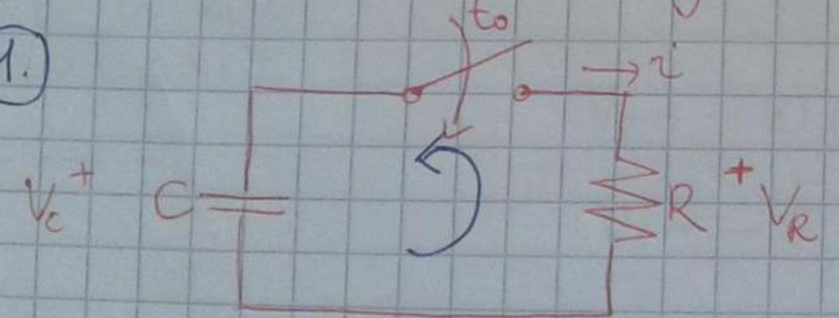
09.26. Diff. egyenlet:

$$\underbrace{a_n \frac{d^n y}{dt^n} + a_{n-1}\frac{d^{n-1} y}{dt^{n-1}} + \dots + a_1 \frac{dy}{dt}}_{\text{áramkör leírása}} = \underbrace{b_m \frac{d^m x}{dt^m} + \dots + b_0 x}_{\text{gerjesztések}}$$

teljes mo = tranziens (homogén) mo + állandósult állapot (partikuláris mo)

- ∀ gerjesztés = 0
- karakterisztikus egyenlet
  ↳ stabil-e?
- csak az áramkörre jell.

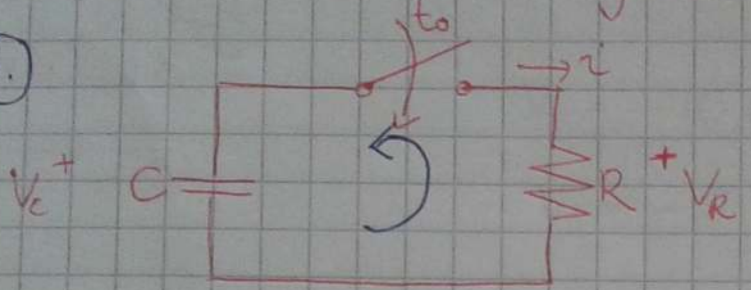
(1.)



Elsőrendű, mert 1 kapacitás és/vagy tekercs van benne (en. tároló elem)

↳ stabil-e?
- csak az áramkörre jell.

1.



Elsőrendű, mert 1 kapacitás és/vagy [illegible] van benne (ún. tároló elem)

Kezdeti felt. $v_C(0-) = V_0$

$$i_C = C\frac{dv}{dt} \Rightarrow v = v(0) + \frac{1}{C}\int_0^t i_C\,dt$$

Mo: 1, huroktegyenlet $t \geq 0$-ra

$$\Sigma v = 0 = v_C - v_R = V_0 + \frac{1}{C}\int_0^t i_C\,d\tau - Ri \Big|_{i_C = -i} =$$

$$= V_0 - \frac{1}{C}\int_0^t i\,d\tau - Ri$$

2, rendszerjellemző DE felírása

$$V_0 - \frac{1}{C}\int_0^t i\,d\tau - Ri = 0 \quad \Big/ \frac{d}{dt}$$

$$-\frac{1}{C}\cdot i - R\frac{di}{dt} = 0$$

$$RC\frac{di}{dt} + i = 0 \qquad i \neq 0$$

Mo: 1, huroktegyenlet $t \geq 0$ - ra

$$\sum v = 0 = V_C - V_R = V_0 + \frac{1}{C}\int_0^t i_C \, d\tau - Ri \Big|_{i_C = -i} =$$

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$$V_0 - \frac{1}{C}\int_0^t i \, d\tau - Ri = 0 \quad \Big/ \frac{d}{dt}$$

