komplexet hidja-e? Descartes-ba váltani

10.03. Gyakorlat

Jövő héten kis zh!

Állandósult állapot figyelembevétele:

- gerjesztést is figyelembe vesszük

Teljes válasz: tranziens + állandósult áll.

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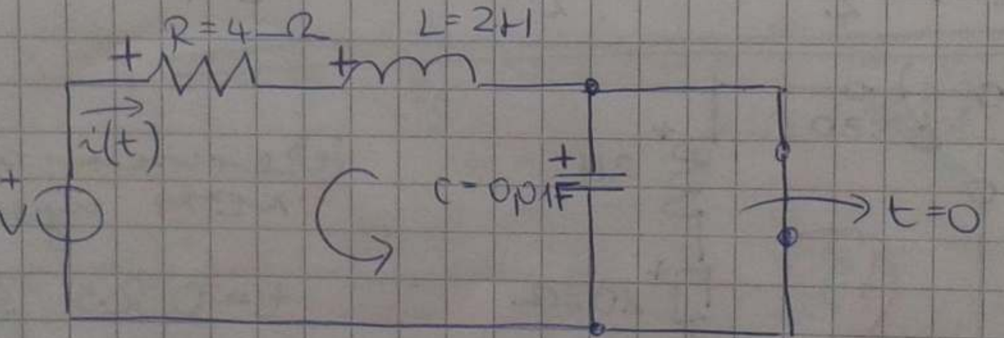

$i(t) = ?$

$V = 20$

1. huroktv. $(t \geq 0)$: $\sum v = 0 = V - Ri - L\frac{di}{dt} - \frac{1}{C}\int_0^t i\,d\tau \quad / \frac{d}{dt}$

2. homogén DE: $R\frac{di}{dt} + L\frac{d^2 i}{dt^2} + \frac{1}{C}i = 0$

3. mo. keresése: $i = Ae^{st}$ alakban

$$\underbrace{Ae^{st}}_{\neq 0}\underbrace{\left(sR + s^2L + \frac{1}{C}\right)}_{= 0} = 0 \qquad i \neq 0$$

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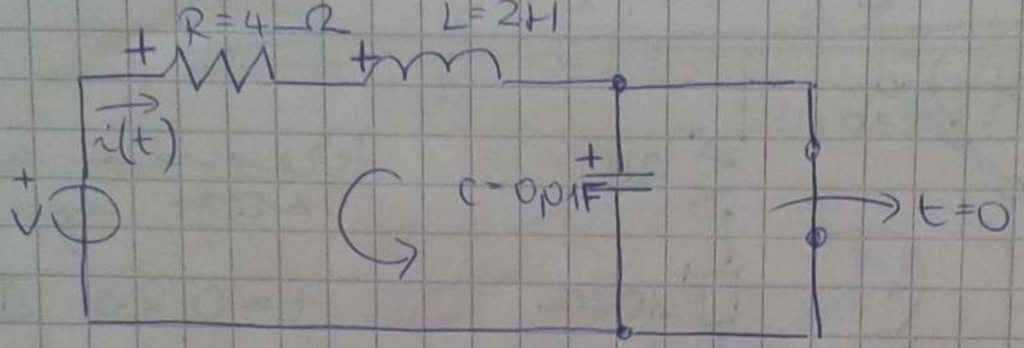

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$$s_{1,2} = \frac{-R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}} = -1 \pm j7 = -\alpha \pm j\omega$$

$\Rightarrow$ Tranziens: $i_t = Ae^{-\alpha t}\sin(\omega t + \varphi) =$

$$= Ae^{-t} \sin(7t + \varphi)$$

4. Állandósult áll.:

$\downarrow$ $\qquad i_g = 0$

- kondi teljesen feltöltött, szakadás, nem folyik áram állandó. áll-ndi
- kapacitás állandó áll-nak rövidzárként viselkedik

Teljes válasz: $i(t) = i_t(t) + i_g(t) = Ae^{-t} \sin(7t + \varphi) + 0$

$\boxed{t = 0+}$

- ind.-on átfo az áram, kap-on a fesz nem válts. pillanatban

$$i = i_L(0-) = i_L(0+) = \frac{V}{R} = \frac{20}{4} = 5A$$

$$5 = i(0+) = Ae^{0} \sin(0 + \varphi) =$$

$$= A \cdot \sin(\varphi) = 5$$

$$L \frac{di}{dt} = 0$$

$$\frac{di}{dt} = \ldots - Ae^{-t} \ldots \sin(7t + \varphi) \ldots e^{-t} \ldots$$

