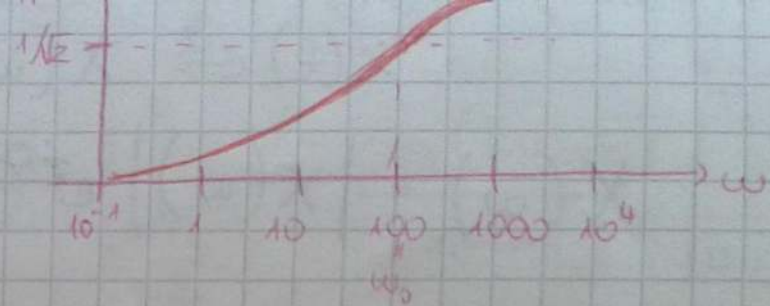
$$|H(j\omega)|\Big|_{\omega\to 0} \to 0$$

$$|H(j\omega)|\Big|_{\omega\to \infty} \to 1$$

$$|H(j\omega)|\Big|_{\omega = \omega_0} \to \frac{1}{\sqrt{2}}$$



Fourier sor

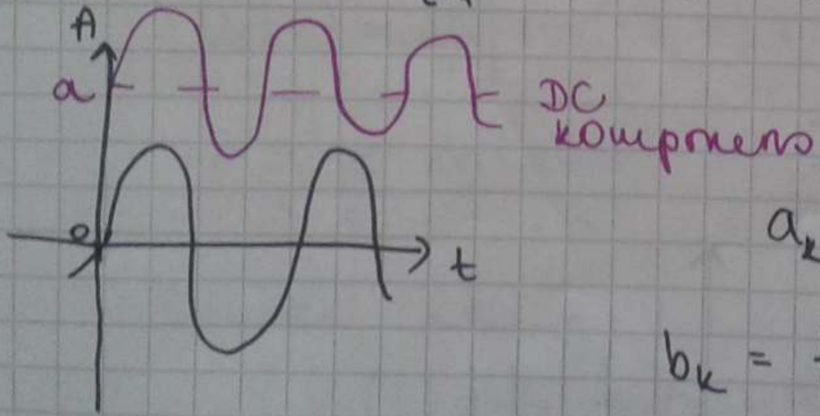
nov. 7.

periodikus jelek, diszkrét spektrumúak

$$x(t+T) = x(t) \quad \forall t$$

$$T_0 = \frac{1}{f_0}$$

$$x(t) = \frac{a_0}{2} + \sum_{k=1}^{\infty} \left(a_k \cos(k\omega_0 t) + b_k \sin(k\omega_0 t) \right)$$



$$a_0 = \frac{2}{T_0} \int_{T_0} x(t)\, dt$$

$$a_k = \frac{2}{T_0} \int_{T_0} x(t) \cos(k\omega_0 t)\, dt$$

$$b_k = \frac{2}{T_0} \int_{T_0} x(t) \sin(k\omega_0 t)\, dt$$

Schaum: Signals and Systems

ha $x(t)$ páros $\Rightarrow b_k = 0$

ha $x(t)$ ptlan $\rightarrow a_k = 0$

Mérnöki alak (harmonic form)

$$x(t) = C_0 + \sum_{k=1}^{\infty} c_k \cos(k\omega_0 t - \vartheta_k)$$

$$C_0 = \frac{a_0}{2} \qquad C_k = \sqrt{a_k^2 + b_k^2}$$

$$\vartheta_k = \operatorname{arctg} \frac{b_k}{a_k}$$

<u>Fourier transzformáció</u>: tetsz. jelekre, a jel spektrum

$$X(\omega) = \mathcal{F}\{x(t)\} = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

$$c_0 = \frac{\ldots}{2} \qquad c_k = \sqrt{a_k^2 + b_k^2}$$

$$\vartheta_k = \operatorname{arctg} \frac{b_k}{a_k}$$

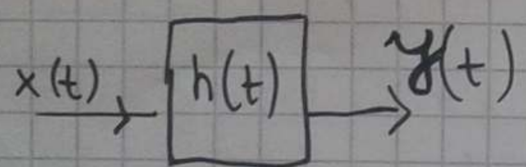
Fourier transzformáció: tetsz. jelekre, a jel spektruma