$$-\frac{1}{C} \cdot i - R\frac{di}{dt} = 0$$

$$RC\frac{di}{dt} + i = 0 \qquad i \neq 0$$

3, megoldás keresése $\quad i = A\exp(st) \quad \frac{di}{dt} = sA\exp(st)$

$$RC\,sAe^{st} + Ae^{st} = 0$$

$$Ae^{st}(RCs + 1) = 0$$

$$RCs + 1 = 0$$

$$s = \frac{-1}{RC} = \frac{-1}{\tau}$$

$\boxed{\tau = RC \text{ időállandó}}$

$i = Ae^{st}$

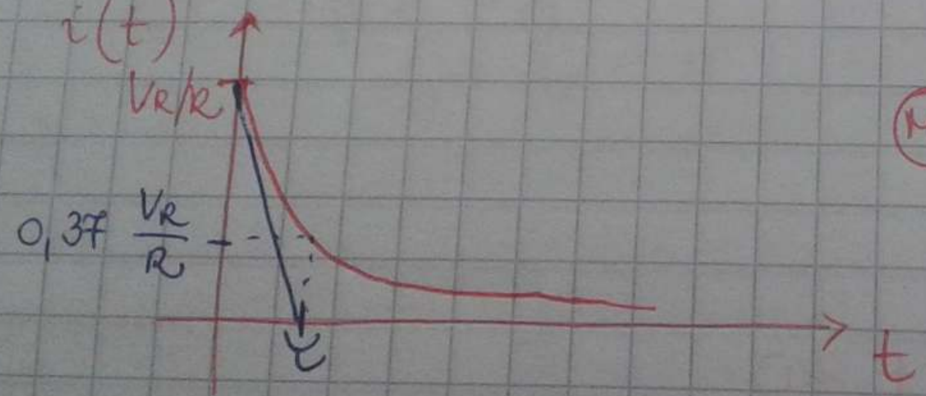
$s = -\frac{1}{\tau}$

$\Big\}\ i = Ae^{-t/\tau} = Ae^{-t/RC}$

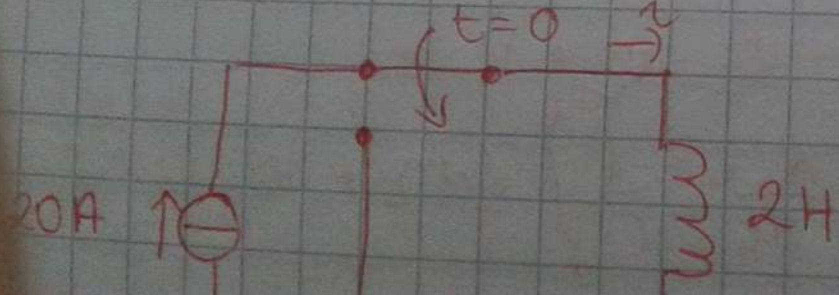
$t = +0$ $\quad$ $i\big|_{t=0} = A \cdot e^{0} = A = \frac{V_R}{R}$, mert