Teljes válasz: $i(t) = i_t(t) + i_g(t) = Ae^{-t}\sin(7t+\varphi) + 0$

$\boxed{t = 0+}$ $\quad i = i_L(0-) = i_L(0+) = \frac{V}{R} = \frac{20}{4} = 5A$

- ind.-on át az áram, kap-on a fesz nem vált. pillanatban

$5 = i(0+) = Ae^{0}\sin(0+\varphi) =$

$= A\cdot\sin(\varphi) = 5$

$L\frac{di}{dt} = 0$

$\frac{di}{dt} = 0 = -Ae^{-t}\{\sin(7t+\varphi) + 7Ae^{-t}\cos(7t+\varphi)$

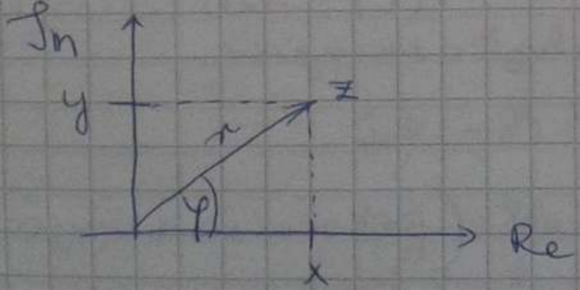
$t = 0$-ban: $-\cancel{A}\sin(\varphi) + 7\cancel{A}\cos(\varphi) = 0$

$tg\,\varphi = 7 \rightarrow \varphi = 81{,}9°$

$\underline{\underline{A}} = \frac{5}{\sin 81{,}9°} = \underline{\underline{5{,}05}}$

$i(t) = 5{,}05e^{-t}\sin(7t + 81{,}9°)$

(2) Komplex számok



algebrai alak:

$z = x + j\cdot y$

$x = Re\{z\}$ $\quad y = Im\{z\}$

$j = \sqrt{-1} \Rightarrow j^2 = -1$

$t = 0$-ban: $-A\sin(\varphi) + 7A\cos(\varphi) = 0$

$$\operatorname{tg}\varphi = 7 \rightarrow \varphi = 81{,}9^\circ$$

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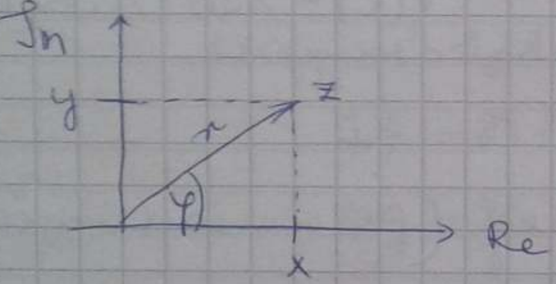
$$i(t) = 5{,}05\, e^{-t} \sin(7t + 81{,}9^\circ)$$

(2) Komplex számok



algebrai alak:

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polár alak:

$$z = |z|\angle\varphi = r\angle\varphi = r \cdot e^{j\varphi}$$

$$r = |z| = \sqrt{x^2 + y^2}$$

$$\varphi = \operatorname{arc\,tg}\frac{y}{x}$$

$$x = r\cos\varphi \qquad y = r\sin\varphi$$

$$Z = x + jy = r\cos\varphi + jr\sin\varphi = re^{j\varphi}$$

$$Z = re^{j\varphi} = |Z|e^{j\varphi}$$

1. $12\angle{-60^\circ} = 12\cdot\cos 60^\circ + 12\sin 60^\circ = 6 - j10{,}39$

2. $$\frac{(2+j5)(8e^{j10})}{2+j4+2\angle{-40^\circ}} = 9{,}672\angle{49{,}66^\circ}$$

$$\left.\begin{array}{l} 2+5j = \sqrt{2^2+5^2}\,\angle{\operatorname{arctg}\frac{5}{2}} = 5{,}385\angle{68{,}2^\circ} \\ 8e^{j10} = 8\angle{10^\circ} \end{array}\right\}(*)$$

