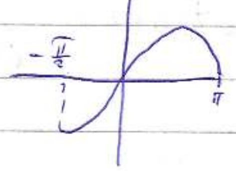
(G-7) 10.20.

(1) $f(x) = \begin{cases} \sin x & -\frac{\pi}{2} \leq x \leq \frac{\pi}{2} \\ 0 & \text{egyéb} \end{cases}$



$f(x) \geq 0 \quad R \to \mathbb{R}^+$

Nem sűrűségfvény

$\int_{-\infty}^{\infty} f(x)dx = 1$

(2) $g(x) = \begin{cases} \frac{1}{x^2} & x > 1 \\ 0 & \text{egyéb.} \end{cases}$

$\int_{-\infty}^{1} 0\,dx + \lim_{\varepsilon \to 0} \int_{1+\varepsilon}^{\infty} \frac{1}{x^2} dx = \lim_{\omega \to \infty} \int_{1}^{\omega} \frac{1}{x^2} dx$

$\lim_{\omega \to \infty} \left[-\frac{1}{x}\right]_1^{\omega} = 0 + 1 = 1 \Rightarrow$ sűrűségfvény ✓

eloszlásfvény: $G(x) = \begin{cases} -\frac{1}{x} + 1 & x > 1 \\ 0 & \text{egyéb} \end{cases}$

$\int_{\text{alsó}}^{x} \frac{1}{x^2} = \left[-\frac{1}{x}\right]_{\text{alsó}}^{x}$

Várható érték

$\int_{-\infty}^{\infty} x\, g(x)\, dx = \int_{-\infty}^{1} x\, 0\, dx + \int_{1}^{\infty} x \frac{1}{x^2} dx = \lim_{\omega \to \infty} [\ln x]_1^{\omega} = \infty - 0$

szórása nincs

(3) $h(x) = \begin{cases} \sin x & 0 \leq x \leq \frac{\pi}{2} \\ 0 & \text{egyébként} \end{cases}$

$\int_0^{\frac{\pi}{2}} \sin x\, dx = [-\cos x]_0^{\frac{\pi}{2}} = 0 + 1$ ✓

eloszlás fvény $\int_0^{x} \sin x\, dx = -\cos x + 1 \rightarrow$ $H(x) \begin{cases} 0 & x \leq 0 \\ -\cos x + 1 & 0 \leq x \leq \frac{\pi}{2} \\ 1 & x \geq \frac{\pi}{2} \end{cases}$

$$E(h) = \int_0^{\frac{\pi}{2}} x \sin x \, dx = \left[-x \cos x\right]_0^{\frac{\pi}{2}} + \int_0^{\frac{\pi}{2}} \cos x \, dx = \left[x \cos x\right]_0^{\frac{\pi}{2}} + \left[\sin x\right]_0^{\frac{\pi}{2}} = 1$$

$$E(h) = 1$$

$$D^2 h = E(h^2) - E^2(h)$$

$$E(h^2) = \int_0^{\frac{\pi}{2}} x^2 \sin x \, dx = \left[-\cos x \cdot x^2\right]_0^{\frac{\pi}{2}} + \int \cos x \cdot 2x \, dx = \cancel{\left[-\cos x \cdot x^2\right]_0^{\pi/2}} + \left[2x \cdot \sin x\right]_0^{\frac{\pi}{2}} - 2\int \sin x \, dx$$

$f' = 2x \quad g = -\cos x$

$$= \pi + 2\left[+\cos x\right]_0^{\frac{\pi}{2}} = \pi - 2$$

$$E(h)^2 - E^2(h) = \pi - 2 - 1 = \pi - 3 = D^2 x \qquad D_x = \sqrt{\pi - 3} = \sqrt{0{,}14}$$

(4) Exponenciális eloszlás

$$\lambda \qquad F(x) = \begin{cases} 1 - e^{-\lambda x} & x > 0 \\ 0 & x \le 0 \end{cases}$$

7 utcai telefonfülke - foglalt mikor odaérek $\lambda = \frac{1}{3}$ exp. eloszlású → örökifjú tulajdonságú

P(5 perc múlva se kerülök sorra)

X - {mennyit várok}

$X = 5 \qquad P(\xi > 5) = 1 - P(\xi \le 5) = 1 - F(5) = 1 - (1 - e^{-\frac{1}{3} \cdot 5}) = e^{-\frac{5}{3}}$

P(tudom, hogy már 2 perce beszélnek, de még 5-öt várni kell)

$$P(\xi > 7 \mid \xi > 2) = \frac{P(\xi > 7)}{P(\xi > 2)} = \frac{1 - 1 + e^{-\frac{1}{3}7}}{1 - 1 + e^{-\frac{1}{3}2}} = e^{-\frac{7}{3} + \frac{2}{3}} = e^{-\frac{5}{3}}$$

mert

$1 - F(7) \qquad 1 - F(2)$

örökifjú tulajdonság

↓

villanykörte

25 · 25
50
125
625

⑤ villanykörte 25% túléli a 4 évet , P(túléli a 8 évet)

$$P(\xi > 4) = 0{,}25 = \cancel{A}\, e^{-\lambda 4} \Rightarrow -\frac{\ln 0{,}25}{4} = +\lambda \qquad \lambda = 0{,}3466$$

$$P(\xi > 8) = e^{-0{,}3466 \cdot 8} = 0{,}0625$$

⑥ $\varphi(x) \Rightarrow$ sűrűségfgv. $\rightarrow \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$

$\Phi(x) \Rightarrow$ eloszlásfvény

$$\int_{-\infty}^{\infty} \varphi(x)\,dx = 1$$

$$\int_{-\infty}^{\infty} x\,\varphi(x)\,dx = 0$$ várható érték

$$\int_{-\infty}^{\infty} x^2 \varphi(x)\,dx = 1$$

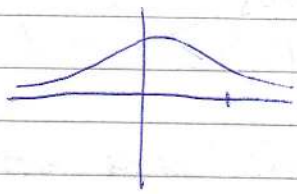
$$D^2(\xi) = 1 - \text{v.é.} = 1$$

$N(0, 1)$

v.é. szórás

$$\boxed{\Phi(-x) + \Phi(x)} = 1$$

$$\int_{-\infty}^{-x} \varphi(-x)\,dx + \int_{-\infty}^{x} \varphi(x)\,dx =$$



$$= \int_{x}^{\infty} \varphi(x)\,dx + \int_{-\infty}^{x} \varphi(x)\,dx = 1$$

(G8) 16.03.

