v_i(k+1) = (1 - \Delta \lambda_i) v_i(k) + \Delta \varrho_i}$$

$\Rightarrow \gamma = 1 - \Delta \lambda_i \Rightarrow v_i(k) = c_i (1 - \Delta \lambda_i)^k + \frac{\varrho_i}{\lambda_i} \;\; \forall i$-re

$\hookrightarrow \bar{w}(k) = \sum v_i(k) \bar{s}_i = \sum c_i (1 - \Delta \lambda_i)^k + \sum \frac{\varrho_i}{\lambda_i} \bar{s}_i$ $\quad \bar{\bar{R}}^{-1} \bar{r}$

$$\Rightarrow \boxed{\bar{w}(k) = \sum c_i (1 - \Delta \lambda_i)^k + w_{opt}}$$ konv. seb.: $O(\max |1 - \Delta \lambda_i|^k)$

$$\Delta_{opt} \to \min_{\Delta} \max_{\lambda \in [\lambda_{min}, \lambda_{max}]} |1 - \Delta \lambda| \Rightarrow \frac{2}{\lambda_{min} + \lambda_{max}} \approx \frac{1}{R(0)}$$

Gershgorin körök

$$\lambda_i \in \left[R_{ii} - \sum_{i,j \neq i} |R_{ij}| \; ; \; R_{ii} + \sum_{j, j \neq i} |R_{ij}| \right]$$

$R(0) - \sum_{n, n \neq 0} R(n) \leq \lambda_{min}$
$R(0) + \sum_{n, n \neq 0} R(n) \geq \lambda_{max}$

mátrix inv. nélkül tudjuk $\bar{w}_{opt}$-t meghatni

X Adaptív szűrés

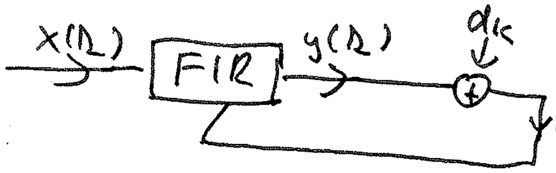
$$J^{(k)} = \{ x(k), \overset{\text{realizáció}}{d(k)}, k = 1..K \} \Rightarrow \text{tanulóhalmaz}$$

$$\overline{w}^{(k)}_{opt} = \min_{\overline{w}} \frac{1}{K} \sum_{k=1}^{K} \left(d_k - \sum_{0}^{J} w_j x_{k-j}\right)^2$$

Robbins – Monroe sztoch approximáció

- ha nem ismert $\overline{\overline{R}}$ és $\overline{r}$, ezzel is tudunk konv. a w_{opt}-hoz

$$w_e(k+1) = w_e(k) - \Delta(k) \{ \underbrace{d_k - \sum_{0}^{J} w_j(k) x_{k-j}}_{e_k} \} x_{k-e}$$



e_k $\quad \overline{w}(k+1) = \overline{w}(k) - \Delta(k) \overline{\Psi}(\overline{d}(k), \overline{x}(k))$

$\lim_{k \to \infty} E\|\overline{w}(k) - \overline{w}^*\|^2 = 0$, azt szeretnénk, hogy $\underline{\overline{w}^* = \overline{\overline{R}}^{-1} \overline{r}}$ legyen

Kushner – Clark tétel (R-M biz.)

Ha $\lim \Delta(k) = 0$; $\sum_{0}^{\infty} \Delta(k) = \infty$; $\sum_{k=0}^{\infty} \Delta^p(k) < \infty$; $p \geq 2$; $\exists \frac{\partial \overline{\Psi}(\overline{w}, \overline{x})}{\partial w_j}$; $\frac{\partial \overline{\Psi}(\overline{w}, \overline{x})}{w_e}$; ${}^{6.}\overline{\Psi}(w) = E_x \overline{\Psi}(\overline{w}, \overline{x}) = \overline{r} - \overline{\overline{R}}\overline{w}$

$${}^{7.}\frac{\partial \overline{w}(t)}{dt} = \overline{\Psi}(\overline{w}(t)) \Rightarrow \exists\, \overline{w}^*: \overline{\Psi}(\overline{w}^*) = 0$$