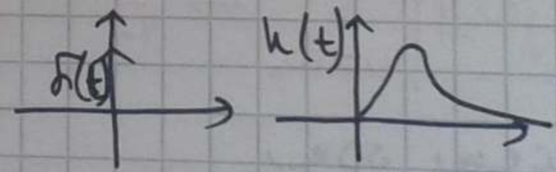
$$X(\omega) = \mathcal{F}\{x(t)\} = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} \, dt$$

$$x(t) = \mathcal{F}^{-1}\{X(\omega)\} = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) e^{j\omega t} \, d\omega$$

$$x(t) \overset{\mathcal{F}}{\Longleftrightarrow} X(\omega)$$

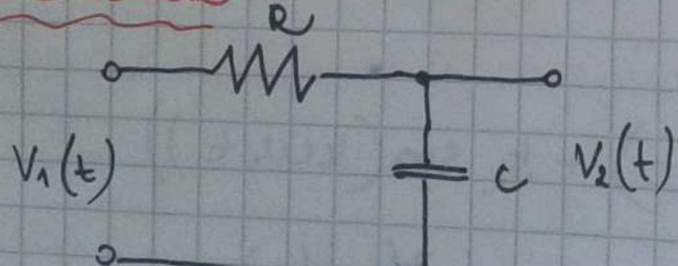
$$X(\omega) = |X(\omega)| e^{j\Theta(\omega)}$$

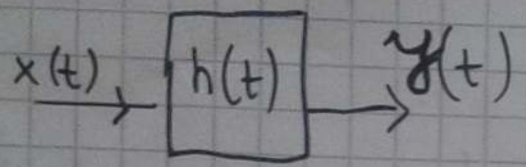


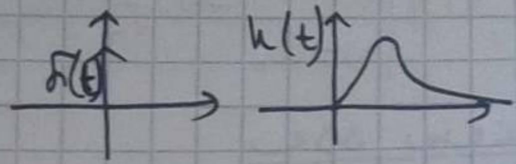


$$y(t) = x(t) * h(t) \longleftrightarrow Y(\omega) = X(\omega) \cdot H(\omega)$$

Feladatok:

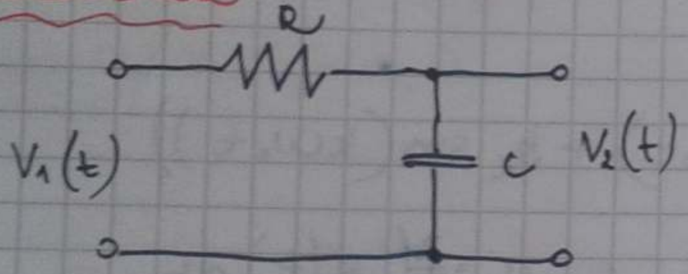




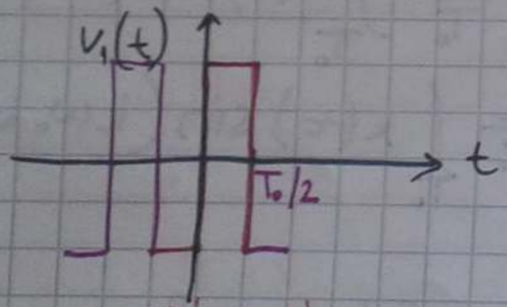


$$y(t) = x(t) * h(t) \longleftrightarrow Y(\omega) = X(\omega) \cdot H(\omega)$$

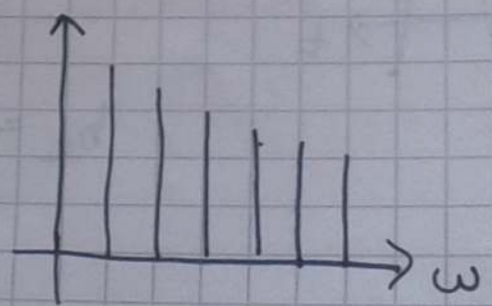
Feladatok:



$$\frac{1}{RC} \ll \frac{2\pi}{T_0}$$



jó periódus



DC komponens

$$a_k = 0 \qquad a_0 = 0$$

(páratlan)

$$a_0 = \frac{2}{T_0}\int_{T_0} v_i(t)\,dt = \frac{2}{T_0}\left[\int_{-T_0/2}^{0}(-1)\,dt + \int_{0}^{T_0/2}(1)\,dt\right]$$

$$\int_{0}^{-T_0/2} -(-1)\,dt$$

$$\int_{0}^{T_0/2} + + (-1)\,dt$$

u. arra kört ∫, ellentétes előjel

⇒ kiejtik egymást

$$b_k = \frac{2}{T_0}\left[\int_{-T_0/2}^{0}(-1)\sin(k\omega_0 t)\,dt + \int_{0}^{T_0/2}(+1)\sin(k\omega_0 t)\,dt\right] =$$

$$= \frac{2}{T_0}\left[-(-1)\cos(k\omega_0 t)\Big|^{0} \ldots \cos(k\omega_0 t)\Big|^{T_0/2}\right]$$

$\circ$

u. arra közt $\int$, ellentétes előjel
$\Rightarrow$ kiejtik egymást

$$b_k = \frac{2}{T_0}\left[\int_{-T_0/2}^{0}(-1)\sin(k\omega_0 t)\,dt + \int_{0}^{T_0/2}(+1)\sin(k\omega_0 t)\,dt\right] =$$

$$= \frac{2}{T_0}\left[-(-1)\frac{\cos(k\omega_0 t)}{k\omega_0}\Bigg|_{-\frac{T_0}{2}}^{0} - \frac{\cos(k\omega_0 t)}{k\omega_0}\Bigg|_{0}^{T_0/2}\right] =$$

$$= \frac{\cancel{2}}{\cancel{T_0}}\,\frac{\cos(0) - \cos\left(-\cancel{2}k\cdot\frac{\cancel{2}\pi}{\cancel{T_0}}\cdot\frac{\cancel{T_0}}{\cancel{2}}\right) - \cos\left(\frac{\cancel{T_0}}{\cancel{2}}\cdot k\cdot\frac{\cancel{2}\pi}{\cancel{T_0}}\right) + \cos(0)}{k\frac{\cancel{2}\pi}{\cancel{T_0}}} =$$

$$= \frac{2\left[1-\cos(k\pi)\right]}{k\pi} = \begin{cases} 0, & \text{ha } k = 2, 4, \dots \\ \frac{4}{k\pi}, & \text{ha } k = 1, 3, \dots \end{cases}$$

~~[illegible]~~:

$$v_1(t) = \frac{4}{\pi}\left[\sin(\omega_0 t) + \frac{\sin(3\omega_0 t)}{3} + \dots + \frac{\sin(k\omega_0 t)}{k} + \dots\right] =$$

$$= \frac{4}{\pi}\sum_k \frac{\sin(k\omega_0 t)}{k}, \quad k = 1, 3, 5, \dots$$

Mérnöki alak:

$$v_1(t) = \frac{4}{\pi}\left[\cos\left(\omega_0 t - \frac{\pi}{2}\right) + \cos\left(3\omega_0 t - \frac{\pi}{2}\right) \dots\right]$$

$$= \frac{2}{T_0}\left[-(-1)\frac{\cos(k\omega_0 t)}{k\omega_0}\Bigg|_{-\frac{T_0}{2}}^{0} - \frac{\cos(k\omega_0 t)}{k\omega_0}\Bigg|_{0}^{T_0/2}\right] =$$

