Pázmány Péter Katolikus Egyetem
Információs Technológia és Bionika Kar

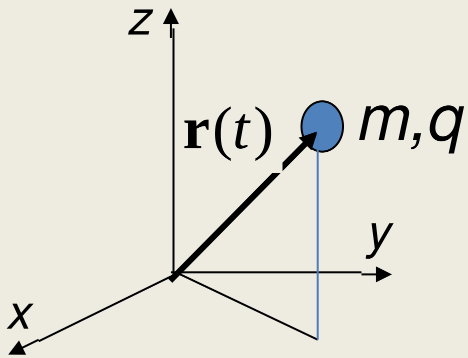
“Fides et Ratio”

Az Információ-technika és a Bionika Fizikája I – 2016 Tavasz – Gyakorlat 2-3

Dr. Csurgay Árpád István

A „tér – idő – test – erő” idealizált modellek

A „tér – idő” Eukleidészi



Descartes $\mathbf{r}(x,y,z)$

Henger $\mathbf{r}(r,\varphi,z)$

Gömbi $\mathbf{r}(r,\vartheta,\varphi)$

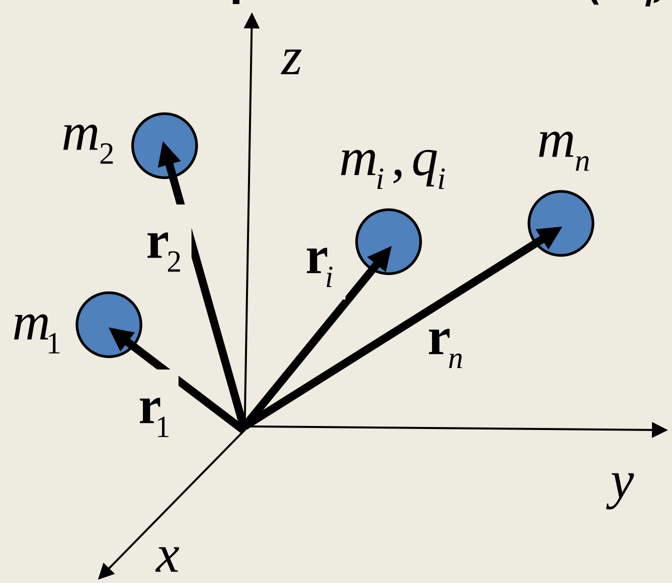
Pontszerű test mozgása

Pálya $\mathbf{r}(t)$

A test

Egy pontszerű test (*m,q*)

Több pontszerű test (*m_i,q_i*)



Merev test mozgása

Tömegközéppont pályája

+

Forgás a tömegközéppont körül

Az erők

$$\mathbf{F}(\mathbf{r}) = m\mathbf{g}(\mathbf{r})$$

$$q\mathbf{E}(\mathbf{r})$$

$$q\left(\mathbf{v}\times\mathbf{B}(\mathbf{r})\right)$$

Erőterek:

$$\mathbf{g}(\mathbf{r});\mathbf{E}(\mathbf{r});\mathbf{B}(\mathbf{r})$$

Vektor terek
Vektor-vektor függvények

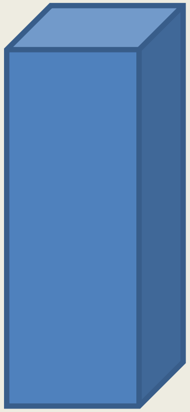
Vektor algebra
Vektoranalízis

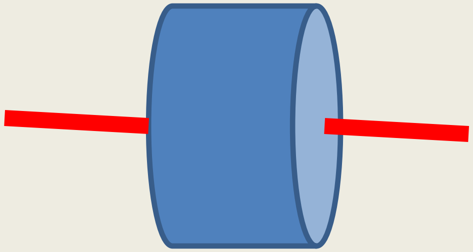
$$\int_L \ldots \mathrm{d}\mathbf{r}; \int_A \ldots \mathrm{d}\mathbf{A}; \int_V \ldots \mathrm{d}V;$$

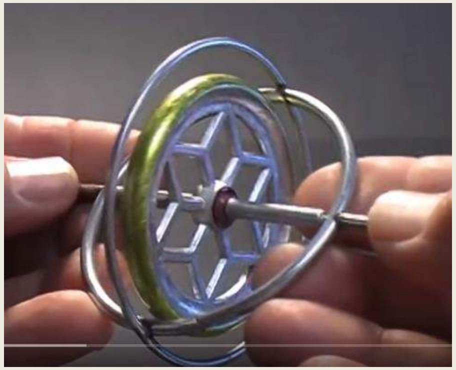
$$\mathrm{grad}\,\varphi(\mathbf{r}) = \nabla\cdot\varphi$$

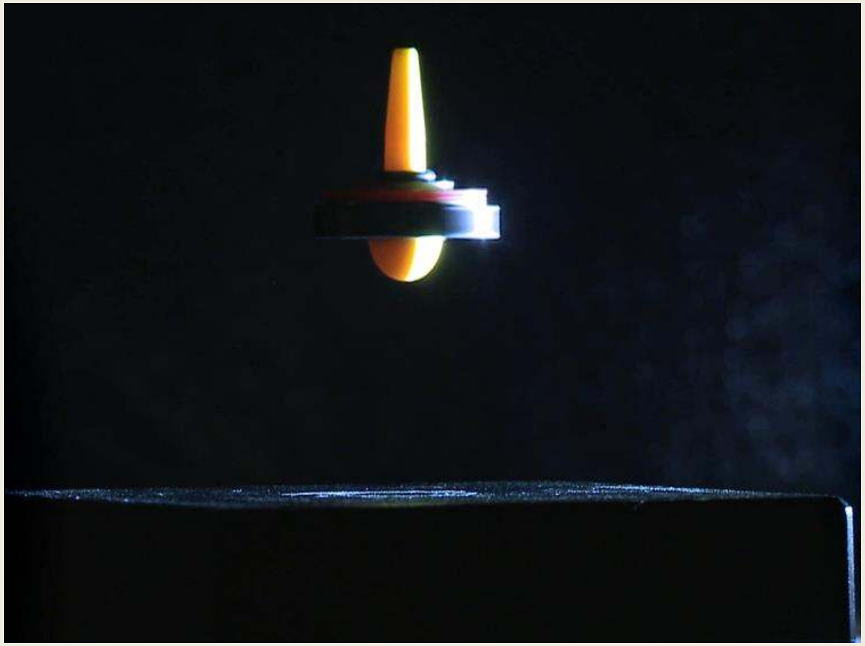
$$\mathrm{div}\,\mathbf{v}(\mathbf{r}) = \nabla\cdot\mathbf{v}$$

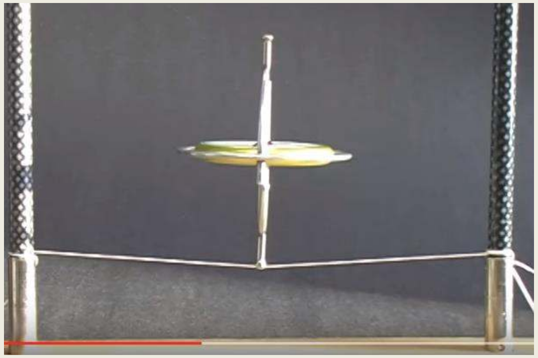
$$\mathrm{rot}\mathbf{v}(\mathbf{r}) = \nabla\times\mathbf{v}$$











KLASSZIKUS MECHANIKA

Konfigurációs tér

$q_1, q_2, q_3, ..., q_f$ Általános koordináták f Szabadságfok

$\dot{q}_1, \dot{q}_2, \dot{q}_3, ..., \dot{q}_f$ Általános sebességek $\dot{q}_i = \mathrm{d}q_i(t) / \mathrm{d}t$

Kinetikus és potenciális energia

$$E_{kin} = E_k(\dot{q}_1, ..., \dot{q}_f, q_1, ..., q_f) \qquad E_{pot} = E_p(q_1, ..., q_f)$$

Lagrange függvény $L = E_{kin} - E_{pot}$

Hamilton elv $\delta \int L \, \mathrm{d}t = 0$

Variációs elv

$$I = \int_{x_1}^{x_2} F \; x, y, y' \; \mathrm{d}x = extremum \Leftrightarrow \quad \frac{\partial F}{\partial y} - \frac{\mathrm{d}}{\mathrm{d}x}\frac{\partial F}{\partial y'} = 0$$

Lagrange mozgásegyenletei

$$\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial L}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i} = 0, \qquad i = 1,\ 2,\ \ldots, f,$$

Általános lendület $p_i = \partial L / \partial \dot{q}_i$

Hamilton függvény $H = \sum_i \dot{q}_i p_i - L$

Hamilton kanonikus mozgásegyenletei

$$\dot{p}_i = -\frac{\partial H\left(p_i, q_i\right)}{\partial q_i}, \qquad \dot{q}_i = \frac{\partial H\left(p_i, q_i\right)}{\partial p_i}. \quad i = 1, 2, \ldots, f$$

Részecske mozgása elektromágneses térben. Lorentz erő

$$\frac{\mathrm{d}\, m\mathbf{v}}{\mathrm{d}t} = q\mathbf{E} + q\mathbf{v} \times \mathbf{B} \qquad \frac{1}{2} m v_1^2 + qU_1 = \frac{1}{2} m v_2^2 + qU_2 .$$

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}; \qquad E_\mathrm{k} = mc^2 - m_0 c^2 .$$