$\Rightarrow$ elért egy stabil állapotot

$$\Rightarrow \lim E\|\overline{w}(k) - \overline{w}^*\|^2 = 0$$

6.

$$\Psi(\overline{w}) = E(d_k - x_{k-e}) - E(\sum w_j x_{k-j} x_{k-e}) = r_e - \sum w_j E(x_{k-j} x_{k-e}) =$$
$$= r_e - \sum R_{ej} w_j = \underline{r_e - \overline{\overline{R}}\overline{w} = \Psi(\overline{w})} \quad 6. \checkmark$$

7.

$$\frac{\partial \overline{w}(t)}{dt} = \overline{r} - \overline{\overline{R}}\overline{w}(t), \quad \overline{\overline{R}}\overline{s}_i = \lambda_i \overline{s}_i, \quad \overline{w}(t) = \sum_{0}^{J} v_j(t) s_j, \quad r = \sum \varsigma_j \overline{s}_j$$

$$\hookrightarrow \frac{d}{dt} \sum v_j(t) s_j = \sum \varsigma_j \overline{s}_j - \overline{\overline{R}} \sum v_j(t) s_j$$

$$\sum \frac{d}{dt} v_j(t) s_j = \sum (\varsigma_j - \lambda_j v_j(t)) s_j \Rightarrow \underline{\frac{d}{dt} v_j(t) = \varsigma_j - \lambda_j v_j(t)}$$

$$\Rightarrow \boxed{v_j(t) = c_j e^{-\lambda_j t} + \frac{\varsigma_j}{\lambda_j}} \Rightarrow \underline{\overline{w}(t) = \sum c_j e^{-\lambda_j t} \overline{s}_j + \sum \frac{\varsigma_j}{\lambda_j} s_j = \overline{\overline{R}}^{-1} \overline{r}}$$

Általános feladatmegoldás

① $\mathcal{T}^{(n)} := \{(x_k, d_k), k = 1..K\}$

② $w_e(k+1) = w_e(k) - \Delta(k)\{d_k - \sum w_j(k)\, x_{k-j}\}\, x_{k-e}$

③ $\lim\limits_{k\to\infty} \mathbb{E}\|\bar{w}(k) - \bar{w}^*\|^2 = 0 \;;\; \bar{\bar{R}}\,\bar{w}^* = \bar{r}$

Spektrális hat. javítása

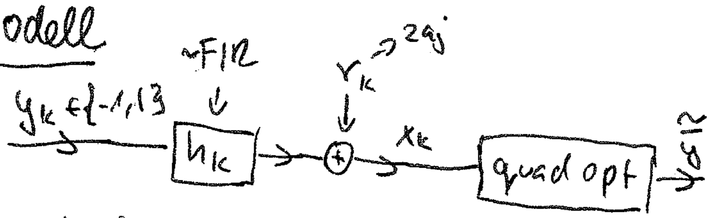
– reflektált jel kiküszöbölésére

① space diversity – csökkenés esetén átter másikra

② frequency diversity – zavarás esetén másik frek.sáv

③ adaptív jelfeldolgozás – chip integrálása mobilba

Modell



$x_k = \sum h_{k-n} y_n + v_k = h_0 y_k + \sum\limits_{n\neq k} h_{k-n} y_n + v_k$

$\Rightarrow$ bitbiba val nagy

– vételi oldalon kell védekezni

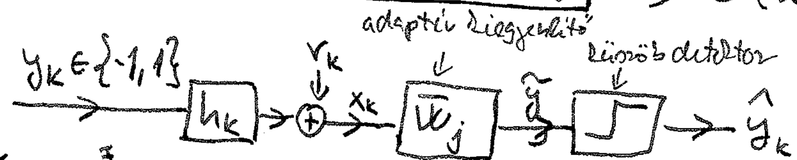
$\bar{x} = \bar{\bar{H}}\,\bar{y} + \bar{v} \;;\; H_{ij} = h_{i-j} \quad \bar{v} \sim N(\bar{0}, \bar{N}_0, \bar{\bar{I}})$

– Bayes döntés – legkisebb hiba val.