$$(*) = 43{,}08\angle{78^\circ}$$

$$2+4j+2\angle{-40^\circ} = 2+j4+2\cos(-40^\circ)+j2\sin(-40^\circ) =$$

$$= 3{,}532 + j2{,}714 = 4{,}454\angle{37{,}54^\circ}$$

Komplex amplitúdók

$$(*) = 43{,}08\angle 78^\circ$$

$$2+4j+2\angle{-40^\circ} = 2+j4+2\cos(-40)+j2\sin(-40^\circ) =$$

$$= 3{,}532 + j2{,}714 = 4{,}454\angle 37{,}54^\circ$$

Komplex amplitúdók

$$v(t) = V_m \cos(\omega t + \varphi) = \mathrm{Re}\left\{ \underbrace{\left[V_{eff}\, e^{j\varphi}\right]}_{\text{komplex amplitúdó}} \left[\sqrt{2}\, e^{j\omega t}\right] \right\}$$

$$\downarrow$$

$$V_{eff}\angle\varphi$$

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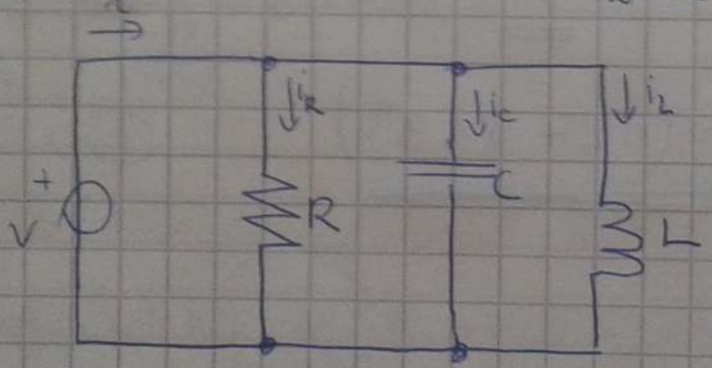


$R = 15\ \Omega$
$C = 83{,}3\ \mu F$
$L = 30\ mH$

$i(t) = ?$

$v(t) = 120\cdot\sqrt{2}\cdot\cos(1000t + 90^\circ)$

1, komplex ampl. tartományba átmegyünk

$$V = 120\angle 90^\circ$$

2, impedanciákat keressük meg (frekvencia függő)

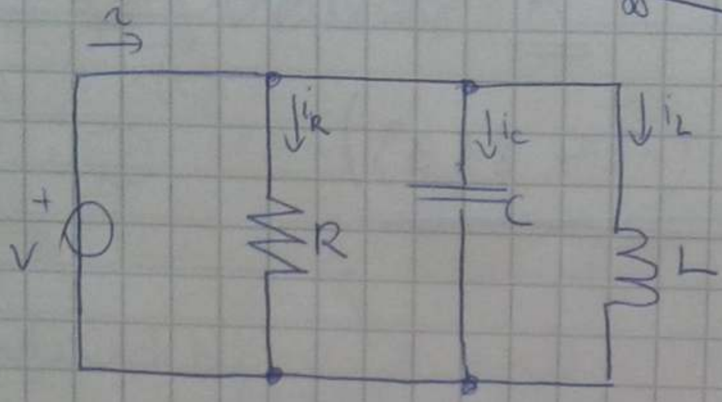
$$\omega = 1000\ \frac{rad}{s}$$

$$Z_R = 15\ \Omega$$

amplitudó

$V_{eff} \angle \varphi$

(1)



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1., komplex ampl. tartományba átmegyünk

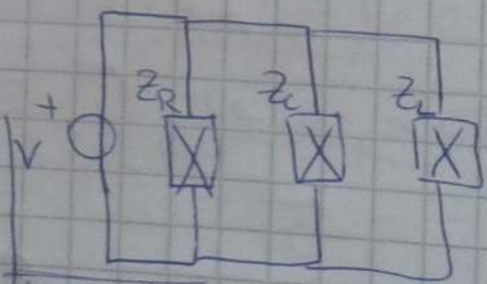
$V = 120 \angle 90^\circ$

2., impedanciákat ~~kat~~ kiírjuk meg (frekvencia függő)

$\omega = 1000\ \frac{rad}{s}$

$Z_R = 15\ \Omega$

$Z_C = \frac{1}{j\omega C} = \frac{1}{j \cdot 1000 \cdot 83{,}3 \cdot 10^{-6}} = -12j = 12 \angle -90^\circ$



$Z_L = j\omega L = j \cdot 1000 \cdot 30 \cdot 10^{-3} = 30 \angle 90^\circ$

3, ~~[illegible]~~ folyó áram impedanciákon

$$I_R = \frac{V}{Z_R} = \frac{120 \angle 90^\circ}{15} = 8 \angle 90^\circ$$

$$I_C = \frac{V}{Z_C} = \frac{120 \angle 90^\circ}{12 \angle -90^\circ} = 10 \angle 180^\circ$$

$$I_L = \frac{V}{Z_L} = \frac{120 \angle 90^\circ}{30 \angle 90^\circ} = 4 \angle 0^\circ$$