Merev test mozgásának Euler egyenletei

$$\mathbf{F} = \dot{\mathbf{p}} = m\ddot{\mathbf{r}}_0; \qquad \mathbf{M} = \dot{\vec{\ell}} = \hat{\mathbf{I}} \times \dot{\boldsymbol{\omega}} + \boldsymbol{\omega} \times \ \mathbf{I} \cdot \boldsymbol{\omega}$$

$$M_x = I_{xx}\dot{\omega}_x - (I_{yy} - I_{zz})\omega_y\omega_z$$

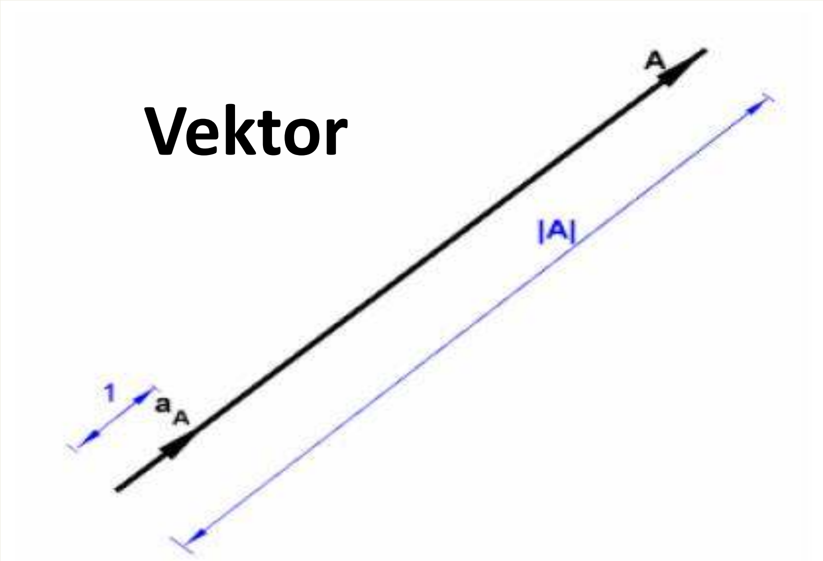
$$M_y = I_{yy}\dot{\omega}_y - (I_{zz} - I_{xx})\omega_x\omega_z$$

$$M_z = I_{zz}\dot{\omega}_z - (I_{xx} - I_{yy})\omega_x\omega_y$$

Vektor algebra

Skalárok (valós, komplex) és Vektorok

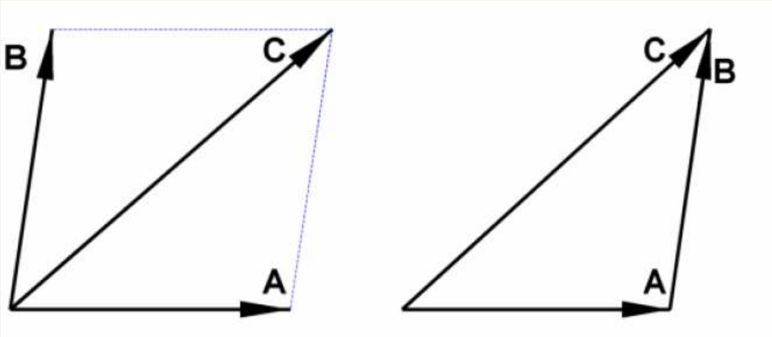
$$\lambda \in R^1; \quad \lambda \in C^1; \qquad \mathbf{A}, \vec{A}$$



Vektor $\mathbf{A}$ $\vec{A}$

Vektor abszolút értéke $|\mathbf{A}|$

Egység vektor $\mathbf{a}_\mathbf{A} = \mathbf{A} / |\mathbf{A}|$



Vektorok összeadása

$$\mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A}$$

$$\mathbf{A} + (\mathbf{B} + \mathbf{C}) = (\mathbf{A} + \mathbf{B}) + \mathbf{C}$$

$$\lambda(\mathbf{A} + \mathbf{B}) = \lambda\mathbf{A} + \lambda\mathbf{B}$$

$$\lambda\mathbf{A} = \lambda|\mathbf{A}|\mathbf{a}_\mathbf{A}$$

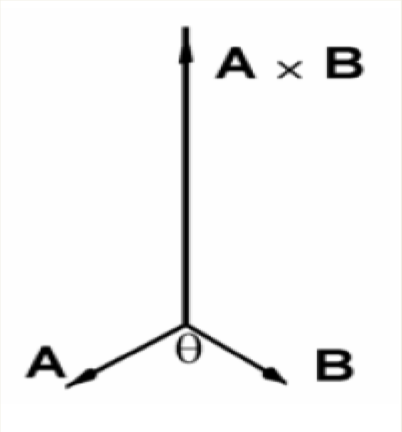
Vektor tér Descartes $\mathbf{A}=A_x\mathbf{a}_x + A_y\mathbf{a}_y + A_z\mathbf{a}_z$

Szokásos jelölés $\mathbf{a}_x \equiv \mathbf{i}; \quad \mathbf{a}_y \equiv \mathbf{j}; \quad \mathbf{a}_z \equiv \mathbf{k}$

Két vektor egynelő $\mathbf{A} = \mathbf{B} \leftrightarrow A_x = B_x; A_y = B_y; A_z = B_z$

Két vektor összege $\mathbf{A}+\mathbf{B} \leftrightarrow (A_x + B_x)\mathbf{a}_x + (A_y + B_y)\mathbf{a}_y + (A_z + B_z)\mathbf{a}_z$

Két vektor skalár szorzata $\mathbf{A}\cdot\mathbf{B} = |\mathbf{A}|\cdot|\mathbf{B}|\cos\theta = A_xB_x + A_yB_y + A_zB_z$

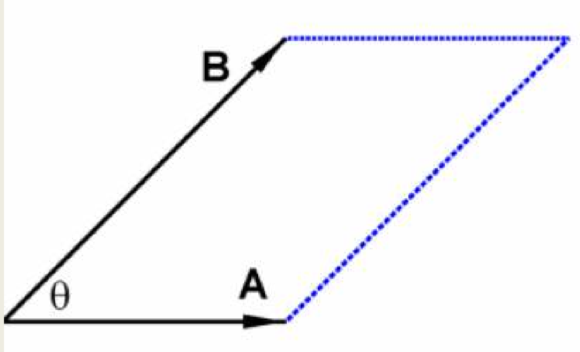


Két vektor vektor szorzata

$$\mathbf{A}\times\mathbf{B} = |\mathbf{A}|\cdot|\mathbf{B}|\sin\theta\cdot\mathbf{a}_{\mathbf{A}\times\mathbf{B}}$$

$$\mathbf{B}\times\mathbf{A} = -\mathbf{A}\times\mathbf{B}$$

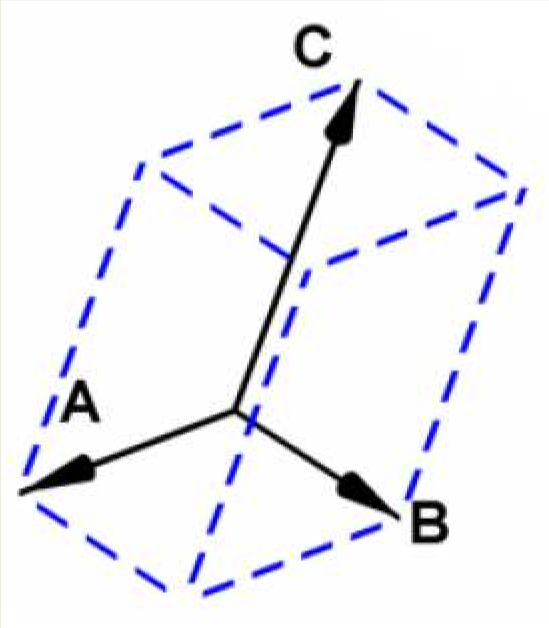
$$\mathbf{A}\times\mathbf{B} = \begin{vmatrix} \mathbf{a}_x & \mathbf{a}_y & \mathbf{a}_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = (A_yB_z - A_zB_y)\mathbf{a}_x + (A_zB_x - A_xB_z)\mathbf{a}_y + (A_xB_y - A_yB_z)\mathbf{a}_z$$



Felület

$$|\mathbf{A}\times\mathbf{B}| = |\mathbf{A}|\cdot|\mathbf{B}|\sin\theta$$

Három vektor vegyes szorzata $\mathbf{A}\cdot(\mathbf{B}\times\mathbf{C}) = \mathbf{B}\cdot(\mathbf{C}\times\mathbf{A}) = \mathbf{C}\cdot(\mathbf{A}\times\mathbf{B})$