$$= \frac{\cancel{2}}{\cancel{T_0}} \frac{\cos(0) - \cos\left(-k\cdot\frac{2\pi}{\cancel{T_0}}\cdot\frac{\cancel{T_0}}{2}\right) - \cos\left(\frac{\cancel{T_0}}{\cancel{2}}\cdot k\cdot\frac{2\pi}{\cancel{T_0}}\right) + \cos(0)}{k\frac{2\pi}{\cancel{T_0}}} =$$

$$= \frac{2\left[1-\cos(k\pi)\right]}{k\pi} = \begin{cases} 0, & \text{ha } k = 2, 4, \dots \\ \frac{4}{k\pi}, & \text{ha } k = 1, 3, \dots \end{cases}$$

Tehát (bipoláris négyszögjelre):

$$v_1(t) = \frac{4}{\pi}\left[\sin(\omega_0 t) + \frac{\sin(3\omega_0 t)}{3} + \dots + \frac{\sin(k\omega_0 t)}{k} + \dots\right] =$$

$$= \frac{4}{\pi}\sum_k \frac{\sin(k\omega_0 t)}{k}, \quad k = 1, 3, 5, \dots$$

Mérnöki alak:

$$v_1(t) = \frac{4}{\pi}\left[\cos\left(\omega_0 t - \frac{\pi}{2}\right) + \frac{\cos\left(3\omega_0 t - \frac{\pi}{2}\right)}{3} + \dots\right] =$$

$$= \frac{4}{\pi}\sum_k \frac{\cos\left(k\omega_0 t - \frac{\pi}{2}\right)}{k}, \quad k = 1, 3, 5, \dots$$

$$V_1(k) = \frac{4\pi}{k} \angle{-\pi/2} \quad , \; k = 1, 3, 5, \dots$$

$$\cancel{H(\omega)\Big|_{\omega = k\omega_0}} \cancel{\gg} \cancel{\frac{1}{c}}$$

$$\cancel{\Downarrow}$$

$$\cancel{\frac{1}{RC}}$$

$$H(\omega) = \frac{1}{1 + j\omega RC}\Bigg|_{\omega = k\omega_0 \gg \frac{1}{RC}} \approx \frac{1}{jk\omega_0 RC} =$$

$$= \frac{1}{k\omega_0 RC} e^{-j\frac{\pi}{2}}$$

$$V_2(k) = V_1(k) \cdot H(k\omega_0) = \frac{4\pi}{k} \cdot e^{-j\frac{\pi}{2}} \cdot \frac{1}{k\omega_0 RC} e^{-j\frac{\pi}{2}} =$$

$$= -\frac{1}{\omega_0 RC} \cdot \frac{4}{\pi} \cdot \frac{1}{k^2}$$

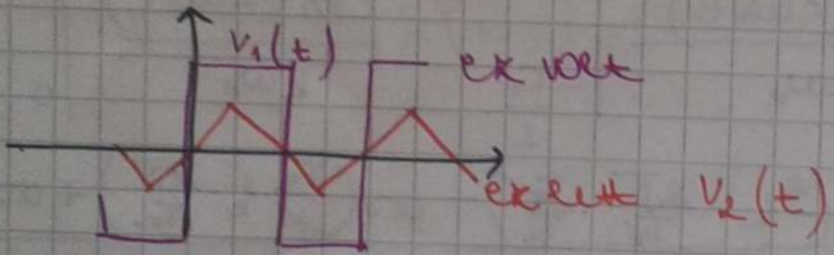
$$\frac{}{RC}$$

$$= \frac{1}{k\omega_0 RC} e^{-j\frac{\pi}{2}}$$

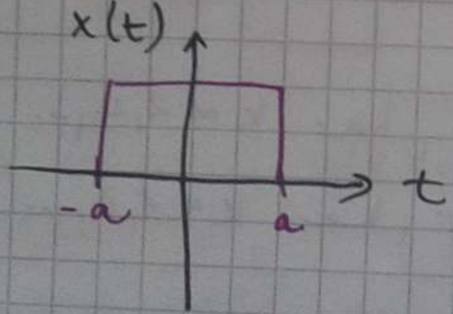
$$V_2(k) = V_1(k) \cdot H(k\omega_0) = \frac{4\pi}{k} \cdot e^{-j\frac{\pi}{2}} \cdot \frac{1}{k\omega_0 RC} e^{-j\frac{\pi}{2}} =$$