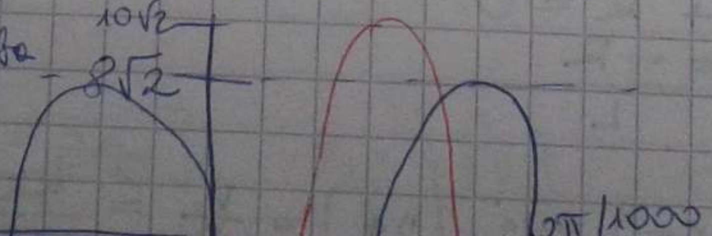
$$I = I_R + I_C + I_L = \underbrace{0 + j8}_{} \underbrace{-10 + j0}_{} + \underbrace{4 + j0}_{} = -6 + 8j =$$

$$= 10 \angle 126{,}9^\circ$$

$$i(t) = 10\sqrt{2} \cdot \cos(1000t + 126{,}9^\circ)$$

$$i_R(t) = 8\sqrt{2} \cdot \cos(1000t + 90^\circ)$$

poz. fáziseltolás : neg. irányba



$$2\pi f = 1000$$

$$\frac{1}{f} = \frac{2\pi}{1000} = T$$

$$J_c = \frac{V}{Z_c} = \frac{\quad}{12\angle -90^\circ} = 10\angle 180^\circ$$

$$J_L = \frac{V}{Z_L} = \frac{120\angle 90^\circ}{30\angle 90^\circ} = 4\angle 0^\circ$$

$$J = J_R + J_c + J_L = \underbrace{0 + j8}_{} - \underbrace{10 + j0}_{} + \underbrace{4 + j0}_{} = -6 + 8j =$$

$$= 10\angle 126{,}9^\circ$$

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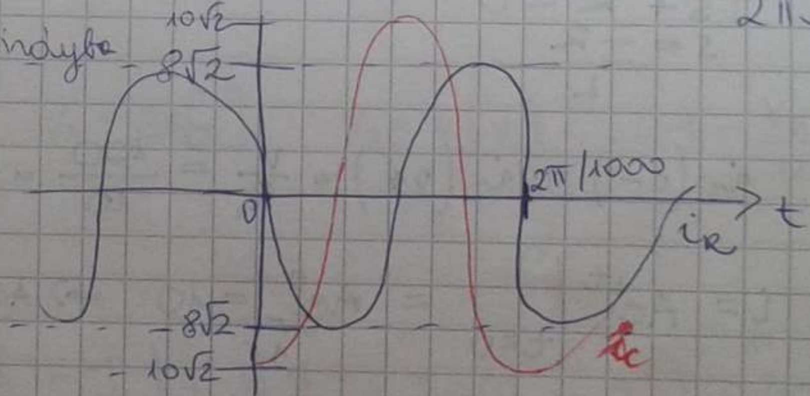

Gyakorlat

okt. 10.

HF zh feladat:

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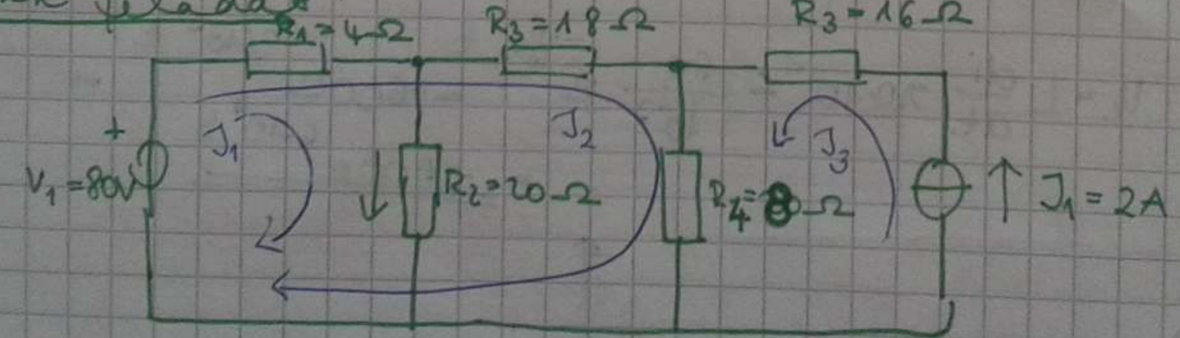

$$J_3 = 2A$$