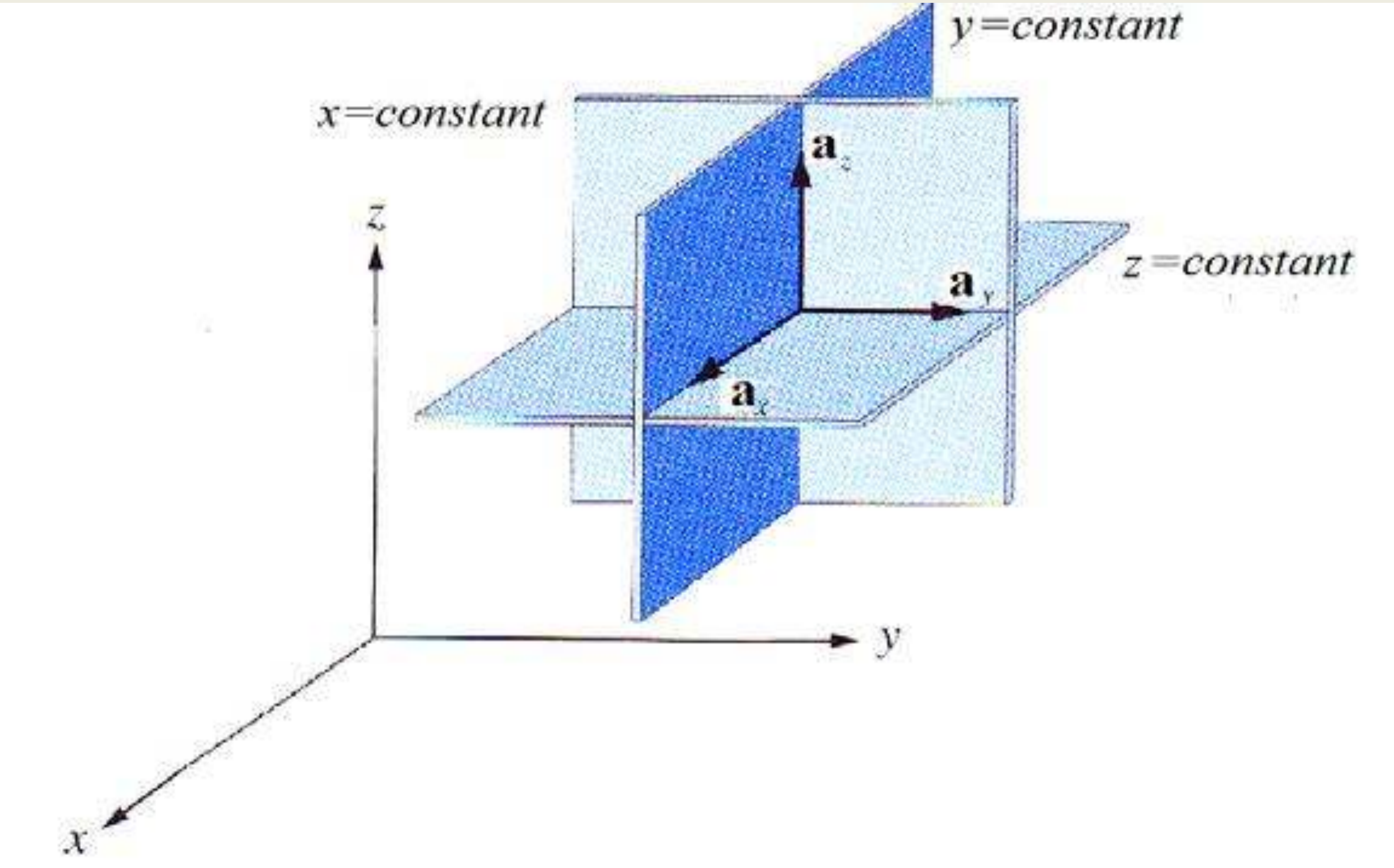
$$\mathbf{A}\times\mathbf{B}\times\mathbf{C} = \begin{vmatrix} A_x & A_y & A_z \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix}$$

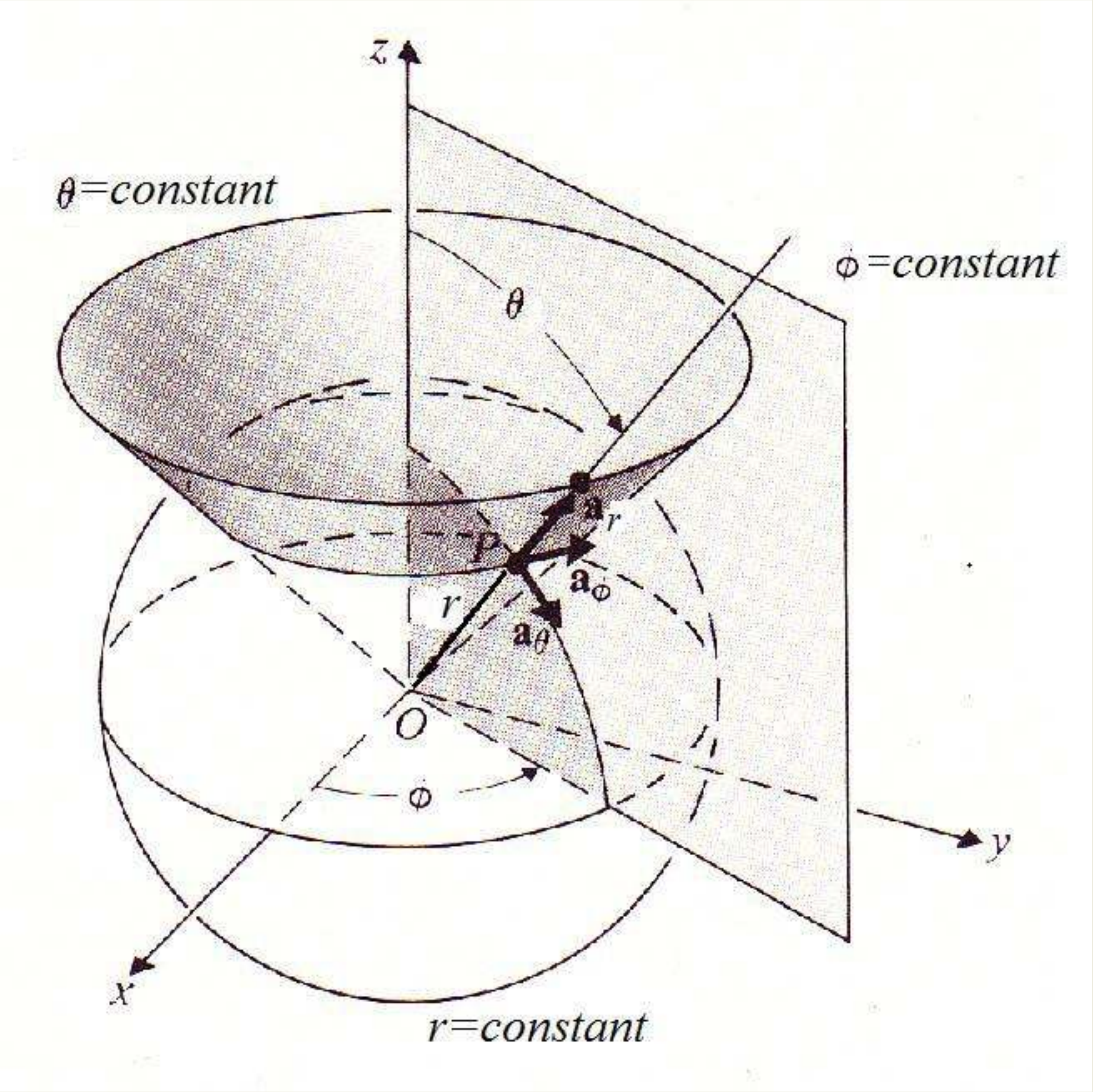
$\mathbf{A},\mathbf{B},\mathbf{C}$, Parallelepipedon térfogata

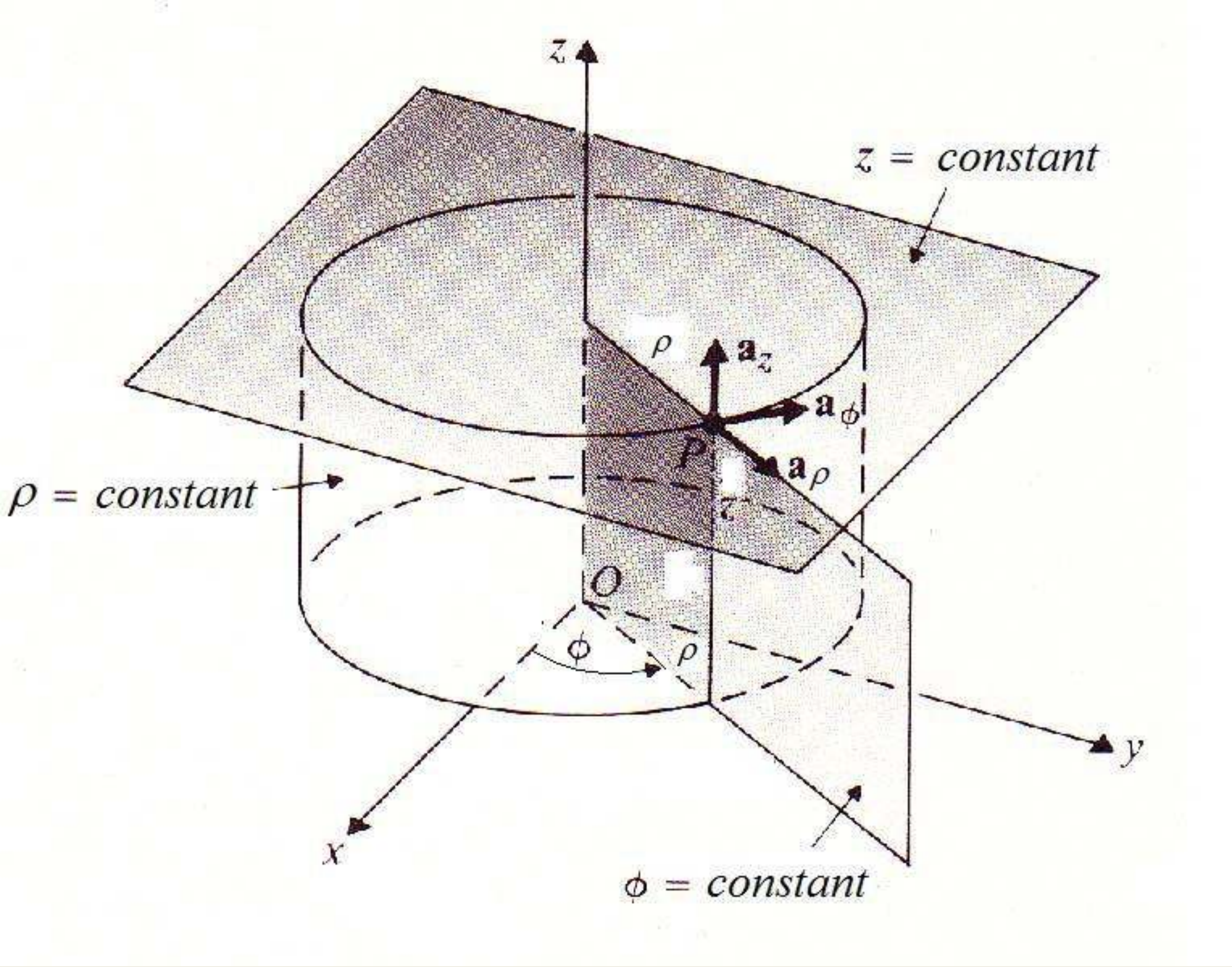
Orthogonalis Koordináta Rendszerek

Descartes, Henger (Cylindrical), Gömbi (Spherical)

