

Laplace transzformációra vonatkozó tételek és mellettük a legfontosabb időfüggvények Laplace transzformáltjai ($t > 0$ -ra)

Property	$f(t)$	$F(s)$	$f(t)$	$F(s)$
Linearity	$a_1 f_1(t) + a_2 f_2(t)$	$a_1 F_1(s) + a_2 F_2(s)$	$\delta(t)$	1
Scaling	$f(at)$	$\frac{1}{a} F\left(\frac{s}{a}\right)$	$u(t)$ <i>egyszerűsítés</i>	$\frac{1}{s}$
Time shift	$f(t-a)u(t-a)$	$e^{-as} F(s)$	e^{-at}	$\frac{1}{s+a}$
Frequency shift	$e^{-at} f(t)$	$F(s+a)$	t	$\frac{1}{s^2}$
Time differentiation	$\frac{df}{dt}$	$sF(s) - f(0^-)$	t^n	$\frac{n!}{s^{n+1}}$
	$\frac{d^2 f}{dt^2}$	$s^2 F(s) - sf(0^-) - f'(0^-)$	$t^n e^{-at}$	$\frac{1}{(s+a)^2}$
	$\frac{d^3 f}{dt^3}$	$s^3 F(s) - s^2 f(0^-) - sf'(0^-) - f''(0^-)$	$t^n e^{-at}$	$\frac{n!}{(s+a)^{n+1}}$
	$\frac{d^n f}{dt^n}$	$s^n F(s) - s^{n-1} f(0^-) - s^{n-2} f'(0^-) - \dots - f^{(n-1)}(0^-)$	$\sin \omega t$	$\frac{\omega}{s^2 + \omega^2}$
Time integration	$\int_0^t f(t) dt$	$\frac{1}{s} F(s)$	$\cos \omega t$	$\frac{s}{s^2 + \omega^2}$
Frequency differentiation	$tf(t)$	$-\frac{d}{ds} F(s)$	$\sin(\omega t + \theta)$	$\frac{s \sin \theta + \omega \cos \theta}{s^2 + \omega^2}$
Frequency integration	$\frac{f(t)}{t}$	$\int_s^\infty F(s) ds$	$\cos(\omega t + \theta)$	$\frac{s \cos \theta - \omega \sin \theta}{s^2 + \omega^2}$
Time periodicity	$f(t) = f(t + nT)$	$\frac{F_1(s)}{1 - e^{-sT}}$	$e^{-at} \sin \omega t$	$\frac{\omega}{(s+a)^2 + \omega^2}$
Initial value	$f(0^+)$	$\lim_{s \rightarrow \infty} sF(s)$	$e^{-at} \cos \omega t$	$\frac{s+a}{(s+a)^2 + \omega^2}$
Final value	$f(\infty)$	$\lim_{s \rightarrow 0} sF(s)$	$\delta(t-a)$	e^{-sa}
Convolution	$f_1(t) * f_2(t)$	$F_1(s)F_2(s)$	$u(t-a)$	$\frac{1}{s} e^{-sa}$

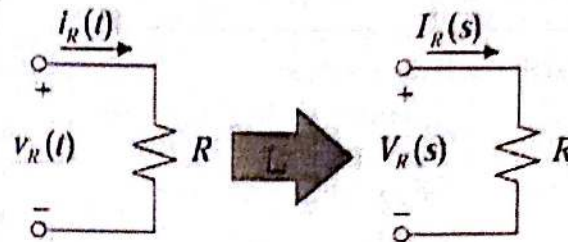
Segítség: Operátoros impedanciák a kezdeti értékekkel (feszültség- majd áramforrás)

Időtartomány

s-tartomány

Ellenállás

$$v_R(t) = R i_R(t)$$

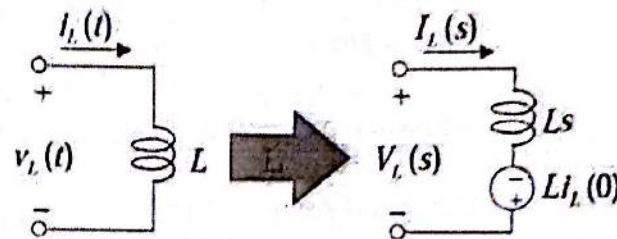


$$V_R(s) = R I_R(s)$$

Induktivitás

$$v_L(t) = L \frac{di_L(t)}{dt}$$

$$i_L(0+) = i_L(0)$$

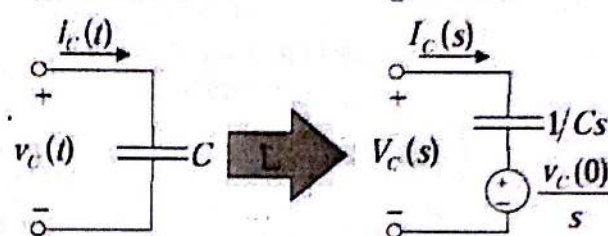


$$V_L(s) = sL I_L(s) - Li_L(0)$$

Kapacitás

$$v_C(t) = \frac{1}{C} \int_0^t i_C(\tau) d\tau + v_C(0)$$

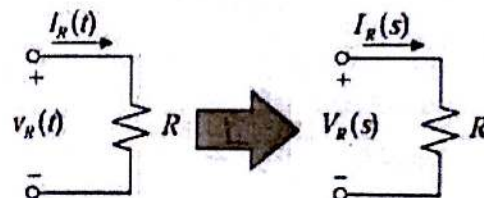
$$v_C(0+) = v_C(0)$$



$$V_C(s) = \frac{1}{sC} I_C(s) + \frac{v_C(0)}{s}$$

Ellenállás

$$i_R(t) = \frac{1}{R} v_R(t)$$

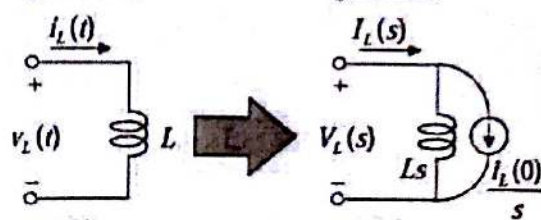


$$I_R(s) = \frac{1}{R} V_R(s)$$

Induktivitás

$$i_L(t) = \frac{1}{L} \int_0^t v_L(\tau) d\tau + i_L(0)$$

$$i_L(0+) = i_L(0)$$

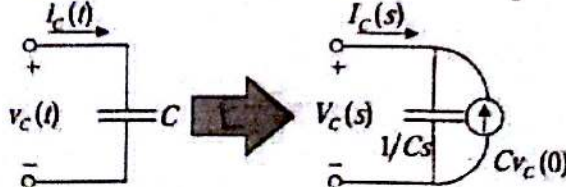


$$I_L(s) = \frac{1}{sL} V_L(s) + \frac{i_L(0)}{s}$$

Kapacitás

$$i_C(t) = C \frac{dv_C(t)}{dt}$$

$$v_C(0+) = v_C(0)$$



$$I_C(s) = sC V_C(s) - C v_C(0)$$

$$Z_R = R$$

$$Z_C = \frac{1}{j\omega C}$$

$$Z_L = j\omega L$$