$V_C(-0) = V_C(+0) = V_R$

$$i = \frac{V_R}{R} \cdot e^{-t/RC}$$



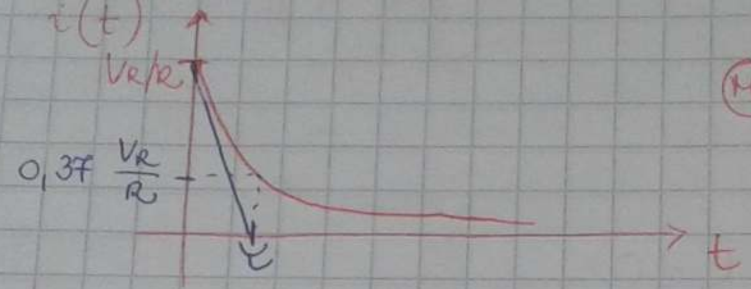
M $e^{-1} = 0.37$



~~$i(t) =$~~ $i(t = 0{,}2s) = ?$

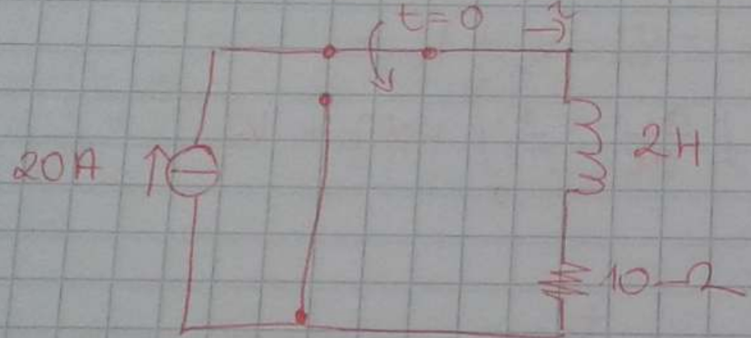
$v_L = L\, di$

$$i = \frac{V_R}{R}\, e^{-t/RC}$$



(M) $e^{-1} = 0.37$

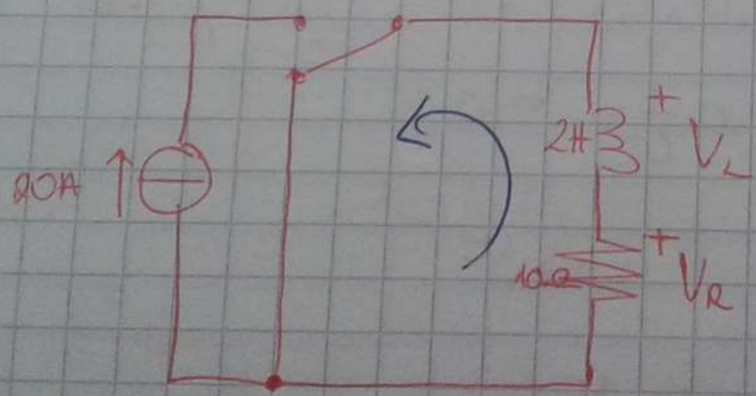
(2.)



$i(t=0{,}2s) = ?$

$v_L = L\frac{di}{dt}$

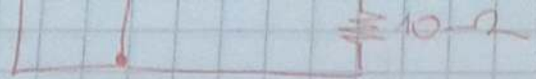
$i_L = i_L(0) + \frac{1}{L}\int_0^t V_{L_0}\, d\tau$



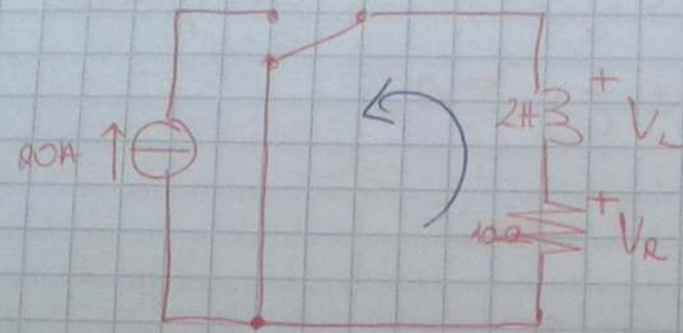
1) $\sum v = 0 = -V_L - V_R = 0 \quad -L\frac{di}{dt} - Ri$

2.) $Ri + L\frac{di}{dt} = 0$

3) uo. rešenje : $i = Ae^{st}$ $\quad \frac{di}{dt} = Ae^{st}$



$$i_L = i_L(0) + \frac{1}{L}\int_0 V_{L0}\, d\tau$$



1) $\sum v = 0 = -V_L - V_R = 0 \quad -L\frac{di}{dt} - Ri$

2) $Ri + L\frac{di}{dt} = 0$

3) m.o. keresése: $i = Ae^{st}$, $\frac{di}{dt} = sAe^{st}$

$$RAe^{st} + LsAe^{st} = 0$$

$$Ae^{st}(R + Ls) = 0 \qquad i \neq 0$$

$$R + Ls = 0$$

$$s = \frac{-R}{L} = \frac{-1}{\tau}$$

$$\tau = \frac{L}{R}$$

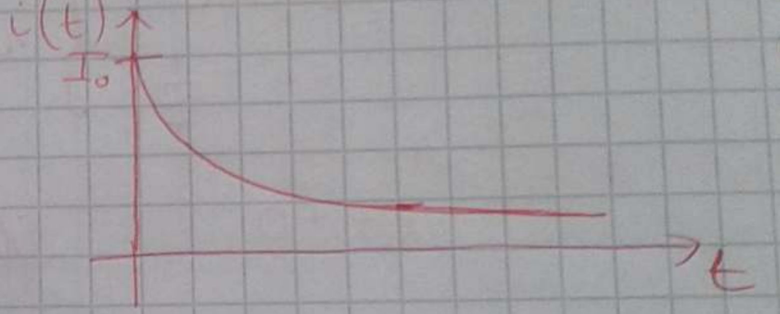
$$i = Ae^{-\frac{R}{L}t}$$

$i(-0) = i(+0)\big|_{t=0} = Ae^{0}$ L árama nem változhat egy pill. alatt (pill. szerint)