(x, y, z) (ρ, φ, z) (r, ϑ, φ)

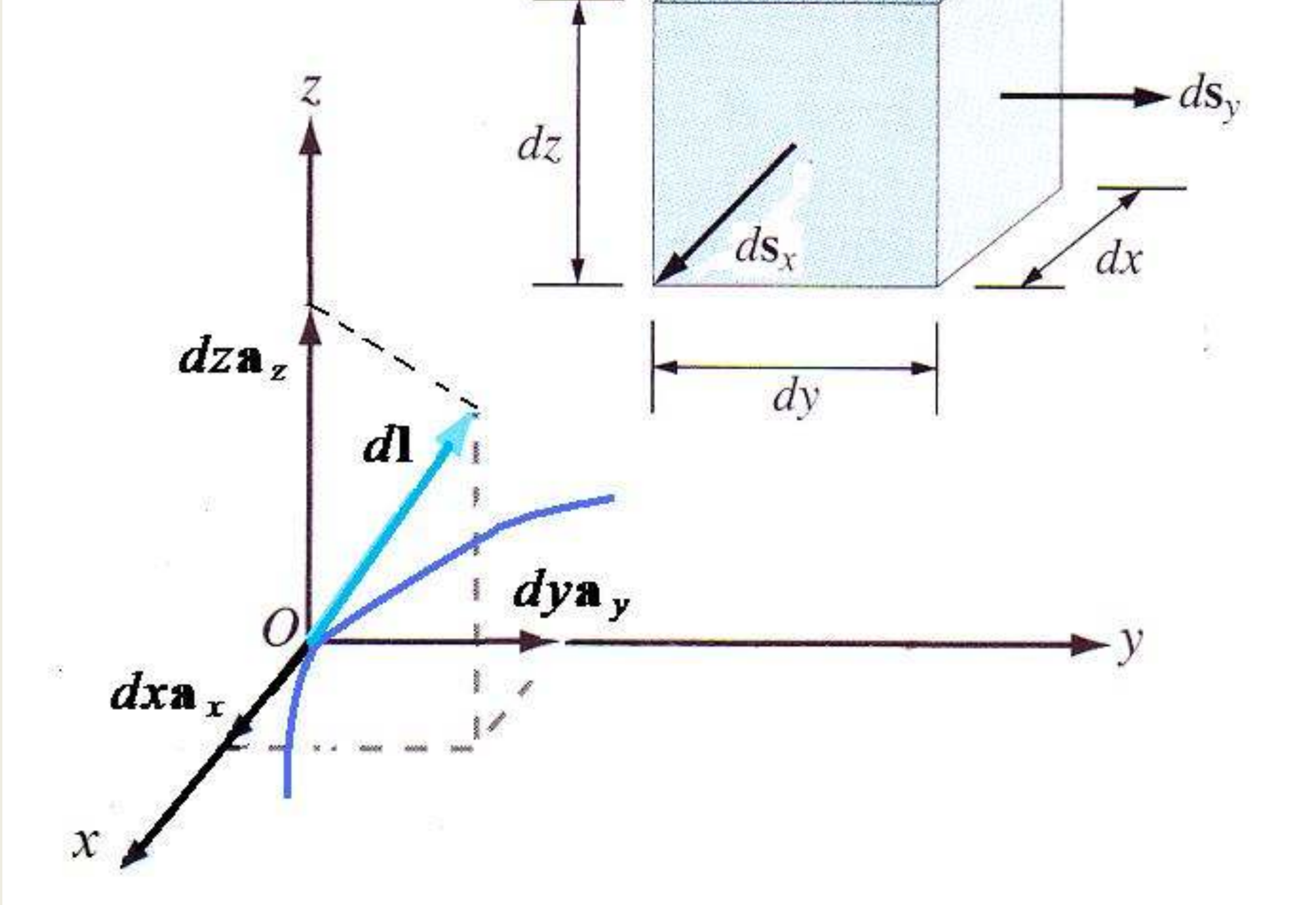



z
θ=constant
φ=constant
θ
P
$\mathbf{a}_r$
$\mathbf{a}_\phi$
$\mathbf{a}_\theta$
r
O
φ
y
x
r=constant


z
z = constant
ρ
$\mathbf{a}_z$
$\mathbf{a}_\phi$
P
$\mathbf{a}_\rho$
ρ = constant
z
O
φ
ρ
y
x
φ = constant

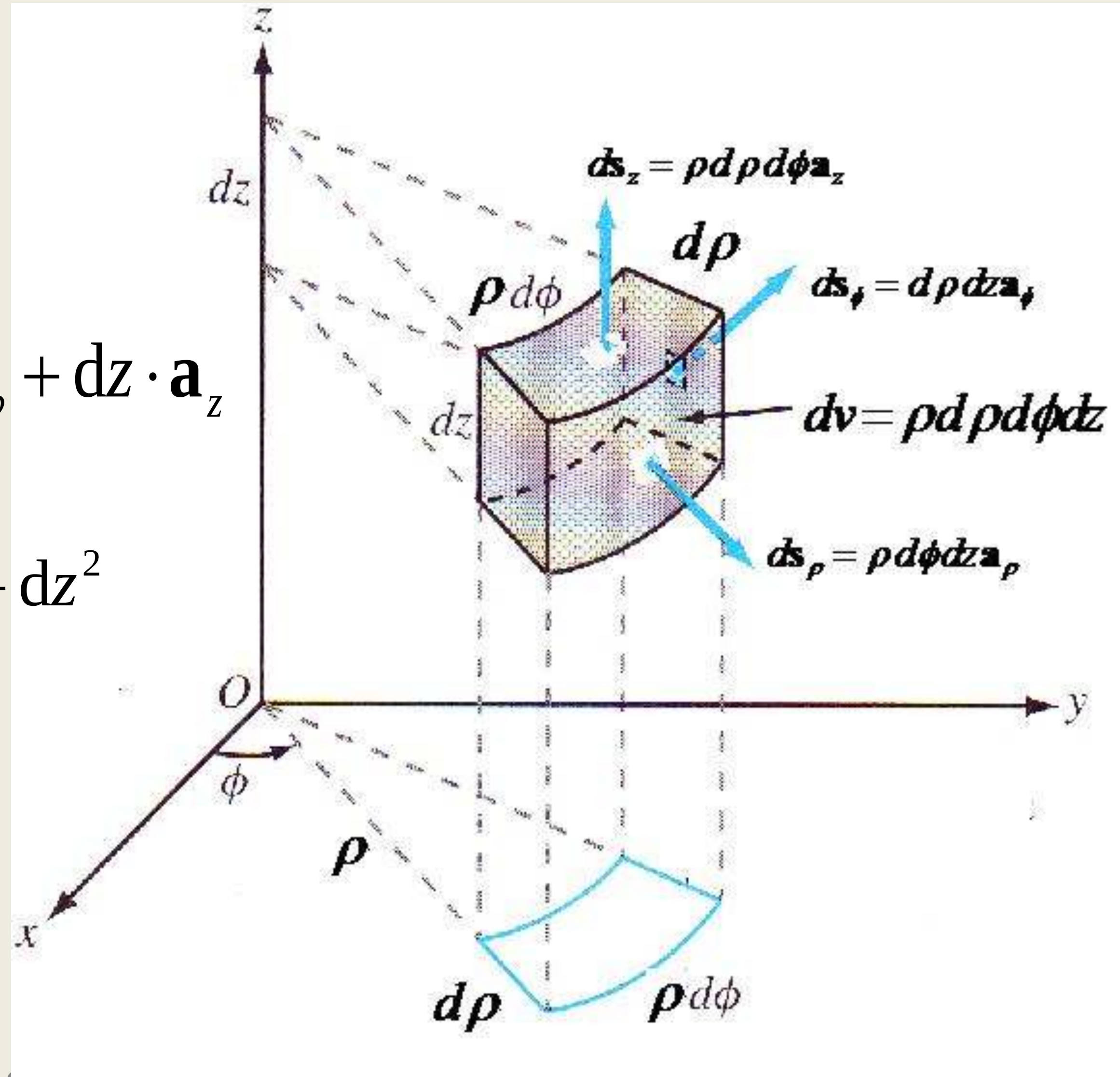
$$\mathrm{d}\mathbf{l}=\mathrm{d}\mathbf{r}=\mathrm{d}x\cdot\mathbf{a}_x+\mathrm{d}y\cdot\mathbf{a}_y+\mathrm{d}z\cdot\mathbf{a}_z$$