(1) 178 cm ≈ átlagos nagasság a populációban

↓

Várható érték $m = 178$

szórás $\delta = 9$

$\boxed{! \quad \phi(x) + \phi(-x) = 1}$

Véletlenszerűen kiválasztott ember 169 – 197

X – testmagasság

$$P(169 < X < 187) = F(187) - F(169)$$

$$\phi\left(\frac{x_1 - m}{\delta}\right) - \phi\left(\frac{x_2 - m}{\delta}\right)$$

$$\phi\left(\frac{187-178}{9}\right) - \phi\left(\frac{169-178}{9}\right)$$

$$\phi(1) - \phi(-1) = \phi(1) - (1 - \phi(1)) = 2\phi(1) - 1 =$$

$$\cancel{1 - 2\phi(-1)} \quad = \underline{0{,}6826}$$

(2) Magasabb 2 m-nél $\quad 1 - F(200)$

$$1 - \phi\left(\frac{22}{9}\right) = 0{,}0073$$

(3) $m = 1{,}2$

$\sigma = 2$

$P(X < 1{,}5) \qquad F(1{,}5)$

$P(-10 < x)$

$P(-5 < x < 8)$

$P(-4{,}2 < x < 1{,}1) \quad x < 0)$

$$\phi_1\left(\frac{1{,}5 - 1{,}2}{2}\right) = \phi_1\left(\frac{0{,}3}{2}\right) = 0{,}5596$$

$$1 - \phi_2\left(\frac{-10 - 1{,}2}{2}\right) = 1 - \phi_2\left(\frac{-11{,}2}{2}\right) = 1 - (1 - \phi(5{,}6)) = 1$$

$$\phi_{31}\left(\frac{6{,}8}{2}\right) - \phi_{32}\left(\frac{-6{,}2}{2}\right) = 0{,}9997 - 1 + 0{,}999 = 0{,}9996$$

$$\overset{A}{P(-4{,}2 < x < 1{,}1} \mid \overset{B}{x < 0)} \qquad \frac{P(A \cap B)}{P(B)}$$

$$\frac{P(-4{,}2 < x < 0)}{P(x<0)} = \frac{\Phi(-0{,}6) - \Phi(-2{,}7)}{\Phi(-0{,}6)} =$$

$$\frac{\cancel{1}-\Phi(0{,}6)-\cancel{1}+\Phi(2{,}7)}{1-\Phi(0{,}6)} = \frac{-0{,}7257+0{,}9965}{1-0{,}7257} = 0{,}9872$$

(4) X [0;1] egyenletes eloszlású

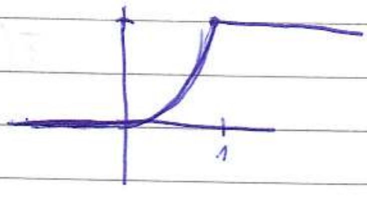
$y = 3x+2$ eloszlása ?

(x elo. fv-e) $F(x) = \begin{cases} 0 & x \leq 0 \\ x & 0 < x \leq 1 \\ 1 & 1 < x \end{cases}$

$$G(y) = \begin{cases} 0 & y \leq 2 \\ \frac{1}{3}y+\frac{2}{3} & 2 < y \leq 5 \\ 1 & 5 < y \end{cases}$$

(5) $z = x^2$

$$H(z) = \begin{cases} 0 & z \leq 0 \\ \sqrt{z} & 0 < z < 1 \\ 1 & 1 < z \end{cases}$$



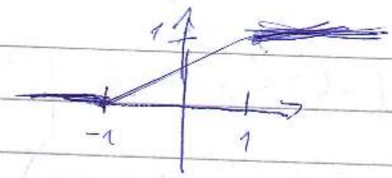
(6) $x \div \lambda = 2$ exp eloszlású

$y = 2x$ $\qquad x = \frac{y}{2}$

$z = x^2$ $\qquad x = \sqrt{z}$

$$F(x) = \begin{cases} 0 & x \leq 0 \\ 1-e^{-2x} & 0 < x \end{cases} \qquad H(z) = \begin{cases} 0 & z \leq 0 \\ 1-e^{-2\sqrt{z}} & 0 < z \end{cases}$$

$$G(y) = \begin{cases} 0 & y \leq 0 \\ 1-e^{-y} & 0 < y \end{cases}$$

HT X $[-1;1]$ -n egyenletes eloszlású 

$z = x^2$

$$H(z) = ? = P(Z^2 < z) = P(-\sqrt{z} < z < \sqrt{z}) =$$

$$F(\sqrt{z}) - F(-\sqrt{z}) = \frac{1}{2}\sqrt{z} + \frac{1}{2} - \left(\frac{1}{2}(-\sqrt{z}) + \frac{1}{2}\right) = \sqrt{z}$$

$$F(x) = \begin{cases} 0 & x \le -1 \\ \frac{1}{2}x + \frac{1}{2} & -1 < x \le 1 \\ 1 & x > 1 \end{cases}$$

$$H(z) = \begin{cases} 0 & z \le 0 \\ \sqrt{z} & 0 < z \le 1 \\ 1 & 1 < z \end{cases}$$

(6)

F \ M	1	2	3	4	5	
1	0	0	0	0	0	0
2	0	0	0	0	0	0
3	0	0	2	0	6	8
4	0	0	1	5	8	14
5	0	0	1	4	4	9
Σ	0	0	4	9	18	

31 Fő

Marginális eloszlások:

$$F(M) = \begin{cases} 0 & x \le 3 \\ 4/31 & 3 < x \le 4 \\ 13/31 & 4 < x \le 5 \\ 1 & 5 < x \end{cases}$$

$$F(F) = \begin{cases} 0 & x \le 3 \\ 8/31 & 3 < x \le 4 \\ 22/31 & 4 < x \le 5 \\ 1 & 5 < x \end{cases}$$

$P_{3,4} = P(M=3;\ F=4) = \frac{1}{31}$

$P_{5,5} = \frac{4}{31}$

(E-9) ea

(1) Első meghibásodásig 5 év
szórás : 1,5 év = σ
normális eloszlás
$X = \{$első meghibásodásig idő (év)$\}$

$$P(X<x) = F(x) = \Phi\left(\frac{x-5}{1,5}\right) = 0,05$$

(b) $P(2<X<3) = \cancel{\Phi\left(\frac{3-5}{1,5}\right) - \Phi\left(\frac{-2}{1,5}\right) - \Phi\left(\frac{-4}{3}\right)} = F(3) - F(2)$

(a) $1 - \Phi\left(\frac{x-5}{1,5}\right) = 1 - 0,05$

$$\Phi\left(\frac{-x+5}{1,5}\right) = 0,95 \qquad \boxed{\frac{-x+5}{1,5} = 1,645}$$