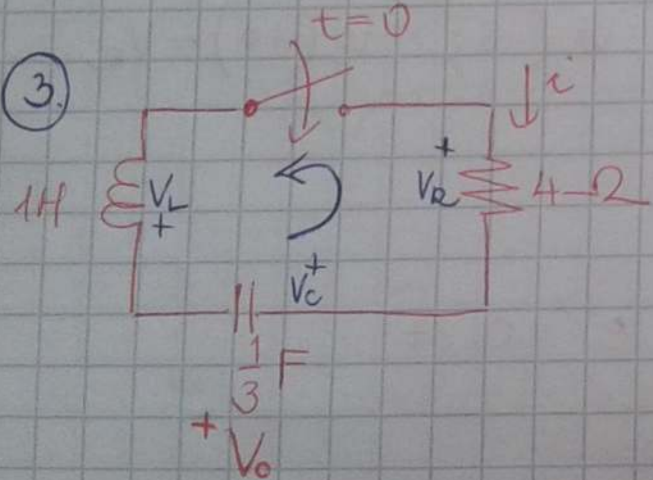
$A = 20 \qquad I_0 = 20A$

$$i(t) = I_0 \cdot e^{-\frac{R}{L}t} = 20 \cdot e^{-5t}$$

$$i(t=0{,}2) = 20e^{-5\cdot 0{,}2} = 7{,}36A$$



3.



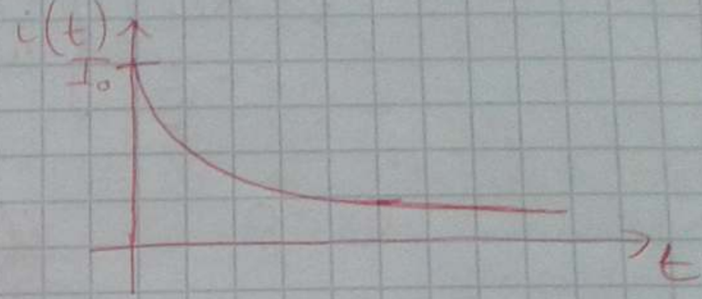
$i(t) = ?$

1.) [illegible]. $t \geq 0$

$$\sum v = 0 = -v_R - v_L - v_C + V_0 =$$

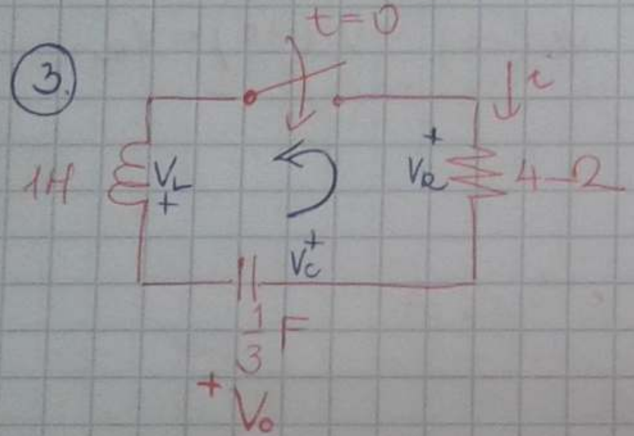
$$= -Ri - L\frac{di}{dt} - \frac{1}{C}\int_0^t i\,d\tau + V_0$$

$i(t) = I_0 \cdot e^{-\frac{R}{L}t} = 20\cdot e^{-5t}$



$i(t=0{,}2) = 20e^{-5\cdot 0{,}2} = 7{,}36A$

3.



$i(t) = ?$

1.) huroktörv. $t \geq 0$

$$\sum v = 0 = -v_R - v_L - v_C + V_0 =$$
$$= -Ri - L\frac{di}{dt} - \frac{1}{C}\int_0^t i\,d\tau + V_0$$

2.) tranziens egy.

$$Ri + L\frac{di}{dt} + \frac{1}{C}\int_0^t i\,d\tau - V_0 = 0 \quad \Big/ \frac{d}{dt}$$

$$R\frac{di}{dt} + L\frac{d^2 i}{dt^2} + \frac{1}{C}i = 0$$

3.) egy. keresése $\quad i = Ae^{st}$

1.) Kirchhoff: $t \geq 0$

$$\sum v = 0 = -v_R - v_L - v_C + V_0 =$$

$$= -Ri - L\frac{di}{dt} - \frac{1}{C}\int_0^t i\,d\tau + V_0$$

2.) tranziens ueo.