$$|\mathrm{d}\mathbf{r}|^2=\mathrm{d}x^2+\mathrm{d}y^2+\mathrm{d}z^2$$



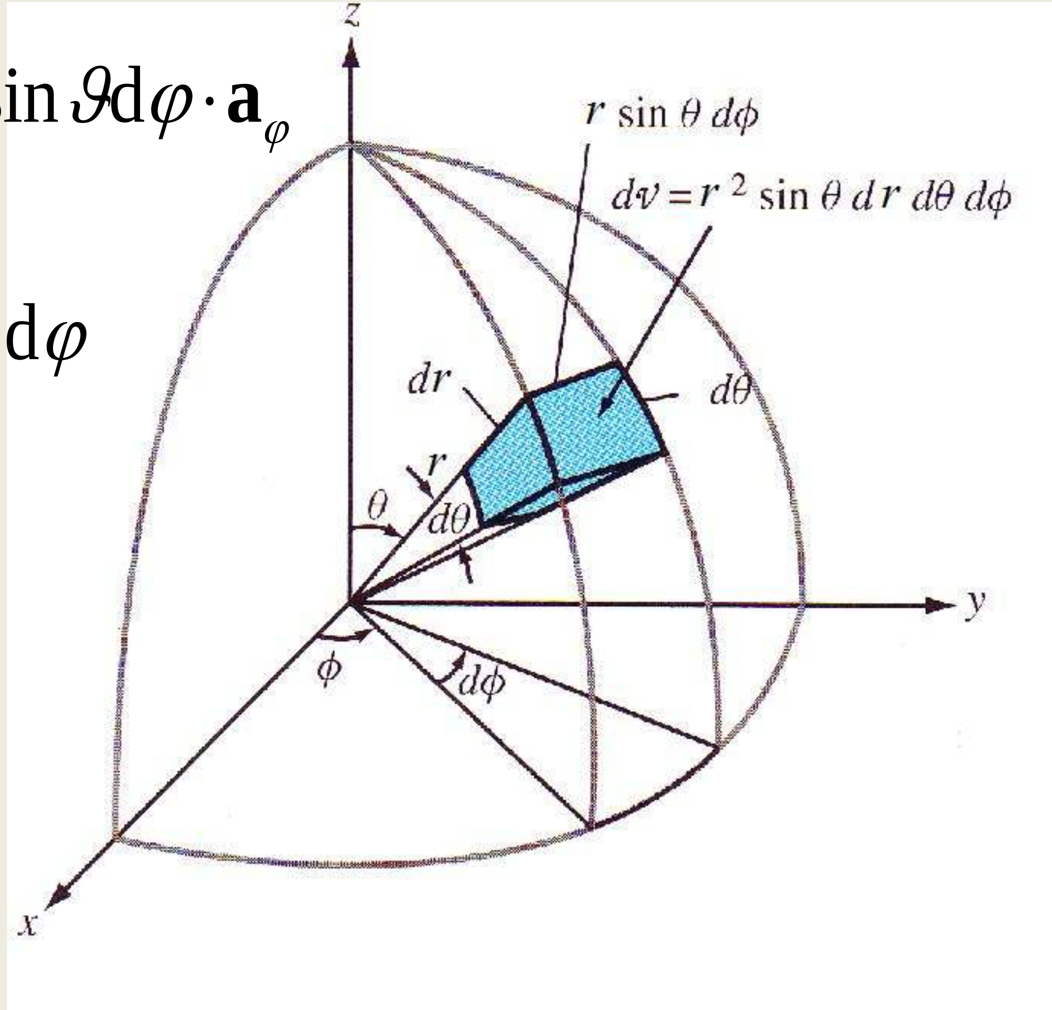
$$\mathrm{d}\mathbf{r} = \mathrm{d}\rho \cdot \mathbf{a}_\rho + \rho \mathrm{d}\varphi \cdot \mathbf{a}_\varphi + \mathrm{dz} \cdot \mathbf{a}_z$$

$$\left|\mathrm{d}\mathbf{r}\right|^2 = \mathrm{d}\rho^2 + \rho^2 \mathrm{d}\varphi^2 + \mathrm{dz}^2$$



$$\mathrm{d}\mathbf{r} = \mathrm{d}r \cdot \mathbf{a}_r + r\mathrm{d}\vartheta \cdot \mathbf{a}_\vartheta + r\sin\vartheta\mathrm{d}\varphi \cdot \mathbf{a}_\varphi$$

$$\mathrm{d}V = r^2 \sin\vartheta \cdot \mathrm{d}r \cdot \mathrm{d}\vartheta \cdot \mathrm{d}\varphi$$



Általános Orthogonalis Koordináta Rendszer

Általában

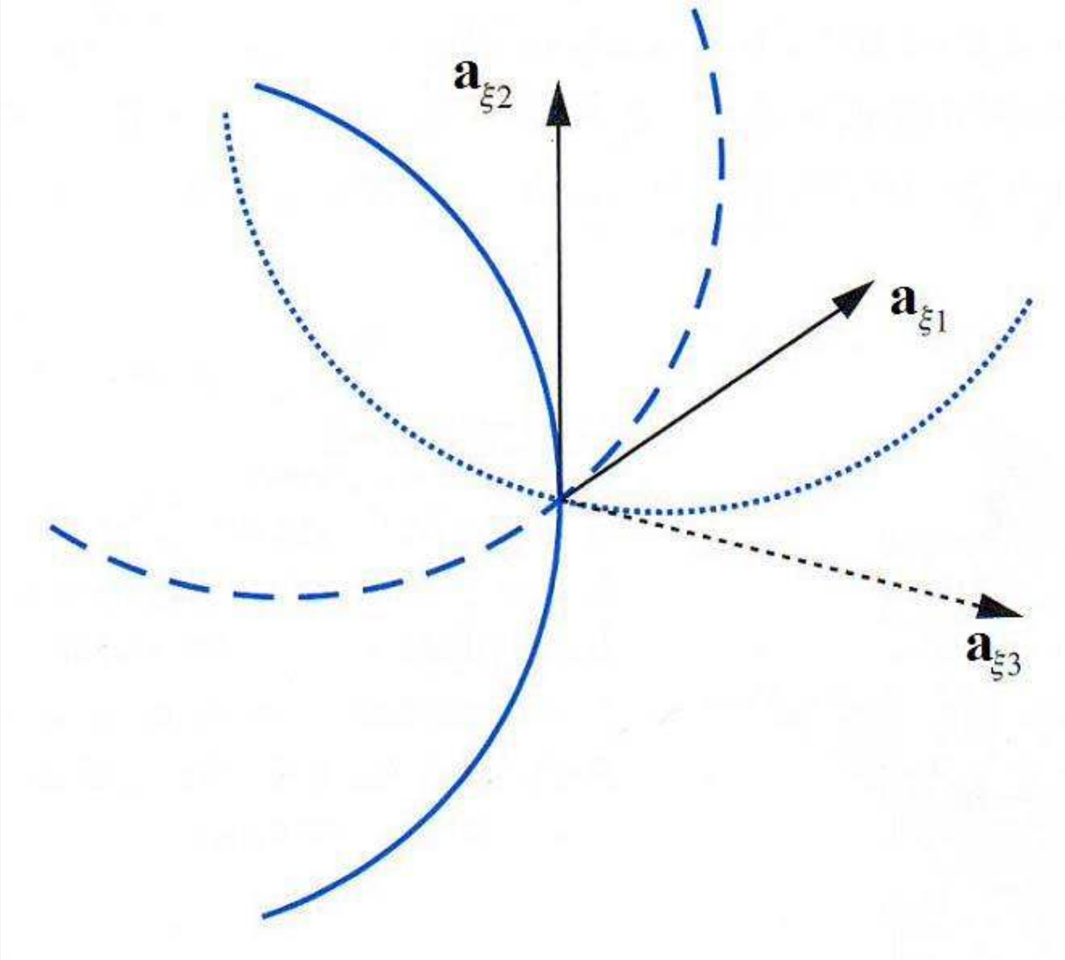
$$\left(x_1, x_2, x_3\right)$$

Három felület

$$x_1 = \text{állandó}$$

$$x_2 = \text{állandó}$$

$$x_3 = \text{állandó}$$



Vektoranalízis – Általános Ortogonális Koordináták

$$\mathbf{r} = x_1\mathbf{a}_1 + x_2\mathbf{a}_2 + x_3\mathbf{a}_3$$

$$\mathbf{a}_1 = \frac{\partial \mathbf{r}}{\partial x_1} / \left|\frac{\partial \mathbf{r}}{\partial x_1}\right|; \qquad \mathbf{a}_2 = \frac{\partial \mathbf{r}}{\partial x_2} / \left|\frac{\partial \mathbf{r}}{\partial x_2}\right|; \qquad \mathbf{a}_3 == \frac{\partial \mathbf{r}}{\partial x_3} / \left|\frac{\partial \mathbf{r}}{\partial x_3}\right|;$$

$$\mathrm{d}\mathbf{r} = g_1 \mathrm{d}x_1 \mathbf{a}_1 + g_2 \mathrm{d}x_2 \mathbf{a}_2 + g_3 \mathrm{d}x_3 \mathbf{a}_3$$

$$\left|\mathrm{d}\mathbf{r}\right|^2 = g_1^2 \mathrm{d}x_1^2 + g_2^2 \mathrm{d}x_2^2 + g_3^2 \mathrm{d}x_3^2$$

$$\psi(\mathbf{r}) = \psi(x_1, x_2, x_3)$$

$$\mathbf{v}(\mathbf{r}) = v_1\mathbf{a}_1 + v_2\mathbf{a}_2 + v_3\mathbf{a}_3;$$

$$v_i(x_1, x_2, x_3) \quad i = 1,2,3$$

	g_1	g_2	g_3
Descartes	1	1	1
Cylindical	1	ρ	1
Spherical	1	r	$r\sin\vartheta$

$$\operatorname{grad}\psi(\mathbf{r}) = \nabla\psi = \frac{1}{g_1}\frac{\partial\psi}{\partial x_1}\mathbf{a}_1 + \frac{1}{g_2}\frac{\partial\psi}{\partial x_2}\mathbf{a}_2 + \frac{1}{g_3}\frac{\partial\psi}{\partial x_3}\mathbf{a}_3$$