(c) $1 - F(9)$ 9 éven belül hány % nem romlik el

$$P(X>9) = 1 - F(9) = 1 - \Phi\left(\frac{4}{1,5}\right)$$

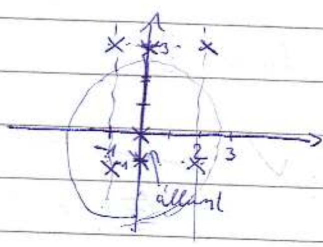
(d) 1000 napos garancia

$$P\left(X < \frac{1000}{365}\right)$$

(2) bolha helyzete 2D-s táblázatban

X\Y	-1	3	$\sum$ (perem eloszlások)
-1	p	2p	3p
0	2p	3p	5p
2	p	p	2p
$\sum$	4p	6p	

$p = 0,1$



körön belül : ha y = -1
$P = 4p = 0,4$

ⓒ Ha tudjuk $x>0$, P(B első negyedben van)

$$P\left(\begin{matrix}x>0\\y>0\end{matrix} \middle| x>0\right) = \frac{p}{2p} = \frac{1}{2}$$

$$P\left(\begin{matrix}x\geq\\y\geq 0\end{matrix} \middle| x\geq 0\right) = \frac{4p}{7p} = \frac{4}{7}$$

③ folytonos

$$F(x;y) = P(\xi < x; \eta < y)$$

$$f(x;y) = \frac{\partial^2 F}{\partial x \partial y}$$

$$\boxed{\text{kell } \begin{matrix} F(x_1; y_0) \leq F(x_2; y_0) & x_1 < x_2 \\ F(x_0; y_1) \leq F(x_0; y_2) & y_1 < y_2 \end{matrix}}$$

parciális deriváltak pozitívak

$$F(x;y) = \int_{-\infty}^{y}\int_{-\infty}^{x} f(u,v)\, du\, dv$$

$$f(x;y) = \begin{cases} 4xy & x \in [0;1];\ y \in [0;1] \\ 0 & \text{máskor} \end{cases}$$

ⓐ sűrűség-e?

① $\int_{-\infty}^{\infty} = 1 \rightarrow \int_0^1\int_0^1 4xy\, dx dy\ 1 \quad \int_0^1 \left[4\frac{x^2}{2}y\right]_0^1 dy = 1$ ✓

② $F' = f \geq 0$ ✓

$\Rightarrow$ sűrűség

ⓑ függetlenek? ha $f(x;y) = f_X(x) \cdot f_Y(y)$

$$\left.\begin{matrix} f_X(x) = \int_{-\infty}^{\infty} f(x;y)\,dy = \int_0^1 4xy\, dy = 2x \\ f_Y(x) = \int_0^1 4xy\, dx = 2y \end{matrix}\right\} \Rightarrow \text{függetlenek!} \rightarrow \text{korreláció} = 0$$

$2x \cdot 2y = 4xy$ ✓

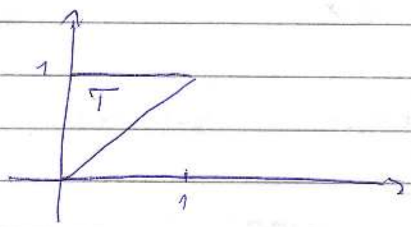
ⓒ Ex és Ey $\quad E(y) = E(x) = \int^1 x f_X(x)\, dx = \int^1 2x^2\, dx = \frac{2}{3}$

$$D^2x = Ex^2 - E^2x = \int_0^1 x^2 f_X(x) dx - \frac{4}{9} = \frac{1}{2} - \frac{4}{9} = \frac{1}{18} \Rightarrow Dx = \frac{1}{\sqrt{18}} = \frac{1}{3\sqrt{2}} = \frac{\sqrt{2}}{6}$$

$\iint_{-\infty}^{\infty} x\cdot y\, f(x,y) = \int_0^1\int_0^1 4x^2y^2\,dx\,dy$ közös $E(x,y)$

(4)

$$f(x,y) = \begin{cases} 6(x-y) & (x,y)\in T \\ 0 & \text{máshol} \end{cases}$$



nem sűrűségfvény

$$f(x,y) = \begin{cases} 6(y-x) & \in T \\ 0 & \text{máshol} \end{cases}$$

sfgvény $\rightarrow \geq 0$ ✓

$$\int_0^1\int_0^y 6y-6x \quad dx\,dy = \int_0^1 6yx - 3x^2\Big|_0^y dy = \int_0^1 6y^2-3y^2dy = 1$$

$$\int_0^1\int_x^1 6y-6x \quad dy\,dx$$ $\Rightarrow$ sfgvény

$$f_X(x) = \int_x^1 6y-6x \quad dy = \left[3y^2-6xy\right]_x^1 = (3-6x)-(3x^2-6x^2) = 3x^2-6x+3$$

$f_Y(y) = 3y^2$ $\Rightarrow$ nem függetlenek

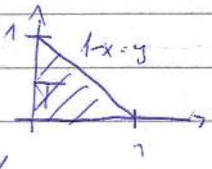
$$E(y) = \int_0^1 y\cdot 3y^2\,dy = \frac{3}{4}$$

$$E(x) = \int_0^1 3x^3-6x^2+3x\,dx = \frac{3}{4}x^4 - \frac{6}{3}x^3 + \frac{3}{2}x^2\Big|_0^1 = \frac{3}{4}-\frac{6}{3}+\frac{3}{2} = \frac{1}{4}$$

$$\int_0^1\int_x^1 6y^2x - 6x^2y\,dy\,dx = \int_0^1 \left[2y^3x - \quad\right]_x^1$$

(5-10)

$$f(x;y)=\begin{cases}60xy^2 & \in T\\ 0 & \notin T\end{cases}$$



$F(x)=P(X<x$

$$\int_0^1\int_0^{1-x} 60xy^2\,dy\,dx$$

$$\int_0^1 60x\,\frac{y^3}{3}\Big|_0^{1-x}dx=\int_0^1 \overset{20}{\cancel{60}}x\,\frac{(1-x)^3}{\cancel{3}}dx=20\int_0^1 x(-x^3+3x^2-3x+1)\,dx$$

$1+3x^2-3x-x^3$

$$=20\left[-\frac{x^5}{5}+\frac{3x^4}{4}-\frac{3x^3}{3}+\frac{x^2}{2}\right]_0^1=20\left(-\frac{1}{5}+\frac{3}{4}-1+\frac{1}{2}\right)=20\left(\frac{-4+15-20+10}{20}\right)=1$$

marginális: $f_1(x)=\int_0^{1-x}60xy^2\,dy=\left[\overset{20}{\cancel{60}}x\frac{y^3}{\cancel{3}}\right]_0^{1-x}=-20x^4+60x^3-60x^2+20x$

$$f_2(y)=\int_0^{1-y}60xy^2\,dx=\left[\overset{30}{\cancel{60}}\frac{x^2}{\cancel{2}}y^2\right]_0^{1-y}=30(1-y)^2y^2=$$
$$=30(1-2y+y^2)y^2=30y^2-60y^3+30y^4$$

független: NEM $\Rightarrow$ szorzatuk nem $60xy^2$

$\Longrightarrow$ kovariancia $\neq 0$

$cov(x;y)=E(x;y)-E(x)E(y)$

$corr(x;y)=\frac{cov(x;y)}{D(x)D(y)}=$

$cov(x,y)\;=-\frac{1}{42}$

$E(x)=\int_0^1 x f_1(x)\,dx\;=\frac{1}{3}$

$E(y)=\int_0^1 y f_2(y)\,dy=\int_0^1 30y^5-60y^4+30y^3\,dy=5y^6-12y^5+10y^3\Big|_0^1=3=\frac{1}{2}$