$\tilde{y} : \max P(\bar{y}, \bar{x}) \sim \max \dfrac{P(\bar{x}|\bar{y})\,\cancel{P(\bar{y})}}{\cancel{P(\bar{x})}} = \max \dfrac{1}{\sqrt{2\pi (N_0)^N}}\, e^{-\frac{1}{2N_0}(\bar{x} - \bar{\bar{H}}\bar{y})^T(\bar{x} - \bar{\bar{H}}\bar{y})}$

$\Rightarrow \min \|\bar{x} - \bar{\bar{H}}\bar{y}\|^2 \sim \min(\bar{y}^T \underbrace{\bar{\bar{H}}^T \bar{\bar{H}}}_{\bar{\bar{A}}} \bar{y}) - 2 \underbrace{\bar{x}\bar{\bar{H}}^T}_{\bar{b}^T} \bar{y}$

$\Rightarrow \boxed{\tilde{y} : \min \bar{y}^T \bar{\bar{A}}\, \bar{y} - 2\, \bar{b}^T \bar{y}} \Rightarrow O(2^N)$



$\tilde{y}_k = \sum\limits_0^{J} w_j x_{k-j} = \sum w_j (\sum h_{k-j-n} y_n + v_{k-j}) = \sum(\sum w_j h_{k-j-n}) y_n + \sum w_j v_{k-j}$

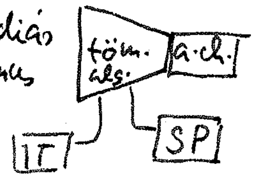
$\bar{w}_{opt} : \min E(y_k - \tilde{y}_k)^2 \sim \min \mathbb{E}(y_k - \sum w_j x_{k-j})^2$

$\mathcal{T}^k = \{(x_k, y_k)\; k = 1..K\}$ $\quad\forall$ infó közlés előtt rövid tanuló fázis, utána jöhet az infó

$w_e(k+1) = w_e(k) - \Delta(k)\{y_k - \sum w_j(k)\, x_{k-j}\}\, x_{k-e}$

XI Adattömörítés

multimédiás algoritmus



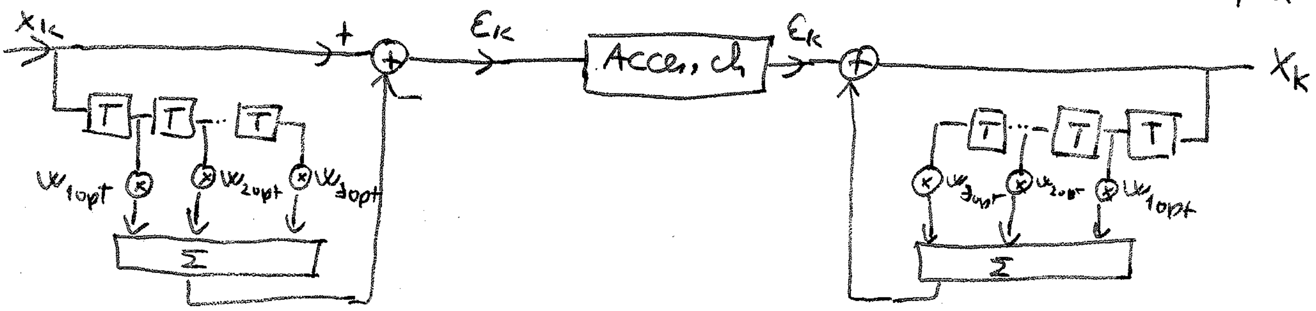
1.) Lossless

2.) Lossy okig
– előírt hűségkrit.-nak megfelelően

① Adaptív prediktív tömörítés (ADPC)

x_k: sztoch. sor.; $R(l) = \mathbb{E}(x_k, x_{k-l})$; $\tilde{x}_k = \sum w_j x_{k-j} \Rightarrow x_{k-J-1}, \underbrace{x_{k-J} \dots x_{k-1}}_{\text{lin. pred.}}, x_k \to \tilde{x}$