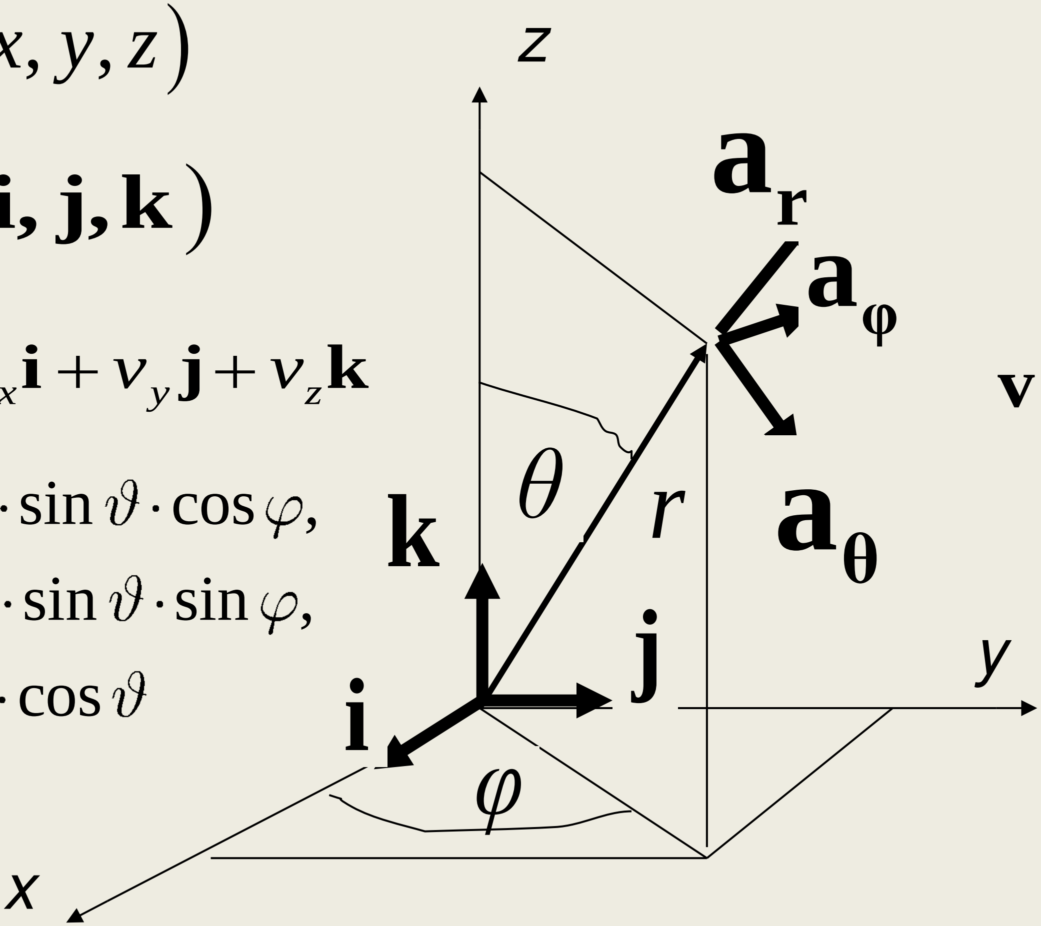
$$\operatorname{div}\mathbf{v}(\mathbf{r}) = \nabla\cdot\mathbf{v} = \frac{1}{g_1 g_2 g_3}\left[\frac{\partial g_2 g_3}{\partial x_1}v_1 + \frac{\partial g_1 g_3}{\partial x_2}v_2 + \frac{\partial g_1 g_2}{\partial x_3}v_3\right]$$

$$\operatorname{rot}\mathbf{v}(\mathbf{r}) = \begin{vmatrix} \dfrac{\mathbf{a}_1}{g_2 g_3} & \dfrac{\mathbf{a}_2}{g_1 g_3} & \dfrac{\mathbf{a}_3}{g_1 g_2} \\ \dfrac{\partial}{\partial x_1} & \dfrac{\partial}{\partial x_2} & \dfrac{\partial}{\partial x_3} \\ g_1 v_1 & g_2 v_2 & g_3 v_3 \end{vmatrix}$$

Descartes és gömbi koordináták

$$\left(x, y, z\right) \qquad \left(r, \vartheta, \varphi\right)$$

$$\left(\mathbf{i}, \mathbf{j}, \mathbf{k}\right) \qquad \left(\mathbf{a}_{\mathbf{r}}, \mathbf{a}_{\vartheta}, \mathbf{a}_{\boldsymbol{\varphi}}\right)$$



$$\mathbf{v} = v_x\mathbf{i} + v_y\mathbf{j} + v_z\mathbf{k}$$

$$\mathbf{v} = v_r\mathbf{a}_r + v_\vartheta\mathbf{a}_\vartheta + v_\varphi\mathbf{a}_\varphi$$

$$x = r\cdot\sin\vartheta\cdot\cos\varphi,$$
$$y = r\cdot\sin\vartheta\cdot\sin\varphi,$$
$$z = r\cdot\cos\vartheta$$

$$r = \sqrt{x^2 + y^2 + z^2},$$
$$\vartheta = \arccos\frac{z}{\sqrt{x^2 + y^2 + z^2}},$$
$$\varphi = \mathrm{arctg}\,\frac{y}{x}$$

$$\mathrm{d}\mathbf{r} = \mathrm{d}x\cdot\mathbf{i} + \mathrm{d}y\cdot\mathbf{j} + \mathrm{d}z\cdot\mathbf{k}$$

$$\mathrm{d}\mathbf{r} = \mathrm{d}r\cdot\mathbf{a}_r + r\mathrm{d}\vartheta\cdot\mathbf{a}_\vartheta + r\sin\vartheta\mathrm{d}\varphi\cdot\mathbf{a}_\varphi$$

$$\left|\mathrm{d}\mathbf{r}\right| = \sqrt{(\mathrm{d}x)^2 + (\mathrm{d}y)^2 + (\mathrm{d}z)^2}$$

$$\left|\mathrm{d}\mathbf{r}\right| = \sqrt{(\mathrm{d}r)^2 + (r\mathrm{d}\vartheta)^2 + (r\sin\theta\mathrm{d}\varphi)^2}$$

	Descartes Derékszögű	Cylindrical Henger	Spherical Gömbi
Változók	x, y, z	r, φ, z	r, ϑ, φ
Vektor komp.	v_x, v_y, v_z	v_r, v_φ, v_z	$v_r, v_\vartheta, v_\varphi$
Egység vektorok	$\mathbf{i}, \mathbf{j}, \mathbf{k}$	$\mathbf{a}_r, \mathbf{a}_\varphi, \mathbf{a}_z$	$\mathbf{a}_r, \mathbf{a}_\vartheta, \mathbf{a}_\varphi$
Metrika g_1, g_2, g_3	$1,1,1$	$1, r, 1$	$1, r, r\sin\vartheta$

$$\mathrm{grad}\,\psi(\mathbf{r}) = \nabla\psi = \frac{\partial\psi}{\partial x}\mathbf{i} + \frac{\partial\psi}{\partial y}\mathbf{j} + \frac{\partial\psi}{\partial z}\mathbf{k} \qquad \text{Descartes}$$

$$\mathrm{grad}\,\psi = \frac{\partial\psi}{\partial r}\mathbf{a}_r + \frac{1}{r}\frac{\partial\psi}{\partial\varphi}\mathbf{a}_\varphi + \frac{\partial\varphi}{\partial z}\mathbf{a}_z \qquad \text{Cylindrical}$$

$$\mathrm{grad}\,\varphi = \frac{\partial\psi}{\partial r}\mathbf{a}_r + \frac{1}{r}\frac{\partial\psi}{\partial\vartheta}\mathbf{a}_\vartheta + \frac{1}{r\sin\vartheta}\frac{\partial\psi}{\partial\varphi}\mathbf{a}_\varphi \qquad \text{Spherical}$$

$$\mathrm{div}\mathbf{v}(\mathbf{r}) = \nabla \cdot \mathbf{v} = \frac{1}{g_1 g_2 g_3}\left[\frac{\partial g_2 g_3}{\partial x_1} v_1 + \frac{\partial g_1 g_3}{\partial x_2} v_2 + \frac{\partial g_1 g_2}{\partial x_3} v_3\right]$$

$$\nabla \cdot \mathbf{v} = \left[\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}\right]$$ Descartes

$$\nabla \cdot \mathbf{v} = \left[\frac{1}{r}\frac{\partial r v_r}{\partial r} + \frac{1}{r}\frac{\partial v_\varphi}{\partial \varphi} + \frac{\partial v_z}{\partial z}\right]$$ Cylindrical Henger

$$\nabla \cdot \mathbf{v} = \left[\frac{1}{r^2}\frac{\partial r^2 v_r}{\partial r} + \frac{1}{r\sin\vartheta}\frac{\partial \sin\vartheta \cdot v_\vartheta}{\partial \vartheta} + \frac{1}{r\sin\vartheta}\frac{\partial v_\varphi}{\partial \varphi}\right]$$ Spherical Gömbi

$$\mathrm{rot}\,\mathbf{v}(\mathbf{r}) = \nabla \times \mathbf{v} = \begin{vmatrix} \dfrac{\mathbf{a}_1}{g_2 g_3} & \dfrac{\mathbf{a}_2}{g_1 g_3} & \dfrac{\mathbf{a}_3}{g_1 g_2} \\ \dfrac{\partial}{\partial x_1} & \dfrac{\partial}{\partial x_2} & \dfrac{\partial}{\partial x_3} \\ g_1 v_1 & g_2 v_2 & g_3 v_3 \end{vmatrix}$$

Descartes

$$\nabla \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ v_x & v_y & v_z \end{vmatrix}$$