$E(x;y)=\int_0^1\int_0^{1-x} xy\,60xy^2\,dy\,dx=\int_0^1 \overset{15}{\cancel{60}}x^2\frac{y^4}{\cancel{4}}\Big|_0^{1-x}=\int_0^1 x^2(1-x)^4\,dx=$

$(1-2x+x^2)^2=1-2x+x^2-2x+4x^2-2x^3+x^2-2x^3+x^4$

$$E(x;y)=\frac{x^3}{3}-\cancel{4}\frac{x^4}{4}+6\frac{x^5}{5}-4\frac{x^6}{6}+\frac{x^7}{7}\Big|_0^1=\frac{1}{3}-1+\frac{6}{5}-\frac{4}{6}+\frac{1}{7}$$

$$E(x^2)=\int_0^1 x^2\cdot -20x^6+60x^5-60x^4+20x^3\,dx=-\frac{20}{7}+10-12+\frac{20}{3}=\frac{-60+210-}{}=\frac{1}{7}$$

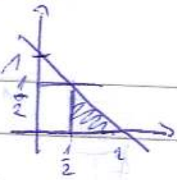
$E(y^2)=\;=\frac{2}{7}$

$D^2x=\frac{2}{63}$

$D^2y=\frac{1}{28}\quad\Rightarrow corr(x;y)=\frac{-\frac{1}{42}}{\sqrt{\quad}}$

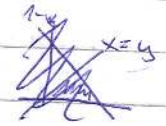
$$P\left(X > \frac{1}{2} \,\middle|\, Y < \frac{1}{2}\right) = \frac{P\left(X > \frac{1}{2};\, Y < \frac{1}{2}\right)}{P\left(Y < \frac{1}{2}\right)}$$

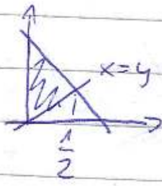
$f = 60xy^2$



$$\frac{\int_{\frac{1}{2}}^{1}\int_{0}^{-x+1} 60xy^2 \, dy\, dx}{\int_0^{\frac{1}{2}} 30y^2 - 60y^3 + 30y^4 \, dy}$$

$P(X < Y)$





$y = -x + 1$

$x = \frac{1}{2}$

$$\int_0^{\frac{1}{2}}\int_x^{-x+1} 60xy^2 \, dy\, dx =$$

Transzformáció

$$f(x;y) = \begin{cases} 4xy & x \in [0;1] \\ & y \in [0;1] \\ 0 & \end{cases}$$

$\Rightarrow$ független? $\Rightarrow$ igen

$\downarrow$

$E(x+y) = Ex + Ey$

$F(X+Y)$

$f_1(x) = 2x$

$f_2(y) = 2y$

$E(x+y) = Ex + Ey = \frac{4}{3} = \frac{2}{3} + \frac{2}{3}$

$$h(x+y) = \int_{-\infty}^{\infty} f_x(\overbrace{z-u}^{x}) f_y(\overbrace{u}^{y})\, du = \int_0^1 2(z-u)\cdot 2u = \int_0^1 4zu - 4u^2 \, du$$

konvolúcióval

$$= 4z\frac{u^2}{2} - \frac{4}{3}u^3 \Big|_0^1 = 2z - \frac{4}{3} \quad \Rightarrow h(z) = \begin{cases} 2z - \frac{4}{3} & z \in [0;1] \\ 0 \end{cases}$$

$u = x$

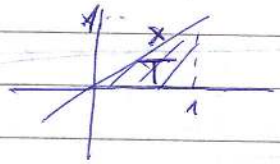
$v = x + y$

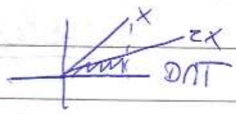
$$H(x+y) = P(X+Y < z) = \iint_T f(x) \circ g(y)\, d(x;y) = \int_0^z\int_0^1 \underset{\substack{\downarrow \\ 2u}}{f(u)}\, \underset{\substack{\downarrow \\ 2(v-u)}}{g(v-u)}\, du\, dv$$

$$\int_0^z \int_0^1 2u \cdot 2(v-u)\, du\, dv = \quad {-4u^2+4uv}$$

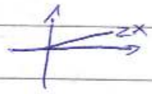
$$H(z) = z^2 - \frac{4}{3}z = \int 2z - \frac{4}{3}$$

(2) $f = \begin{cases} 6(x-y) & T \\ 0 \end{cases}$





$z = \frac{y}{x}$ - nem mi az eloszlása? $\quad y = zx \rightarrow$ egyenes



$D = \{(x;y) : (\frac{y}{x} < z)\}$

$$F(z) = P\left(\frac{y}{x} < z\right) = \iint_{D \cap T} 6x - 6y \, d(x;y) = \int_0^1 \int_0^{zx} 6x - 6y \, dy\, dx =$$

$$= \int_0^1 6xy - \frac{6}{2}y^2 \Big|_0^{zx} dx = \int_0^1 6x^2 z - 3z^2x^2 \, dx = 2x^3 - z^2x^3 \Big|_0^1 = 2z - z^2 = F(z)$$

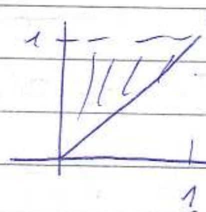
$$F(z) \begin{cases} 0 & z \leq 0 \\ 2z - z^2 & 0 < z \leq 1 \\ 1 & 1 < z \end{cases}$$

Sfgv:

$$F' = f(z) = \begin{cases} 2 - 2z & z \in [0;1] \\ 0 \end{cases}$$

(3)

$$f = \begin{cases} 2 & (x;y) \in T \\ 0 & \text{egyébként} \end{cases}$$



$y \in [0;1]$
$x \in [0; x]$

$D = \{(x;y) \in \mathbb{R}^2 : xy < z\}$
$\{y < \frac{z}{x}\} \quad x > 0$
$\quad y > \frac{z}{x} \quad x < 0$