$\overset{\text{hiba}}{\varepsilon_k} = x_k - \tilde{x}_k \Rightarrow$ ez kerül átvitelre

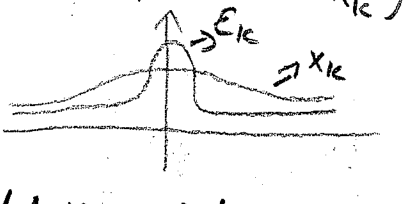


$\bar{w}_{opt}$: $\min \mathbb{E}\left(x_k - \sum_{j=1}^{J} (x_k - \sum_{1}^{J} w_j x_{k-j}\right)^2$

$$\boxed{\mathbb{E}(\varepsilon_k)^2} = \mathbb{E}\left(x_k - \sum_{j=1}^{\infty} w_{jopt} x_{k-j}\right)^2 = \mathbb{E}(x_k^2) - 2\sum w_{jopt}\mathbb{E}(x_k, x_{k-j}) + \sum\sum w_{jopt} w_{iopt} \mathbb{E}(x_{k-j}, x_{k-i})$$

$$= R(0) - 2\bar{w}_{opt}^T \underset{\bar{\bar{R}} w_{opt}}{\bar{r}} + \bar{w}_{opt}^T \bar{\bar{R}} \bar{w}_{opt} = \underline{R(0) - \bar{w}_{opt}^T \bar{\bar{R}} \bar{w}_{opt}} > 0$$

$\mathbb{E}(e_k^2) \ll \mathbb{E}(x_k^2)$

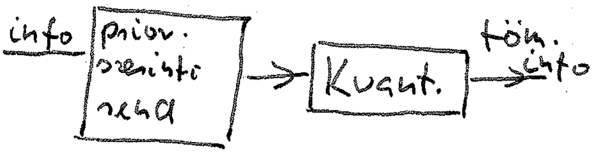


prob: korr. fv nem ismert

$w_l(k+1) = w_l(k) - \delta(k)\{x_k - \sum w_j x_{k-j}\} x_{k-l}$

$\lim \mathbb{E}\|\bar{w}(k) - \bar{w}_{opt}\|^2$ ⇓ tanulófázis

② Képtömörítés

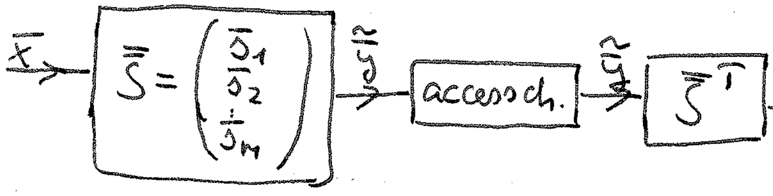


gy. stac. sztoch $\bar{x}$; $\dim(\bar{x}) = 480.000$

frame: $\bar{x} = \sum_{1}^{N} y_i \bar{s}_i$; $y_i = \bar{s}_i^T \bar{x}$

adott $\bar{\bar{R}} \to \lambda$ is $\Rightarrow \underset{\bar{s}_1}{\lambda_1} > \underset{\bar{s}_2}{\lambda_2} > \dots \underset{\bar{s}_M}{\lambda_M} > \lambda_{M+1} \dots > \lambda_N$

$\Rightarrow y_i = \bar{s}_i^T \bar{x}$ $i = 1..M$



$\tilde{\bar{x}} = \sum_{i=1}^{M} y_i \bar{s}_i$

$s_i^T \mathbb{E}(\bar{x}\bar{x}^T)\bar{s}_j = \bar{s}_i^T \bar{\bar{R}} \bar{s}_j = \lambda_j \bar{s}_i^T \bar{s}_j = \lambda_j \delta_{ji}$