Cylindrical

$$\mathrm{rot}\,\mathbf{v}(\mathbf{r}) = \nabla \times \mathbf{v} = \left(\frac{1}{r}\frac{\partial v_z}{\partial \varphi} - \frac{\partial v_\varphi}{\partial z} \right)\mathbf{a}_r + \left(\frac{\partial v_r}{\partial z} - \frac{\partial v_z}{\partial r} \right)\mathbf{a}_\varphi + \frac{1}{r}\left(\frac{\partial r v_\varphi}{\partial r} - \frac{\partial v_r}{\partial \varphi} \right)\mathbf{a}_z$$

Spherical

$$\nabla \times \mathbf{v} = \frac{1}{r \sin\vartheta}\left(\frac{\partial \sin\vartheta \cdot v_\varphi}{\partial \vartheta} - \frac{\partial v_\vartheta}{\partial \varphi} \right)\mathbf{a}_r + \frac{1}{r}\left(\frac{1}{\sin\vartheta}\frac{\partial v_r}{\partial \varphi} - \frac{\partial r v_\varphi}{\partial r} \right)\mathbf{a}_\vartheta + \frac{1}{r}\left(\frac{\partial r v_\vartheta}{\partial r} - \frac{\partial v_r}{\partial \vartheta} \right)\mathbf{a}_\varphi$$

Az Elektromágneses Erőtér

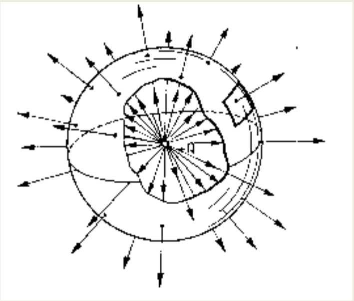
$\mathbf{E}^{\mathrm{V/m}}, \quad \mathbf{D}^{\mathrm{As/m^2}} \quad \mathbf{B}^{\mathrm{Vs/m^2}} \quad \mathbf{H}^{\mathrm{A/m}}. \qquad \mathbf{E}(\mathbf{r},t), \mathbf{D}(\mathbf{r},t), \mathbf{B}(\mathbf{r},t), \mathbf{H}(\mathbf{r},t)$

Vákuumban $\quad \mathbf{D} = \varepsilon_0 \cdot \mathbf{E}, \qquad \mathbf{B} = \mu_0 \cdot \mathbf{H}.$

$\varepsilon_0 = 8.854 \cdot 10^{-12}\ \mathrm{As/Vm} \qquad \mu_0 = 1.256 \cdot 10^{-6}\ \mathrm{Vs/Am}$

„Egyszerű" anyagokban $\quad \mathbf{D} = \varepsilon \cdot \mathbf{E}, \qquad \mathbf{B} = \mu \cdot \mathbf{H}. \qquad \mathbf{J} = \sigma\left(\mathbf{E} + \mathbf{E}_{gen}\right)$

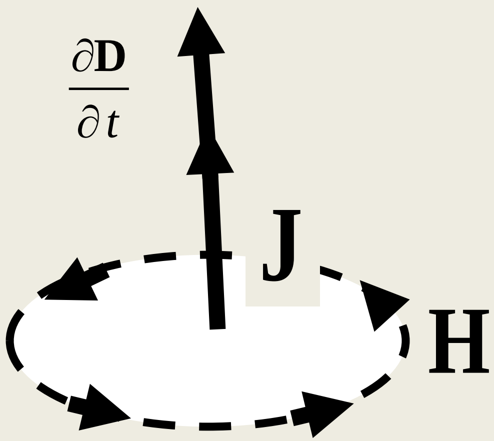
A $\mathbf{D}$ erővonalak („Eltolási vektor") „forrásai" az elektromos töltések



$$\oint_A \mathbf{D} \cdot \mathrm{d}\mathbf{A} = \sum_i Q_i = \int_V \rho \cdot \mathrm{d}V \rightarrow \mathrm{div}\mathbf{D} = \nabla \cdot \mathbf{D} = \rho$$

A $\mathbf{B}$ mágneses indukció vektornak nincsenek forrásai:

$$\oint_A \mathbf{B} \cdot \mathrm{d}\,\mathbf{A} = 0 \rightarrow \mathrm{div}\mathbf{B} = \nabla \cdot \mathbf{B} = 0$$



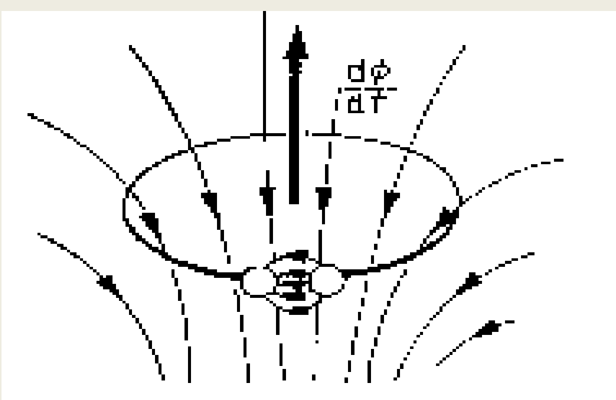
Az áramsűrűség $\mathbf{J}$ és az eltolási áram $\frac{\partial \mathbf{D}}{\partial t}$

maga körül mágneses örvényeket gerjeszt

$$\oint_L \mathbf{H} \cdot \mathrm{d}\,\mathbf{l} = \int_A \left(\mathbf{J} + \frac{\partial \mathbf{D}}{\partial t} \right) \cdot \mathrm{d}\,\mathbf{A} \rightarrow \mathrm{rot}\mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$$

Ampere törvénye

Az időben változó mágneses tér (mágneses indukció)
maga körül az elektromos térerősség örvényeit generálja



$$\oint_L \mathbf{E} \cdot \mathrm{d}\,\mathbf{l} = -\frac{\partial}{\partial t} \int_A \mathbf{B} \cdot \mathrm{d}\,\mathbf{A} \rightarrow \mathrm{rot}\mathbf{E} = \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

Faraday törvénye

Maxwell egyenletek

Integrál

I. Ampere törvény

$$\oint_L \mathbf{H}\cdot d\mathbf{l} = \int_A \mathbf{J}\cdot d\mathbf{A} + \frac{\partial}{\partial t}\int_A \mathbf{D}\cdot d\mathbf{A}$$

II. Faraday törvény

$$\oint_L \mathbf{E}\cdot d\mathbf{l} = -\frac{\partial}{\partial t}\int_A \mathbf{B}\cdot d\mathbf{A}$$

III. Az elektromos tér Gauss törvénye

$$\oint_A \mathbf{D}\cdot d\mathbf{A} = \int_V \rho\cdot dV$$

IV. A mágneses tér Gauss törvénye

$$\oint_A \mathbf{B}\cdot d\mathbf{A} = 0$$

V. Anyagok (Egyszerű)

$$\mathbf{D} = \varepsilon\cdot\mathbf{E}$$

$$\mathbf{B} = \mu\cdot\mathbf{H}$$

$$\mathbf{J} = \gamma\cdot\left(\mathbf{E}+\mathbf{E}_{gen}\right)$$

VI. Energia sűrűség

$$w = \frac{1}{2}\varepsilon\mathbf{E}^2 + \frac{1}{2}\mu\mathbf{H}^2$$

Lorentz erő

$$\mathbf{F} = q\cdot\mathbf{E} + q\cdot\left(\mathbf{v}\times\mathbf{B}\right)$$

Lokális

$$\nabla\times\mathbf{H} = \mathbf{J} + \frac{\partial\mathbf{D}}{\partial t}$$

$$\nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t}$$

$$\nabla\cdot\mathbf{D} = \rho$$

$$\nabla\cdot\mathbf{B} = 0$$

$$\mathbf{D} = \varepsilon\cdot\mathbf{E}$$

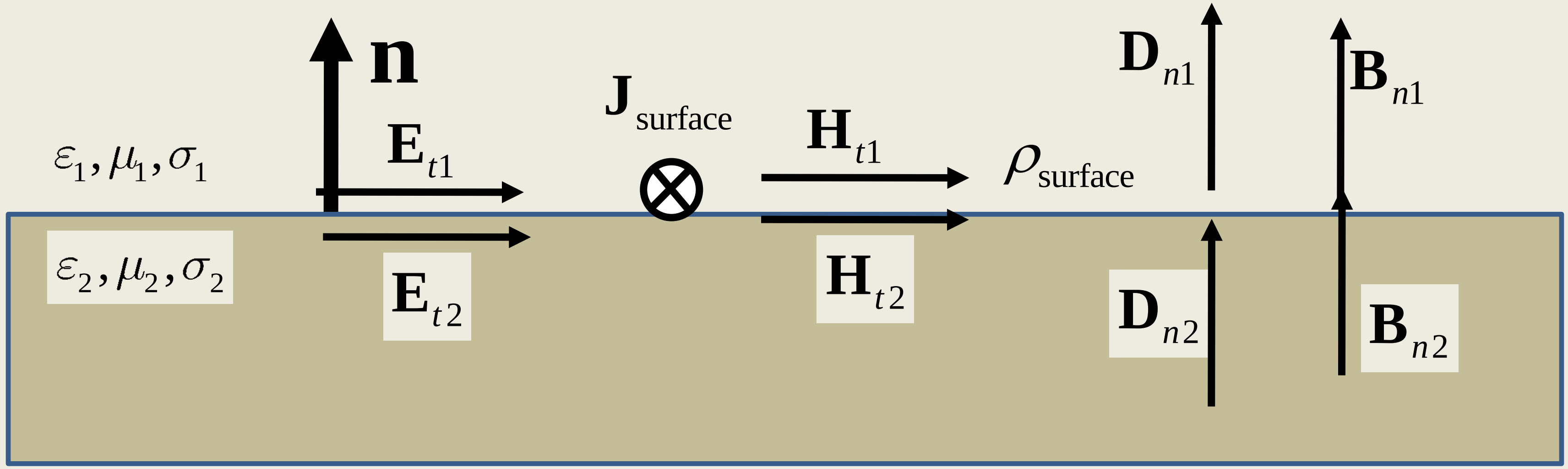
$$\mathbf{B} = \mu\cdot\mathbf{H}$$

$$\mathbf{J} = \gamma\cdot\left(\mathbf{E}+\mathbf{E}_{gen}\right)$$

$$w = \frac{1}{2}\varepsilon\mathbf{E}^2 + \frac{1}{2}\mu\mathbf{H}^2$$

$$\mathbf{F} = q\cdot\mathbf{E} + q\cdot\left(\mathbf{v}\times\mathbf{B}\right)$$

A térerősségek két „egyszerű” közeg határoló felületén



$$\mathbf{n}\times(\mathbf{E}_1-\mathbf{E}_2)=0$$

$$\mathbf{n}\cdot(\mathbf{B}_1-\mathbf{B}_2)=0$$

$$\mathbf{n}\times(\mathbf{H}_1-\mathbf{H}_2)=\mathbf{J}_{\text{surface}}$$

$$\mathbf{n}\cdot(\mathbf{D}_1-\mathbf{D}_2)=\rho_{surface}$$

Az **E** tangenciális komponense folytonosan megy át a felületen.