$$= -\frac{1}{\omega_0 RC} \cdot \frac{4}{\pi} \cdot \frac{1}{k^2}$$

$$V_2(t) = \frac{1}{\omega_0 RC}\left(-\frac{4}{\pi}\left[\cos(\omega_0 t) + \frac{\cos(3\omega_0 t)}{9} + \frac{\cos(5\omega_0 t)}{25} + \dots\right]\right)$$



2.



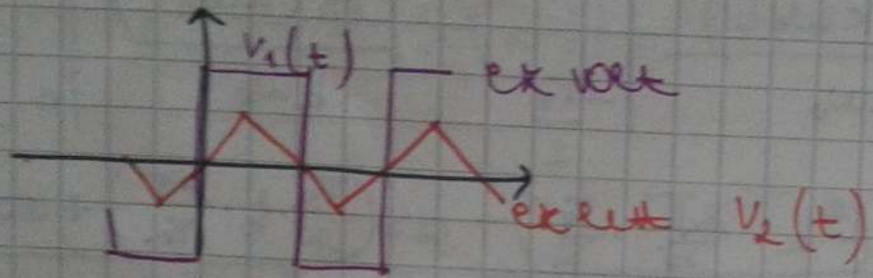
$$x(t) = p_0(t) = \begin{cases} 1 & |t| < 0 \\ 0 & |t| > 0 \end{cases}$$

$$X(\omega) = \int_{-\infty}^{\infty} p_0(t) e^{-j\omega t} dt =$$

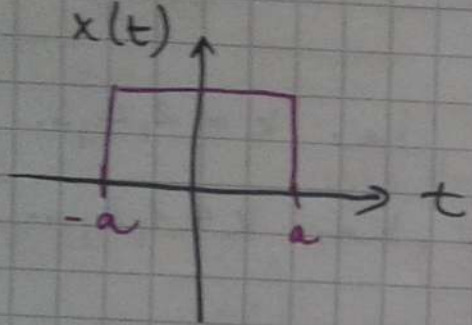
$$= \int_{-a}^{a} 1 \cdot e^{-j\omega t} dt = -\frac{1}{j\omega}\left[e^{-j\omega t}\right]_{-a}^{a} = -\frac{1}{j\omega}\left[e^{-j\omega a} - e^{-j\omega(-a)}\right] =$$

$$= \frac{1}{\ } \left[e^{j\omega} - e^{-j\omega a}\right]$$

$$\ldots \frac{\ldots}{9} \ldots \frac{\ldots}{25} + \ldots \Big] \Big)$$



(2.)



$$x(t) = p_0(t) = \begin{cases} 1 & |t| < 0 \\ 0 & |t| > 0 \end{cases}$$

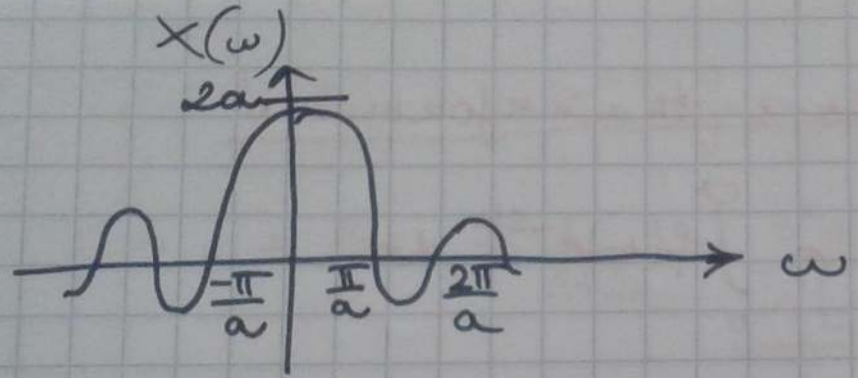
$$X(\omega) = \int_{-\infty}^{\infty} p_0(t) e^{-j\omega t} \, dt =$$