$$Ri + L\frac{di}{dt} + \frac{1}{C}\int_0^t i\,d\tau - V_0 = 0 \quad \Big/ \frac{d}{dt}$$

$$R\frac{di}{dt} + L\frac{d^2i}{dt^2} + \frac{1}{C}i = 0$$

3.) ueo. keresése $\qquad i = Ae^{st}$

$$\frac{d^2i}{dt^2} = s^2Ae^{st}$$

$$Ae^{st}\left(sR + s^2L + \frac{1}{C}\right) = 0 \qquad i \neq 0$$

$$sR + s^2L + \frac{1}{C} = 0$$

$$s_{1,2} = \frac{-R \pm \sqrt{R^2 - 4\frac{L}{C}}}{2L} \Rightarrow 2\text{ gyök}$$

$$\Rightarrow i = A_1e^{s_1t} + A_2e^{s_2t}$$

$\vdots$

$$s_1 = -1 \qquad s_2 = -3$$

$$i(t) = A_1e^{-t} + A_2e^{-3t}$$

$$R\frac{di}{dt} + L\frac{d^2 i}{dt^2} + \frac{1}{C}i = 0$$

3.) mo. ~~[illegible]~~ $i = Ae^{st}$

$$\frac{d^2 i}{dt^2} = s^2 A e^{st}$$

$$Ae^{st}\left(sR + s^2 L + \frac{1}{C}\right) = 0 \qquad i \neq 0$$

$$sR + s^2 L + \frac{1}{C} = 0$$

$$s_{1,2} = \frac{-R \pm \sqrt{R^2 - 4\frac{L}{C}}}{2L} \Rightarrow 2 \text{ gyök}$$

$$\Rightarrow i = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

$\vdots$

$$s_1 = -1 \qquad s_2 = -3$$

$$i(t) = A_1 e^{-t} + A_2 e^{-3t}$$

$$V_C(0-) = V_C(0+) = -V_0$$

$t = 0$: $i(0-) = i(0+) = 0 \Rightarrow V_R(0+) = Ri(0+) = \emptyset$

$$0 = -v_R - v_L - v_C + V_0 = 0 - V_L - 0 + V_0$$

$$V_0 = V_L$$
$$\left.\begin{array}{l} V_0 = V_L \\ V_L = L\dfrac{di}{dt} \end{array}\right\} L\frac{di}{dt} = V_0 \Rightarrow \frac{di}{dt} = \frac{V_0}{L}$$

$$\frac{di}{dt} = -A_1 e^{-t} - 3A_2 e^{-3t}$$

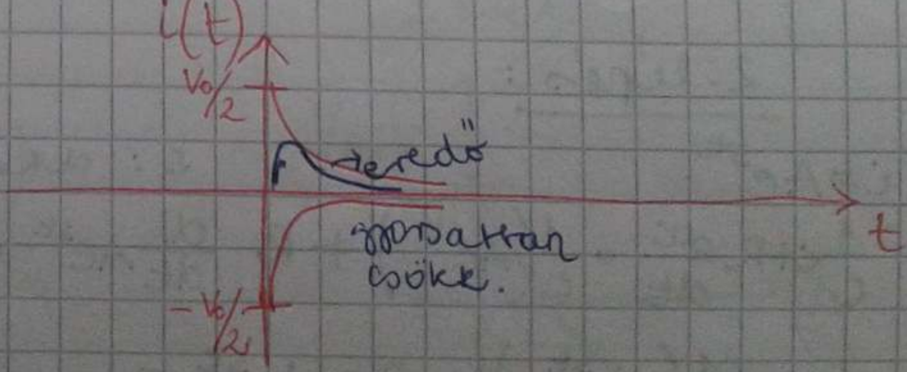
$$t=0: \quad \left.\begin{array}{l} -A_1 - 3A_2 = \dfrac{V_0}{L} \\ A_1 = -A_2 \end{array}\right\}$$