Idő tartomány

$(\mathbf{r},t) \Leftrightarrow \mathbf{r},\omega$

Frekvencia tartomány

$\mathbf{E}(\mathbf{r},t), \mathbf{D}(\mathbf{r},t), \mathbf{H}(\mathbf{r},t), \mathbf{B}(\mathbf{r},t),$
$\rho(\mathbf{r},t), \mathbf{J}(\mathbf{r},t)$

$$\nabla\times\mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$$

$$\nabla\times\mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla\cdot\mathbf{B} = 0$$

$$\nabla\cdot\mathbf{D} = \rho$$

$\mathbf{D} = \varepsilon\cdot\mathbf{E}$ $\quad$ $\mathbf{B} = \mu\cdot\mathbf{H}$

$$\mathbf{J} = \sigma(\mathbf{E} + \mathbf{E}_{gen})$$

$$\mathbf{E}(\mathbf{r},t) = \mathrm{Re}\left[\mathbf{E}(\mathbf{r})\cdot e^{j\omega t}\right]$$

$$e^{j\omega t}$$

$$\mathbf{E}(\mathbf{r},t) \Leftrightarrow \mathbf{E}(\mathbf{r})$$

„Komplex amplitúdók világa”

$\mathbf{E}(\mathbf{r}), \mathbf{D}(\mathbf{r}), \mathbf{H}(\mathbf{r}), \mathbf{B}(\mathbf{r}),$
$\rho(\mathbf{r}), \mathbf{J}(\mathbf{r})$

$$\nabla\times\mathbf{H} = \mathbf{J} + j\omega\mathbf{D}$$

$$\nabla\times\mathbf{E} = -j\omega\mathbf{B}$$

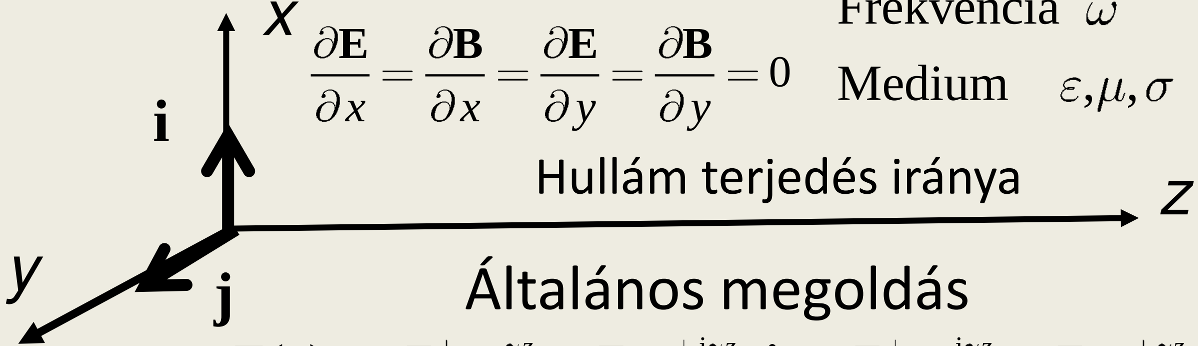
$$\nabla\cdot\mathbf{B} = 0$$

$$\nabla\cdot\mathbf{D} = \rho$$

$\mathbf{D} = (\varepsilon)\cdot\mathbf{E}$ $\quad$ $\mathbf{B} = \mu\cdot\mathbf{H}$

$$\mathbf{J} = \sigma(\mathbf{E} + \mathbf{E}_{\mathrm{gen}})$$

Síkhullámok veszteséges közegben



$$\frac{\partial \mathbf{E}}{\partial x}=\frac{\partial \mathbf{B}}{\partial x}=\frac{\partial \mathbf{E}}{\partial y}=\frac{\partial \mathbf{B}}{\partial y}=0$$

Frekvencia ω

Medium ε,μ,σ

$$\gamma=\alpha+\mathrm{j}\beta=\mathrm{j}\omega\sqrt{\mu\left(\varepsilon-\mathrm{j}\frac{\sigma}{\omega}\right)}$$

$$\eta=\sqrt{\mu/\left(\varepsilon-\mathrm{j}\frac{\sigma}{\omega}\right)}$$

Általános megoldás

$$\mathbf{E}(z)=\;E_x^+e^{-\gamma z}+E_x^-e^{+\mathrm{j}\gamma z}\;\mathbf{i}+\;E_y^+e^{-\mathrm{j}\gamma z}+E_y^-e^{+\gamma z}\;\mathbf{j}$$

Komplex amplitúdók

$E_x^+, E_x^-, E_y^+, E_y^-$

$$\mathbf{H}(z)=\left(\frac{E_x^+}{\eta}e^{-\gamma z}-\frac{E_x^-}{\eta}e^{+\gamma z}\right)\mathbf{j}+\left(\frac{E_y^+}{\eta}e^{-\mathrm{j}\gamma z}-\frac{E_y^-}{\eta}e^{+\mathrm{j}\gamma z}\right)\mathbf{i}$$

$$\mathbf{E}(z,t)=\mathrm{Re}\;\;\mathbf{E}(z)\cdot e^{\mathrm{j}\omega t} \qquad \mathbf{H}(z,t)=\mathrm{Re}\;\;\mathbf{H}(z)\cdot e^{\mathrm{j}\omega t}$$

Síkhullám veszteséges közegben

Lineáris polarizáció $\mathbf{i}$ Síkhullám +z irányban terjed

$$\mathbf{E}\;\;z\;\;=E_x^+e^{-\alpha z}e^{-\mathrm{j}\beta z}\;\mathbf{i}\rightarrow$$

$$\mathbf{H}\;\;z\;\;=\frac{E_x^+}{\eta}e^{-\gamma z}\;\mathbf{j}\rightarrow$$

$$\mathbf{E}\;\;z,t\;\;=\left|E_x^+\right|e^{-\alpha z}\cos\;\;\omega t-\beta z+\varphi_x^+\;\;\mathbf{i}$$

$$\mathbf{H}\;\;z,t\;\;=\frac{\left|E_x^+\right|}{\left|\eta\right|}e^{-\alpha z}\cos\;\;\omega t-\beta z+\varphi_x^+-\varphi_\eta\;\;\mathbf{j}$$

Síkhullám áthaladása két közeget határoló felületen

Merőleges beesés

$$T = \frac{E_{m2}^{+}}{E_{m1}^{+}} = \frac{2\eta_2}{\eta_1 + \eta_2}; \quad \Gamma = \frac{E_{m1}^{-}}{E_{m1}^{+}} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}; \quad 1 + \Gamma = T$$

Ferde beesés – Reflexió és Transzmisszió

$$\theta_r = \theta_i; \quad \frac{\sin\theta_t}{\sin\theta_i} = \frac{\beta_1}{\beta_2} = \frac{\sqrt{\varepsilon_1\mu_1}}{\sqrt{\varepsilon_2\mu_2}}; \quad \eta_1 = \sqrt{\frac{\mu_1}{\varepsilon_1}}; \quad \eta_2 = \sqrt{\frac{\mu_2}{\varepsilon_2}}$$

Fresnel egyenletek

$$\Gamma_{\parallel} = \frac{\hat{E}_{\parallel m}^{r}}{\hat{E}_{\parallel m}^{i}} = \frac{\eta_2\cos\theta_t - \eta_1\cos\theta_i}{\eta_1\cos\theta_i + \eta_2\cos\theta_t}; \quad T_{\parallel} = \frac{\hat{E}_{\parallel m}^{t}}{\hat{E}_{\parallel m}^{i}} = \frac{2\eta_2\cdot\cos\theta_i}{\eta_1\cdot\cos\theta_i + \eta_2\cdot\cos\theta_t}$$

$$\Gamma_{\perp} = \frac{\hat{E}_{\perp m}^{r}}{\hat{E}_{\perp m}^{i}} = \left(\frac{\eta_2\cos\theta_i - \eta_1\cos\theta_t}{\eta_2\cos\theta_i + \eta_1\cos\theta_t}\right); \quad T_{\perp} = \frac{\hat{E}_{\perp m}^{t}}{\hat{E}_{\perp m}^{i}} = \left(\frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_i + \eta_1\cos\theta_t}\right)$$

Brewster szög

$$\theta_{iB} = \tan^{-1}\sqrt{\frac{\varepsilon_2}{\varepsilon_1}};$$

Kritikus szög

$$\theta_i \geq \theta_{critical} = \sin^{-1}\sqrt{\frac{\varepsilon_2}{\varepsilon_1}}$$