Minősége

$\mathbb{E}(\sum x_i^2) = \sum R_{ii} = \sum \lambda_i \quad \mathbb{E}(\bar{x}\bar{x}^T)\bar{s}_i = \bar{\bar{R}}\bar{s}_i = \lambda_i \bar{s}_i$

$$\mathbb{E}\|\bar{x}-\tilde{\bar{x}}\|^2 = \mathbb{E}\|\bar{x} - \sum y_i \bar{s}_i\| = \mathbb{E}\|\bar{x}\|^2 - 2\sum \mathbb{E}(\bar{x} y_i)\bar{s}_i + \mathbb{E}(\sum\sum y_i y_j \bar{s}_i^T \bar{s}_j) =$$

$$= \sum_{i=1}^{N} \lambda_i - 2\sum \lambda_i (\bar{s}_i^T s_i) + \sum\sum \lambda_i \delta_{ij} \bar{s}_i^T \bar{s}_j = \sum_1^N \lambda_i - 2\sum_1^M \lambda_i + \sum_1^M \lambda_i =$$

$$= \sum_{i=M+1}^{N} \lambda_i \Rightarrow \min$$

$\sim O(a^{-i})$ $\Rightarrow$ hiba hamar ε alá $\downarrow$

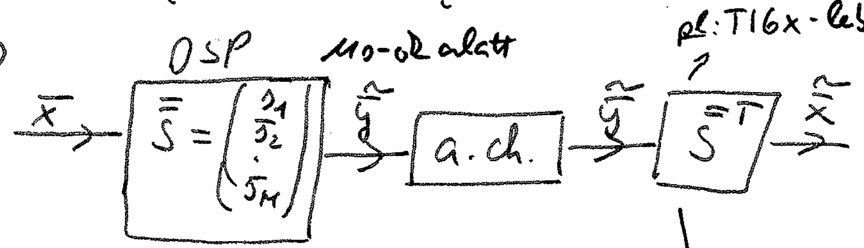
Összefoglalva

① $\bar{\bar{R}}\bar{s}_i = \lambda_i \bar{s}_i \Rightarrow \det(\bar{\bar{R}} - \lambda \bar{\bar{I}}) = 0 \Rightarrow \lambda_1 > \lambda_2 \ldots > \lambda_M > \lambda_{M+1} \ldots > \lambda_N$

$\bar{s}_1 \quad \bar{s}_2 \ldots \bar{s}_M$

② $M: \mathbb{E}\|\bar{x} - \tilde{\bar{x}}\|^2 = \sum \lambda_i \leq \varepsilon$

③



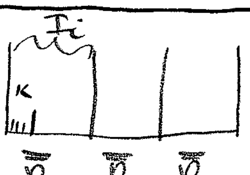
gyors, olcsó, hatékony, de framenként új $\bar{\bar{S}}$ kell

$y_i = \bar{s}_i^T \bar{x}$

$\tilde{\bar{y}} = \bar{\bar{S}}\bar{x}$

$$\tilde{\bar{x}} = \sum_1^M y_i \bar{s}_i = \bar{\bar{S}}^T \tilde{\bar{y}} \Rightarrow (\tilde{\bar{x}}) = (\bar{S}_1^T \ \bar{S}_2^T \ldots \bar{S}_M^T)(\tilde{\bar{y}})$$

③ Kernel based PCA



T_i [10; 30]ms $\in$ időablakról időablakra újra kell számolni T alatt nem megy

$\Rightarrow$ megfigyeljük K framesorozatot; becsüljük R-et

$\bar{x}^{(k)}$; $k = 1 \ldots K \Rightarrow \tilde{\bar{\bar{R}}} = \frac{1}{K}\sum_1^K \bar{x}^{(k)} \bar{x}^{(k)T}$