$$i\Big|_{t=0} = A_1 e^0 + A_2 e^0 = 0$$
$$A_1 + A_2 = 0$$

$$A_1 = \frac{V_0}{2L}\Bigg|_{L=1} = \frac{V_0}{2}$$

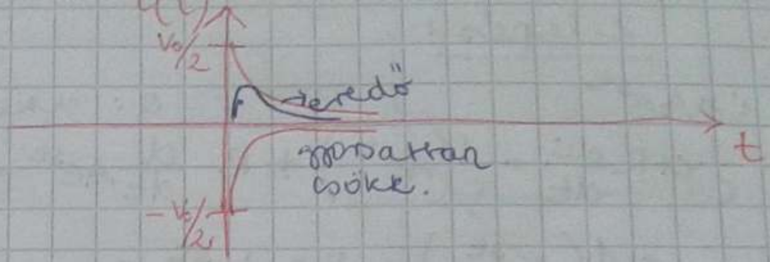
$$A_2 = \frac{-V_0}{2L}$$

$$i(t) = \frac{V_0}{2} e^{-t} - \frac{V_0}{2} e^{-3t}$$

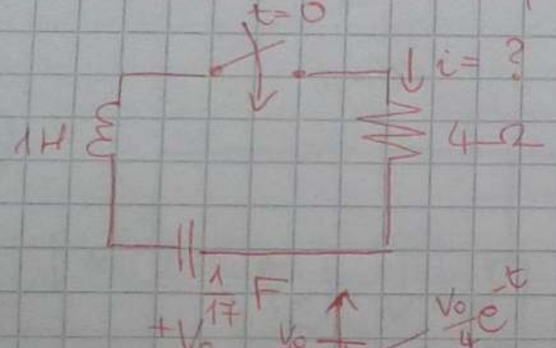


4.

$$i(t) = \frac{V_0}{2} e^{-t} - \frac{V_0}{2} e^{-3t}$$



(4.) áramkör u. az, értékek mások



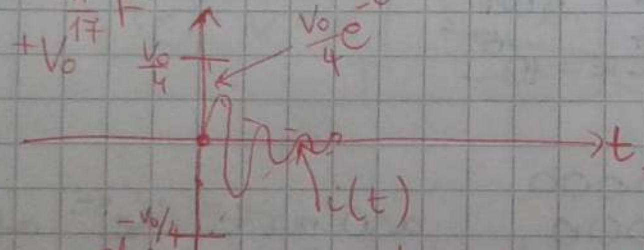
$$s_{1,2} = -1 \pm \sqrt{-16} = -1 \pm j4$$

$$s_1 = -\alpha + j\omega$$

$$s_2 = -\alpha - j\omega$$

néd meg a gyar. anyagban (nex)

$$i = Ae^{-\alpha t} \sin(\omega t + \varphi)$$



$$i = Ae^{-t} \sin(4t + \varphi)$$

$$\frac{di}{dt} = -Ae^{-t} \sin(4t + \varphi) + 4Ae^{-t} \cos(4t + \varphi)$$

$t = +0$) $\quad 0 = -v - \underset{\downarrow \, 0}{V_R} - \underset{\downarrow \, 0}{V_C} + V_0 = -V_L + V_0 = 0$

$$V_0 = V_L = L \frac{di}{dt} \qquad i_L = 0 = i$$

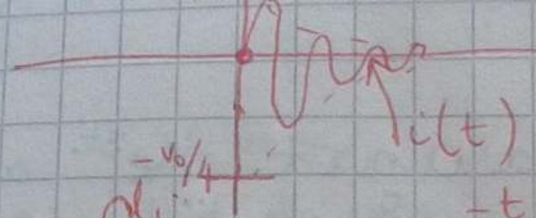
$$i(t = +0) = Ae^0 \sin(0 + \varphi) = \underbrace{A}_{\neq 0} \underbrace{\sin \varphi}_{= 0} = 0$$

$$\varphi = 0$$

nem amper

$$\frac{di}{dt} = \frac{V_0}{L} = -Ae^0 \sin 0 + 4Ae^0 \cos 0 = 4A$$

$$A = \frac{V_0}{4} = \frac{V_0}{...} \qquad i(t) = \frac{V_0}{4} e^{-t} \sin(4t)$$



$i = Ae^{-t}\sin(4t+\varphi)$

$$\frac{di}{dt} = -Ae^{-t}\sin(4t+\varphi) + 4Ae^{-t}\cos(4t+\varphi)$$

$t=0$) $\quad 0 = -v - \underset{0}{V_R} - \underset{0}{V_C} + V_0 = -V_L + V_0 = 0$

$V_0 = V_L = L\frac{di}{dt}$ $\qquad i_L = 0 =$

$$i(t=+0) = Ae^{0}\sin(0+\varphi) = \underbrace{A}_{\neq 0}\,\underbrace{\sin\varphi}_{=0} = 0$$

$\varphi = 0$ new amper

$$\frac{di}{dt} = \frac{V_0}{L} = -Ae^{0}\sin 0 + 4Ae^{0}\cos 0 = 4A$$

$A = \frac{V_0}{4L}\Big|_{L=1} = \frac{V_0}{4}$ $\qquad \underline{\underline{i(t) = \frac{V_0}{4}e^{-t}\sin(4t)}}$