$x \cdot y$ eloszlása

$$F(z) = P(Z < z) = P(XY < z) = \iint_{D \cap T} f(x;y)\, d(x;y) =$$

$$\int_0^z \int_x^1 2 \, dy\, dx + \int_z^{\sqrt{z}} \int_x^{\frac{z}{x}} 2 \, dy\, dx =$$

$$= \int_0^z 2 - 2x \, dx + \int_z^{\sqrt{z}} \frac{2z}{x} - 2x \, dx =$$

$$= 2z - z^2 + \left(2z \ln|x| - x^2\right)\Big|_z^{\sqrt{z}} = 2z - \cancel{z^2} + \overbrace{2z\ln\sqrt{z} - z - 2z\ln z + z^2}^{z\ln z}$$

$$F(z) = z - z\ln z \Rightarrow f = 1 - \ln z - z \cdot \frac{1}{z} = -\ln z$$

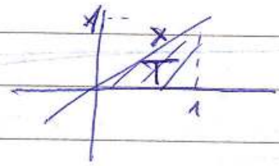
$P_1 \Rightarrow x = z$
$P_2 \Rightarrow x = \sqrt{z}$

$$\int_0^z\int_0^1 2u\cdot 2(v-u)\,du\,dv = \qquad -4u^2+4uv$$

$$H(z) = z^2 - \frac{4}{3}z = \int 2z - \frac{4}{3}$$

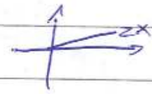
(2)

$$f = \begin{cases} 6(x-y) & T \\ 0 & \end{cases}$$



$z = \frac{y}{x}$ -nem mi az eloszlása?

$y = zx \to$ egyenesek



$$D = \{(x;y) : (\tfrac{y}{x} < z)\}$$

$$F(z) = P\left(\frac{y}{x} < z\right) = \iint_{D\cap T} 6x - 6y\, d(x,y) = \int_0^1\int_0^{zx} 6x - 6y\, dy\, dx =$$

$$= \int_0^1 6xy - \frac{6}{2}y^2 \Big|_0^{zx} dx = \int_0^1 6x^2 z - 3z^2x^2\, dx = 2zx^3 - z^2x^3 \Big|_0^1 = 2z - z^2 = F(z)$$

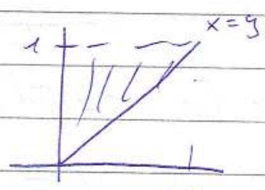
$$F(z)\begin{cases} 0 & z \le 0 \\ 2z - z^2 & 0 < z \le 1 \\ 1 & 1 < z \end{cases}$$

Sfgv:

$$F' = f(z) = \begin{cases} 2 - 2z & z \in [0;1] \\ 0 & \end{cases}$$

(3)

$$f = \begin{cases} 2 & (x;y) \in T \\ 0 & \text{egyébként} \end{cases}$$



$y \in [0;1]$

$x \in [0;x]$

$x \cdot y$ eloszlása

$$D = \{(x;y) \in \mathbb{R}^2 : xy < z\}$$

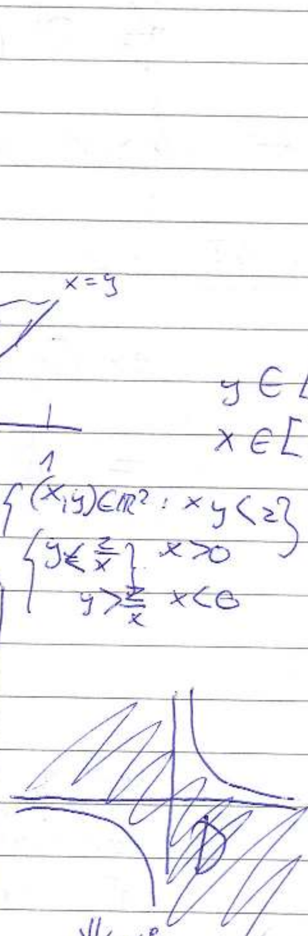
$$\begin{cases} y < \frac{z}{x} & x > 0 \\ y > \frac{z}{x} & x < 0 \end{cases}$$

$$F(z) = P(Z < z) = P(XY < z) = \iint_{D\cap T} f(x;y)\, d(x;y) =$$

$$\int_0^z\int_x^1 2\, dy\, dx + \int_z^{\sqrt{z}}\int_x^{\frac{z}{x}} 2\, dy\, dx =$$

$$= \int_0^z 2 - 2x\, dx + \int_z^{\sqrt{z}} \frac{2z}{x} - 2x\, dx =$$

$$= 2z - z^2 + \left(2z\ln|x| - x^2\right)\Big|_z^{\sqrt{z}} = 2z - z^2 + \overbrace{2z\ln\sqrt{z}}^{z\ln z} - z - 2z\ln z + z^2$$



$P_1 \Rightarrow x = z$

$P_2 \Rightarrow x = \sqrt{z}$

$$F(z) = z - z\ln z \Rightarrow f = 1 - \ln z - z\cdot\frac{1}{z} = -\ln z$$

(6-11) 11.24.

(1) Van két exp. eloszl. v.v.

X Y

sfgv. $f(x) = \begin{cases} \lambda_1 e^{-\lambda_1 x} & x \geq 0 \\ 0 & \text{máshol} \end{cases}$

$f(y) = \begin{cases} \lambda_2 e^{-\lambda_2 y} & y \geq 0 \\ 0 & \text{máshol} \end{cases}$

~~conv~~ konvolúció $= h(x+y) = h_{X+Y}(z) =$

$z = \cancel{x+y}\, X+Y$

$$\int_{-\infty}^{\infty} f_X(z-u)\cdot g_Y(u)\,du = \int_0^{\infty} \lambda_1 e^{-\lambda_1(z-u)}\cdot \lambda_2 e^{-\lambda_2(u)}\,du =$$

$$= \int_0^{\infty} \lambda_1\lambda_2 e^{-\lambda_1 z}\left(e^{+u(\lambda_1-\lambda_2)}\right)du = \lambda_1\lambda_2 e^{-\lambda_1 z}\cdot \frac{e^{-u(\lambda_2-\lambda_1)}}{-\lambda_2+\lambda_1}\Bigg|_0^{\infty}$$

$$= \boxed{+\frac{\lambda_1\lambda_2}{\lambda_2-\lambda_1} e^{-\lambda_1 z}} \to$$

sűrűségfv.: $\int_{-\infty}^{z} dz$

$$H(X+Y) = \int_0^z \frac{\lambda_1\lambda_2}{\lambda_2-\lambda_1} e^{-\lambda_1 z}\,dz = \frac{\lambda_1\lambda_2}{\lambda_2-\lambda_1}\,\frac{e^{-\lambda_1 t}}{-\lambda_1}\Bigg|_0^z$$

$$H(X+Y) = \frac{\lambda_2}{\lambda_1-\lambda_2}\left(e^{-\lambda_1 z}-1\right)$$