$\bar{\bar{R}}$ diadikus; rangja K

$\quad \hookrightarrow \bar{x}\cdot\bar{x}^T$

$$\tilde{\bar{s}}^{(i)} = \sum_1^K a_m^{(i)} \bar{x}^{(m)} \to \bar{a}^{(i)} \Rightarrow \bar{\bar{R}} \sum_1^K a_m^{(i)} \bar{x}^{(m)} = \lambda_i \sum_1^K a_m^{(i)} \bar{x}^{(m)}$$

$$\Rightarrow \sum_1^K a_m^{(i)} \overbrace{\frac{1}{K}\sum_1^K \bar{x}^{(k)} \bar{x}^{(k)T}}^{\bar{\bar{R}}} \bar{x}^{(m)} = \lambda_i \sum_1^K a_m^{(i)} \bar{x}^{(m)} \qquad / \cdot \bar{x}^{(\ell)T}$$

$$\Rightarrow \sum a_m^{(i)} \sum \bar{x}^{(\ell)T} \bar{x}^{(k)} x^{(k)T} x^{(m)} = \lambda_i K \sum a_m^{(i)} \bar{x}^{(\ell)T} \bar{x}^{(m)} \qquad \bar{\bar{G}} \Rightarrow G_{\ell m} = \bar{x}_\ell^T \bar{x}_m$$

$$\sum a_m^{(i)} \sum G_{\ell k} G_{km} = \lambda_i K \sum a_m^{(i)} G_{\ell m} \qquad / \cdot \bar{\bar{G}}^{-1}$$

$$\Rightarrow \boxed{\bar{\bar{G}} a^{(i)} = \lambda_i K a^{(i)}}$$

Következmények

$\bar{\bar{G}}_{K\times K} \to$ SÉ-ei $K \cdot \lambda_i \Rightarrow$ arányos $\bar{\bar{R}}$-el

SV-ai $a^{(i)} \Rightarrow \bar{s}^i = \sum a_m^i \bar{x}^{(m)}$

$\bar{\bar{G}}$ sajátértékfeladata ekv. azzal, mintha $\bar{\bar{R}}$-et oldanánk meg

$type(\bar{\bar{R}}): N \times N$ $K \ll N$

↳ 100~200 → nagyságrenddel kisebb

Algoritmus

① $\bar{x}^{(k)}$ $k = 1 \ldots K$

② elkészítjük $\bar{\bar{G}}$-t ; $G_{ij} = \bar{x}^{(i)T} \bar{x}^{(j)}$ $i,j = 1 \ldots K$

③ $\bar{\bar{G}}\, \bar{a}^{(i)} = K \lambda_i\, \bar{a}^{(i)}$ $\lambda_1 > \lambda_2 \ldots > \lambda_m > \lambda_{m+1} \ldots > \lambda_K$

④ $M: \sum_{M+1}^{N} \lambda_i \leq \varepsilon$

⑤ $\tilde{\bar{s}}^{(i)} = \sum a_m^{(i)} \bar{x}^{(m)}$; $i = 1 \ldots M$

⑥ $\tilde{\bar{y}} = \tilde{\bar{\bar{S}}}\, \bar{x}$

⑦ $\tilde{\bar{x}} = \tilde{\bar{\bar{S}}}^T \tilde{\bar{y}}$

klasszikus PCA-ként működik

- online, hatékony

I Digitális jelek leírása

- Dirac delta $\delta(n) = \begin{cases} 1 & \text{ha } n=0 \\ 0 & \text{ha } n \neq 0 \end{cases}$
- Egységugrás $u(n) = \begin{cases} 1 & \text{ha } n \geq 0 \\ 0 & \text{ha } n < 0 \end{cases}$
- Egységnyeb. ugr. $v(n) = \begin{cases} n & \text{ha } n \geq 0 \\ 0 & \text{ha } n < 0 \end{cases}$
- $x(n) = a^n \cdot u(n)$
- $x(n) = A\cos(2\pi f n + \Theta) = A\cos(2\pi f(n+N) + \Theta)$, ha $f = \frac{k}{N}$

$f' = A\cos(2\pi f' n + \Theta) \neq f \neq A\cos(2\pi f n + \Theta)$ $f' = f + l$

↳ megkülönböztethetetlen

$\Rightarrow$ alapfrekv. tart

$f' = f + l \Rightarrow -\frac{1}{2} \leq f \leq \frac{1}{2}$

- időeltolás:

$S^k x(n) = x(n-k)$

- időinvariáns

$\mathfrak{T} S^k x(n) = S^k \mathfrak{T} x(n)$

- lineáris

$\mathfrak{T}\{a_1 x_1(n) + a_2 x_2(n)\} = a_1 \mathfrak{T} x_1(n) + a_2 \mathfrak{T} x_2(n)$

II Példák TI-re

① $y(n) = a x(n) - x(n-1)$

$\mathfrak{T} x(n-1) = a x(n-1) - x(n-2)$

$S^1(a x(n) - x(n-1)) = a x(n-1) - x(n-2)$ $\Rightarrow$ TI ✓

② $y(n) = n\, x(n)$

$\mathfrak{T} x(n-1) = n x(n-1)$

$S^1\{n x(n)\} = (n-1) x(n-1)$ $\Rightarrow$ TI ↯

③ $y(n) = x(-n)$

$\mathfrak{T} x(n-1) = x(-n+1)$

$S^1\{x(-n)\} = x(-n-1)$ $\Rightarrow$ TI ↯

III Lineáris differenciaegyenletek

① Megoldás lépései

① $\sum_{i=0}^{N} a_i \lambda^{N-i} = 0 \rightarrow \lambda_1 \dots \lambda_N$

② $y_h(n) = \sum_{i=1}^{N} c_i \lambda_i^n$

③ $y_{ip}(n)$ (⇒ kereső fv. behelyettesítése)

④ a kezdeti értékekből iterálva $n \geq \overset{x}{M} - \overset{y}{N} + 1$

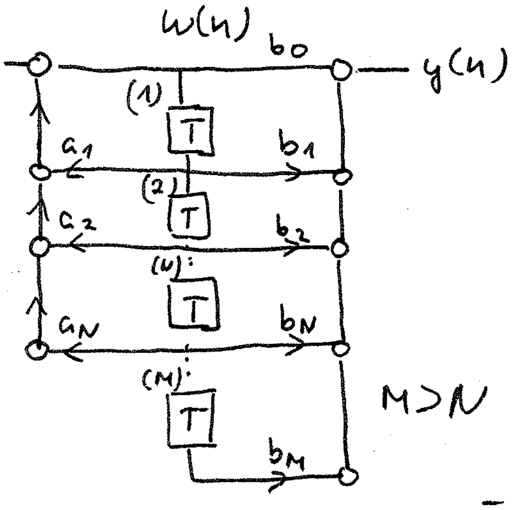
⑤ $\bar{\bar{\Lambda}}\, \bar{c} = \bar{g}$ $y(n) = \sum_{i=1}^{N} c_i \lambda_i^n + y_{ip}(n)$

- multiplikitás, rezonancia!

② y_{ip} alakjai

$g(n)$	$y_{ip}(n)$
$M\, u(n)$	$K\, u(n)$
α^n	$K \alpha^n$
n^M	$K_M + K_{M-1} n + K_{M-2} n^2 + \dots K_0 n^M$
$\alpha^n n^M$	$\alpha^n (K_M + K_{M-1} n + \dots K_0 n^M)$
$\cos \omega_0 n$	$K_1 \cos \omega_0 n + K_2 \sin \omega_0 n$
$\sin \omega_0 n$	$K_1 \cos \omega_0 n + K_2 \sin \omega_0 n$

IV Optimalizálás implementálása



Gyakorlat

V Komplex azonosságok

$$k = r \cdot (\cos\varphi + i\sin\varphi) \; ; \; r = \sqrt{a^2+b^2} \quad \varphi = \begin{cases} \operatorname{arctg}\frac{b}{a} & b \geq 0 \\ \pi + \operatorname{arctg}\frac{b}{a} & b < 0 \end{cases}$$

$$e^{i\varphi} = \cos\varphi + i\sin\varphi$$

$$e^{i\alpha} + e^{-i\alpha} = 2\cos\alpha \; ; \; e^{i\alpha} - e^{-i\alpha} = 2\sin\alpha$$