$$= \int_{-a}^{a} 1 \cdot e^{-j\omega t} \, dt = -\frac{1}{j\omega} \left[e^{-j\omega t} \right]_{-a}^{a} = -\frac{1}{j\omega} \left[e^{-j\omega a} - e^{-j\omega(-a)} \right] =$$

$$= \frac{1}{j\omega} \left[e^{j\omega} - e^{-j\omega a} \right] = 2 \frac{\sin(\omega a)}{\omega} =$$

$$= 2a \frac{\sin(\omega a)}{\omega a}$$

$$\sin(\omega a) = \frac{e^{j\omega a} - e^{-j\omega a}}{2j}$$



$\sin(\omega a) = 0$

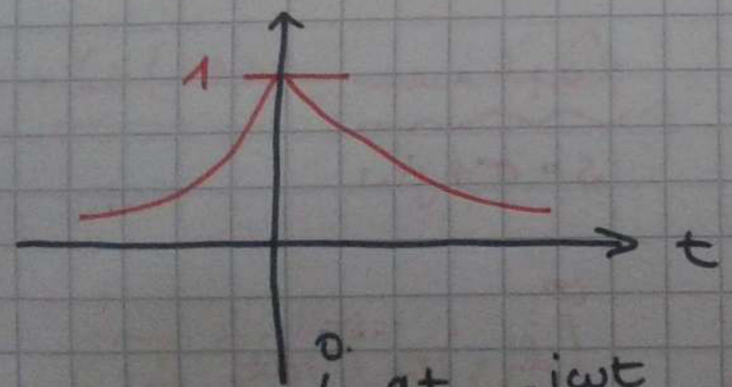
$\omega a = k\pi$

$\omega = \frac{k\pi}{a}$

③

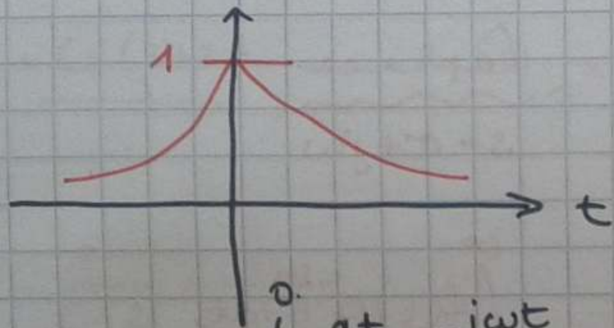
$x(t) = e^{-a|t|} \qquad a > 0$

$x(t) = e^{-a|t|} = \begin{cases} e^{-at} & t > 0 \\ e^{at} & t < 0 \end{cases}$



$X(\omega) = \int^{0} e^{at} e^{-j\omega t} \, dt + \int^{\infty} e^{-at} e^{-j\omega t} \, dt =$

$$x(t) = e^{-a|t|} = \begin{cases} e^{-at} & t > 0 \\ e^{at} & t < 0 \end{cases}$$



$$X(\omega) = \int_{-\infty}^{0} e^{at} e^{-j\omega t}\, dt + \int_{0}^{\infty} e^{-at} e^{-j\omega t}\, dt =$$

$$= \int_{-\infty}^{0} e^{(a-j\omega)t}\, dt + \int_{0}^{\infty} e^{-(a+j\omega)t}\, dt = \text{dábeárat}$$

$$= \frac{1}{a-j\omega} + \frac{1}{a+j\omega} = \frac{2a}{a^2+\omega^2}$$