② X sűrűség $\to f(x) = \begin{cases} \lambda_1 e^{\lambda_1 x} & x \geq 0 \\ 0 \end{cases}$

Y: $[0;1]$ en egyenletes eloszlás $\qquad$ $(\text{eloszlás})' = \text{sűr}$

$\downarrow$

$g(y) = \begin{cases} 1 & [0;1] \\ 0 & \text{egyébként} \end{cases}$

$Z = X + Y \Rightarrow h(z)$

$h(X \pm Y) = \int f_X(x)\, f_Y(z-x)\, dx$ $\qquad$ $Y=1 \quad X=Z-1$; $Y=0 \quad X=Z$

$= \int_{z}^{z-1} \lambda e^{-\lambda x} \cdot 1 \, dx =$

$= -e^{-\lambda x} \Big|_{z}^{z-1} = \underline{\underline{-e^{-\lambda(z-1)} + e^{-\lambda z} = h(z)}}$

$f_X(z-y) \cdot f_Y(y)$-ra * másor az integrálási határ

CHT - NSZT

$\downarrow$ $\to$ hasonlít egy normális eloszláshoz

~~Sor folytat~~

① $P(6000$ (kockadobások)-ból a hatosok száma $\in [970; 1050])$

$P(970 \leq X \leq 1050)$

$F(x) = P(X < x) = P\left(\frac{X - np}{\sqrt{np(1-p)}} < \frac{x - np}{\sqrt{np(1-p)}}\right)$

$\Downarrow$

$P\left(\underbrace{\frac{970 - 6000 \cdot \frac{1}{6}}{\sqrt{6000 \cdot \frac{1}{6} \cdot \frac{5}{6}}}}_{**} < \frac{X - 6000 \cdot \frac{1}{6}}{\sqrt{6000 \cdot \frac{5}{36}}} < \underbrace{\frac{1050 - 6000 \cdot \frac{1}{6}}{\sqrt{6000 \cdot \frac{5}{36}}}}_{*}\right)$

$\phi(*) - \phi(**) = \Phi(1{,}73) - \Phi(-1{,}04) =$

$= \Phi(1{,}73) - (1 - \Phi(1{,}04)) =$

$\Phi(1{,}73) + \Phi(1{,}04) - 1 = 0{,}9582 + 0{,}8508 - 1$

$= 0{,}809$

(2) Egy gyár ~~terméke~~ terméke elfogadható 0,95 valséggel
hüvelhező 150-ből legfeljebb 10 nem elfog.

$X = $ ξ elfog. termékek száma

$p = 0{,}85$

$P(X \geq 140)$

$$1 - P(X \leq 140) = 1 - P\left(\frac{X - 150 \cdot 0{,}95}{\sqrt{150 \cdot 0{,}95 \cdot 0{,}05}} < \frac{140 - 150 \cdot 0{,}95}{\sqrt{150 \cdot 0{,}95 \cdot 0{,}05}}\right)$$

$$1 - \Phi(s) = 1 - \left(1 - \Phi\left(\frac{+2{,}5}{2{,}67}\right)\right) = \Phi(0{,}9366) = 0{,}8264$$

(3) Kocka → feldobáljuk, míg $\sum x \geq 300$

P(legalább 80 dobásra van szükség)

$$\boxed{CHT}\ P\left(\frac{X_1 + \dots + X_n - n\mu}{\sqrt{n} \cdot \underbrace{Dx}_{\sigma}} < x\right) = \Phi(x)$$

$$\mu = \sum_1^6 x_i p_i = \frac{1}{6} + \frac{2}{6} + \dots + \frac{6}{6} = 3{,}5 \qquad EX^2 = \frac{1}{6}(1 + 4 + 9 + 16 + 25 + 36) = \frac{91}{6}$$

$$\sigma = \sqrt{EX^2 - (EX)^2} = \sqrt{\frac{91}{6} - \frac{49}{4}} = \sqrt{\frac{35}{12}}$$

$$P(S_{79} < 300) = P\left(\frac{S_{79} - 79 \cdot 3{,}5}{\sqrt{79 \cdot \frac{35}{12}}} < \frac{300 - 79 \cdot 3{,}5}{\sqrt{79 \cdot \frac{35}{12}}}\right)$$

$$= \Phi\left(\frac{300 - 79 \cdot 3{,}5}{\sqrt{}}\right) = \Phi\left(\frac{23{,}5}{15{,}1795}\right) = 0{,}9394$$

(4) 2 fajta érme $\Rightarrow$ Van egy ismeretlen érme

igazságos $p_f = 0,5$

55% - fej
45% -
$p_f = 0,55$

1000 dobás - 525 legalább fej $\rightarrow$ hamis

1000 - 525 < nem fej $\rightarrow$ igaz ~~p_f~~

P(téved a teszt)

jó az érme

$$P(X \geq 525) = 1 - P(X < 525) = 1 - \Phi\left(\frac{525 - 1000 \cdot 0,5}{\sqrt{1000 \cdot 0,5 \cdot 0,5}}\right)$$

hamis az érme

$$P(X < 525)$$

$$\Phi\left(\frac{525 - 1000 \cdot 0,55}{\sqrt{1000 \cdot 0,55 \cdot 0,45}}\right)$$

(5) K egész szám = ?

P(400 dobásból K < fej < 195 $\rightarrow p = 0,5$)

$$P(195 < X < k) = 0,5$$

$$\Phi\left(\frac{k - 400 \frac{1}{2}}{\sqrt{400 \cdot \frac{1}{4}}}\right) - \underbrace{\Phi\left(\frac{195 - 400 \cdot \frac{1}{2}}{\sqrt{100}}\right)}_{1 - \Phi(0,5)} = 0,5$$

$$\Phi\left(\frac{k - 200}{10}\right) = 0,5 + 1 - \Phi(0,5) = \cancel{0,8085} \; 0,8085$$

$$\frac{k - 200}{10} \approx 0,87$$

$$k \approx 208,7 \Rightarrow k = 209$$

(6) 100 égő

↳ exp ⇒ 5 h várh. é. folyamatosan cseréljük →
525 h - után van működő égőnk

$E(x) = 5 = D(x)$

$$P(S_{100} \geq 525) = 1 - P(S < 525) = 1 - P\left(\frac{S_{100} - 100 \cdot 5}{\sqrt{100} \cdot 5} \ldots\right)$$

$$1 - \Phi\left(\frac{525 - 500}{50}\right) = 3{,}088$$

(7) Rulett → 0 zöld
↘ 18 piros
18 fekete

∀ körben 10 peták
100 játék után = −300 peták

$P(S_{100} \leq 300)$

(G1)

(A) $(\xi_1; \eta)$ 2D-s folyt

$$f = \begin{cases} 4xy & (x;y) \in [0,1] \times [0,1] \\ 0 & \text{máskor} \end{cases}$$

(1) ξ felt. sűrűsége $\eta = y$ feltétel mellett?
(2) Mi a ξ-t legjobban közelítő $g(\eta)$ fgvény (legkisebb négyzetes értelemben)?