VI "Z" transzformáltak

$x(n)$	$X(z)$
$\delta(n)$	1
$u(n)$	$\frac{z}{z-1} = \frac{1}{1-z^{-1}}$
$u(n)\,a^n$	$\frac{z}{z-a} = \frac{1}{1-az^{-1}}$
$\delta(n-k)$	z^{-k} — pl. $\delta(n+1) \Rightarrow z$; $\delta(n) \Rightarrow 1$
$u(n)\,n \cdot a^{n-1}$	$\frac{z}{(z-a)^2} = \frac{z^{-1}}{(1-az^{-1})^2}$
$e^{j\beta n}\,u(n)$	$\frac{1}{1-e^{j\beta n}z^{-1}}$
$u(n)\cos\beta n$	$\frac{1-\cos\beta z^{-1}}{1-2\cos z^{-1}\beta + z^{-2}}$

VII Ablakolás

négyszög $w_k(n) = \begin{cases} 1 & \text{ha } \lvert n\rvert \leq k \\ 0 & \text{ha } \lvert n\rvert > k \end{cases}$

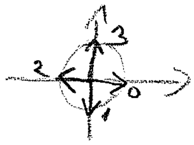
Bartlett $w_B(k) = \begin{cases} 1-\frac{\lvert k\rvert}{M} & \lvert k\rvert \leq M \\ 0 & \lvert k\rvert > M \end{cases}$

Hanning ill. Hamming

$$w_H(k) = \left(a + 2b\cos\frac{2\pi k}{2k+1}\right) w_R(k)$$

Hamming: $a = 0{,}54$; $b = 0{,}46$

Hanning: $a = 0{,}5$; $b = 0{,}5$



VIII DFT számolás mátrixos módszerrel

$W_{kn} = e^{knj\frac{2\pi}{N}}$

pl: $N=4$

$x(n) = [3\ 2\ 1\ 0]$

$$\overline{\overline{W_n}} = \begin{bmatrix} W_4^0 & W_4^0 & W_4^0 & W_4^0 \\ W_4^0 & W_4^1 & W_4^2 & W_4^3 \\ W_4^0 & W_4^2 & W_4^0 & W_4^2 \\ W_4^0 & W_4^3 & W_4^2 & W_4^1 \end{bmatrix} \quad X(k) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 3 \\ 2 \\ 1 \\ 0 \end{bmatrix}$$

IX Szűrőtervezés

$$H(\omega) = \sum_{-\infty}^{\infty} h_d(n) e^{-j\omega n}$$

$$\hookrightarrow \frac{1}{2\pi} \int_{-\pi}^{\pi} H_d(\omega) e^{j\omega n} d\omega$$

FIR transzf fv

$$H(z) = \sum_{n=0}^{N-1} h(n) z^{-n} \quad ; \quad \text{frekv átv fv} \quad H(\omega) = \sum h(n) e^{-jn\omega}$$

$N-1 = 2R$

$$\hookrightarrow H(z) = z^{-R} \sum_{n=0}^{N-1} h(n) z^{-(n-R)} = z^{-R} G(z)$$

$$G(z) = \sum_{n=0}^{N-1} h(n) z^{-(n-R)} = \sum_{k=-R}^{R} h(k+R) z^{-k} = \sum g(k) z^{-k}$$

$$H(\omega) = e^{-j\omega R} G(\omega)$$

$$\hookrightarrow G(\omega) = \sum_{k=-R}^{R} g(k) e^{-jk\omega} \sim H_d(\omega); \; g(k) = \begin{cases} d(k) & |k| \le R \\ 0 & |k| > R \end{cases}$$

Méretezési lépések

1. $H_d(\omega)$ megrajzolása
2. $h_d(k)$ $K = 0, 1 \ldots R$
3. $g(k) = h_d(k)$; $h(n) = g(n-R)$
4. Ell: $|H(\omega)| = |e^{-jk\omega}| |G(\omega)| = |G(\omega)|$
5. Jó-e, ha igen, egyébként
6. realizáció