~~$f_{\xi|\eta=y}(x) =$~~

$$f_\eta(y) = \int_{-\infty}^{\infty} f \, dx = \cancel{2xy} \; 2y$$

ξ és η függetlenek

$$f_\xi(x) = 2x$$

$$f_{\xi|\eta=y}(x) = \begin{cases} \frac{f(x;y)}{f_\eta(y)} & \text{, ha } f_\eta(y) \neq 0 \\ 0 & \text{máskor} \end{cases}$$

$$f_{\xi|\eta=y} = \frac{f_\eta(x) \cdot \cancel{f_\eta(y)}}{\cancel{f_\eta(y)}} = f_\eta(x)$$

$$E[\xi | \eta = y] = \int_{\mathbb{R}} x \, f_{\xi|\eta=y} \, dx = \int_0^1 2x^2 \, dx = \frac{2}{3} x^3 = \frac{2}{3}$$

$\Downarrow$ mert függenek

$\|$

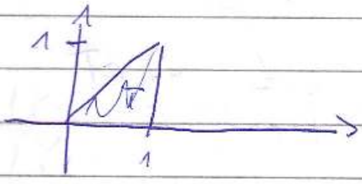
$E\xi$

(2) regressziós fgvény $g(y) = E[\xi | \eta = y]$

$$g(y) = \begin{cases} \frac{2}{3} & \text{, ha } y \in [0,1] \\ 0 & \text{ha } \notin \end{cases}$$

(2)

$$f(x,y) = \begin{cases} 6(x-y), & \text{ha } (x,y) \in T \\ 0 & \text{egyébként} \end{cases}$$



$x \in [0,1]$, $y \in [0,x]$ $\qquad$ $y \in [0,1]$, $x \in [y,1]$

$$f_\eta(y) = \int_y^1 f \, dx = \int 6x - 6y \, dx = 3x^2 - 6yx \Big|_y^1 =$$

$$= 3 - 3y^2 - 6y + 6y^2 = 3(y-1)^2$$

$$f_{\xi|\eta=y}(x) = \begin{cases} \dfrac{\cancel{6}x - \cancel{6}y}{\cancel{3}(y-1)^2} \quad y \in [0,1],\ x \in [y,1] \\ 0 \end{cases}$$

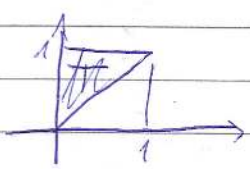
$\cancel{x} \in [y,1]$

$$E[\xi \mid \eta = y] = \int_y^1 x \cdot \frac{2x-2y}{(y-1)^2} \, dx$$

$$= \frac{1}{(y-1)^2} \cdot \left(\frac{2}{3}x^3 - \frac{2}{2} y x^2 \right) \Big|_y^1 = \frac{1}{(y-1)^2} \cdot \left[\left(\frac{2}{3} - y \right) + \left(\cancel{\frac{2}{3}y^3 - y^3} \right) \right] \quad + \frac{1}{3}y^3$$

$$\frac{\frac{1}{3}y^3 - y + \frac{2}{3}}{(\quad)^2} = \frac{2 - y - y^2}{3(1-y)}$$

(3)



$$f(x,y) = \begin{cases} 2 & y, x \in T \\ 0 \end{cases}$$

$y \in [0;1]$, $x \in [0;y]$ $\qquad$ $x \in [0;1]$, $y \in [x;1]$

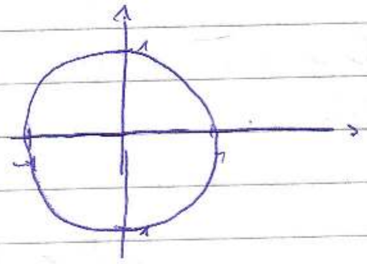
$$f_\eta(y) = \int_0^y 2 \, dx = 2x \Big|_0^y = \underline{\underline{2y}}$$

$$f_{\xi|\eta=y} = \begin{cases} \dfrac{1}{y} & y \in [0;1],\ x \in [0;y] \\ 0 \end{cases}$$

$$E[\xi \mid \eta = y] = \int_0^y x \frac{1}{y} \, dx = \frac{1}{y} \frac{x^2}{2} \Big|_0^y = \frac{y}{2}$$

regressziós függvény

$E[\xi - g(\eta)]^2$ a legkisebb!

(4) T: 

$T = \{x^2 + y^2 \le 1\}$

$(\xi;\eta)$ 2D-s sűrűség:

$f: \begin{cases} \frac{1}{\pi} & \text{ha } \in T \\ 0 & \end{cases}$

$y \in [-1;1]$
$x \in [-\sqrt{1-y^2}; \sqrt{1-y^2}]$

$$f_\eta(y) = \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} \frac{1}{\pi}\,dx = \frac{1}{\pi}\,x \Big|_{-\sqrt{\ }}^{\sqrt{\ }}$$

$$\frac{1}{\pi}\sqrt{1-y^2} + \frac{1}{\pi}\sqrt{1-y^2} = \frac{2}{\pi}\sqrt{1-y^2} = f_\eta(y)$$

$$f(\xi \mid \eta = y) \begin{cases} \dfrac{\frac{1}{\pi}}{\frac{2}{\pi}\sqrt{1-y^2}} = \dfrac{1}{2\sqrt{1-y^2}} & y \in [-1;1] \quad x \in [-\sqrt{\ };\sqrt{\ }] \\ 0 & \notin \end{cases}$$

szimmetrikus a sűrűség

$$E[\xi \mid \eta = y] = \int_{-\sqrt{\ }}^{\sqrt{\ }} \frac{x}{2\sqrt{\ }}\,dx = \frac{1}{2\sqrt{\ }}\left(\frac{x^2}{2}\right) = \cancel{\frac{1}{1}}\ \cancel{\frac{1}{2}}\ 0$$

(5) Diszkr. együttes

x \ y	−1	3
−1	0,1	0,2
0	0,2	0,3
2	0,1	0,1

P(X

	0,6	1
−1	0,2	$\frac{1}{3}$
0	0,3	$\frac{1}{2}$
2	0,1	$\frac{1}{6}$

$R(x) = \{-1, 0, 2\}$

$P(X = -1 \mid Y = 3) = \frac{1}{3}$
$P(\ \ 0 \mid \ \ 3) = \frac{1}{2}$
$P(\ \ 2 \mid \ \ 3) = \frac{1}{6}$

$E[X \mid Y=3]$
$E[X \mid Y=-1]$
$E[X]$

$E[X \mid Y=3] = -1 \cdot P(X=-1 \mid Y=3) + 0 \cdot P(X=0 \mid Y=3) + 2 P(X=2 \mid Y=3) = 0$
$E[X \mid Y=-1] = -1 \cdot P(X=-1 \mid Y=-1) + 0 \cdot \quad + 2 P(X=2 \mid Y=-1) = \frac{1}{4}$ ℓ
$EX = 0{,}1$

(5) Múlt órai feil!

$$\int_{-\infty}^{\infty} f(z-x)\, g(x)\, dx = \int_0^{z} \lambda_1 e^{-\lambda_1 (z-u)} \cdot \lambda_2 e^{-\lambda_2 u}\, du$$

$z-u>0$, $u>0$

$$f \begin{cases} \lambda_1 e^{-\lambda_1 x} & x \geq 0 \\ 0 \end{cases}$$

$$e^{-\lambda_1 z} \cdot \lambda_1 \lambda_2 \int_0^z e^{-u[\lambda_2 - \lambda_1]}\, du$$

$$\frac{\lambda_1 \lambda_2}{-\lambda_2 + \lambda_1} \left[e^{-\lambda_2 z} - e^{-\lambda_1 z} \right]$$

$$\int_{-\infty}^{\infty} f(u)\, g(z-u)\, du$$

$$\frac{\lambda_1 \lambda_2}{-\lambda_1 + \lambda_2} \left[e^{-\lambda_1 z} - e^{-\lambda_2 z} \right]$$

(6) $m = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$ $\quad \Sigma = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$

$E(x) = 3$ $\quad D^2 x = 2$ $\quad cov(X, y) = 1$
$E(y) = 1$ $\quad D^2 y = 3$

$$kor(X, y) = \frac{cov(x, y)}{D(x) D(y)} = \frac{1}{\sqrt{6}}$$

Exponenciális eloszlás $Ex = Dx$

$E(X+Y) = E(x) + E(y) = 4$

mivel normális eloszlású

$E(xy) = cov(x,y) + E(x)E(y) = 1+3 = +2$

$D(X+Y) = \sqrt{D^2(x) + D^2(y) + 2cov(x,y)} = \sqrt{2+3+2\cdot 1} = \sqrt{7}$

$E(2x-3Y) = 2Ex - 3Ey = 3$

$D(2x-3Y) = \sqrt{D^2(2x-3y)} = \sqrt{4D^2(x) + 9D^2(y) + \underbrace{2cov(2x-3y)}_{+2\cdot 2\cdot(-3)cov(x,y)}}$

$E(XY+Y^2) = \underset{+2}{E(XY)} + \underset{4}{E(Y^2)} = 6$

$\underset{3}{D^2Y} = E(Y^2) - \underset{1}{(EY)}^2$

7. Csebisev: $Ex = 10$ $Dx = 2$ normális

~~Legalább~~ $P(X \in$ $P_{min}([5;15] \ni x)$

a) $P(5 \leq X \leq 15) = P(|X-10| < 5) = 1 - \underbrace{P(|X-10| \geq 5)}_{\leq \frac{D^2(x)}{C^2}}$ $Dx-15$, $Dx-5$, C

$\geq 1 - \frac{4}{25} = \frac{21}{25} = 0{,}84$

$P(x \text{ benne van}) > 0{,}84$

ha normális

b) $P(5 \leq x \leq 15) = F(15) - F(5) = \Phi\left(\frac{15-10}{2}\right) - \Phi\left(\frac{5-10}{2}\right)$

~~ha exponenciális~~ exponen

⑧ átl. 4 levél / nap $D_x = 2$ $E_x = 4$

P(legfeljebb 1 levelet érkezik hozzá)

$P(X \leq 1)$ $P(|X-4| \geq 3) \leq \frac{4}{9}$

ez szar becslés de jobb nincs

⑨ füreggy. görhenc karék → használhatatlan ha a mérete ±1mm-el

$E_x = 80$ mm

$D(x) = 0{,}5$ mm   legfeljebb ?% selejtes

$$1 - P(79 < X < 81) = P(|X-80| \geq 1) \leq \frac{0{,}5^2}{1} = 0{,}25$$

⑩ áruház → 50% ár leszállítás

hány főt 95% ~~...~~

kiderítik, meg, hogy |vásárló — megsejtett| $\leq 0{,}05$ → hány embert kell megkérdezni a jó sejtéshez

$$P\left(\left|\frac{k}{n} - p\right| < \underset{\sqrt{\varepsilon^2}}{0{,}05}\right) \geq 1 - \frac{1}{4\varepsilon^2 n} = 1 - \frac{1}{4 \cdot 0{,}05^2 \cdot n} \geq 0{,}95$$ | CHT-val

$\frac{k}{n}$: megkérdezettek közötti átlag; p: igazi

$$0{,}05 > \frac{1}{4 \cdot 25 \cdot 10^{-4}\, n} \qquad n > \frac{10^4}{4 \cdot 25 \cdot 0{,}05} = \quad n > 2000$$

legalább 2000 ember.

⑪ dohányzók aránya   ? megfigyelés, hogy az arány a valódi aránytól 0,9 valséggel legfeljebb 0,01-dal térjen el

$$P\left(\left|\frac{k}{n} - p\right| < 0{,}01\right) > 1 - \frac{1}{4 \cdot 0{,}01^2 n} > 0{,}9$$

(12) 11 szám komplikált → $\frac{9,7}{10}$ | 9,7 | 10 | 9,6 | 10,1 | 9,8 | 10,2 | 9,5 |
10,3 | 9,9 | 9,4 | 10,4

tapasztalati várható érték = 9,9
tapasz. szórás
$$\hookrightarrow \frac{\sqrt{\sum (x_i - \bar{x})^2}}{n} = s_n$$

(13) koleszterin mint mérő   20 mg – átlagos hiba
↳ hiba szórás

$n = 15 \quad \bar{\xi}_{15} = 153 \text{ mg}$ 98% szimmetrikus konfiden. inter. az ismeretlen adatra

$$\left[\bar{\xi}_n - \frac{\sigma}{\sqrt{n}} \phi^{-1}\left(\frac{1+p}{2}\right)\right]$$

→ 0,995 → visszafelé kell olvasni

$$\frac{\sigma}{\sqrt{n}} \phi^{-1}\left(\frac{1+p}{2}\right) = \frac{20}{\sqrt{15}} \phi^{-1}\left(\frac{1+0,99}{2}\right)$$

$$= \frac{20}{\sqrt{15}} \cdot 2,58 = 13,32$$

$[153 - 13,32 \,;\, 153 + 13,32] \rightarrow$ Konf. int $[139,68 \,;\, 166,32]$