1. $\sum_{k=1}^{\infty} \frac{5k+2}{k^2}$ konvergens-e

a) gyök-kritériummal: $\lim_{k\to\infty} \sqrt[k]{\frac{5k+2}{k^2}} = \lim_{k\to\infty} \sqrt[k]{\frac{5k}{k^2} + \frac{2}{k^2}} = \lim_{k\to\infty} \sqrt[k]{\frac{5}{k} + \frac{2}{k^2}} =$

$= \lim_{k\to\infty} \sqrt[k]{\frac{1}{k}\left(5 + \frac{2}{k}\right)} = \lim_{k\to\infty} \sqrt[k]{\frac{1}{k}} \sqrt[k]{\left(5+\frac{2}{k}\right)} = \lim_{k\to\infty} \frac{1}{\sqrt[k]{k}} \cdot \sqrt[k]{5 + \frac{2}{k}} = 1 \to$ nem tudjuk gyökkritériummal megállapítani

$\sqrt[k]{k} \to 1$, $\frac{2}{k} \to 0$, $\sqrt[k]{5} \to 1$

b) hányados-kritériummal: $\lim_{k\to\infty} \frac{\frac{5(k+1)+2}{(k+1)^2}}{\frac{5k+2}{k^2}} = \lim \frac{5(k+1)+2}{(k+1)^2} \cdot \frac{k^2}{(5k+2)} = \lim_{k\to\infty} \frac{(5k+5)+2}{(5k+2)} \cdot \frac{k^2}{(k+1)^2} =$

$= \lim_{k\to\infty} \frac{5k+7}{5k+2} \cdot \frac{k^2}{k^2+2k+1} = \lim_{k\to\infty} \underbrace{\frac{k}{k}}_{=1} \cdot \frac{5+\frac{7}{k}}{5+\frac{2}{k}} \cdot \underbrace{\frac{k^2}{k^2}}_{=1} \cdot \frac{1}{1+\frac{2}{k}+\frac{1}{k^2}} = 1 \to$ további vizsgálatot igényel

$\frac{7}{k} \to 0$, $\frac{2}{k} \to 0$, $\to 1$

2. $\sum_{k=1}^{\infty} \frac{k^2+2}{k^k}$ konvergens-e

~~a) gyök-kritériummal:~~ $\cancel{\lim_{k\to\infty} \sqrt[k]{\frac{k^2+2}{k^k}} = \lim_{k\to\infty} \frac{\sqrt[k]{k^2+2}}{\sqrt[k]{k^k}} = \lim_{k\to\infty} \frac{\sqrt[k]{k^2+2}}{k} = \lim_{k\to\infty} \frac{1}{k}(k^2+2)^{\frac{1}{k}}}$

$\cancel{= \lim_{k\to\infty} \frac{1}{k} \cdot \sqrt[k]{k^2+2} = \lim_{k\to\infty} \frac{1}{k} \sqrt[k]{k^k\left(\frac{1}{k^{k-2}} + \frac{2}{k^k}\right)} = \lim_{k\to\infty} \frac{1}{k} \sqrt[k]{k^k} \cdot \sqrt[k]{\left(\frac{1}{k^{k-2}} + \frac{2}{k^k}\right)} =}$

$\cancel{= \lim_{k\to\infty} \frac{1}{k} \cdot k \cdot \sqrt[k]{\frac{1}{k^{k-2}} + \frac{2}{k^k}}}$

a) gyök-kritériummal: $\lim_{k\to\infty} \frac{k^2+2}{k^k} \Rightarrow$ Öngyilkosság :)

b) hányados kritériummal: $\lim_{k\to\infty} \frac{\frac{(k+1)^2+2}{(k+1)^{k+1}}}{\frac{k^2+2}{k^k}} = \lim_{k\to\infty} \frac{[(k+1)^2+2]}{(k+1)^{k+1}} \cdot \frac{k^k}{(k^2+2)} =$

$= \lim_{k\to\infty} \frac{[(k^2+2k+1)+2]}{(k^2+2)} \cdot \frac{k^k}{(k+1)^{k+1}} = \lim \frac{[(k^2+2)+(2k+1)]}{(k^2+2)(k+1)} \cdot \frac{k^k}{(k+1)^k} = \lim_{k\to\infty} \left[\frac{k^2+2}{(k^2+2)(k+1)} + \frac{2k+1}{(k^2+2)(k+1)}\right] \cdot \left(\frac{k}{k+1}\right)^k =$

$= \lim_{k\to\infty} \left[\frac{1}{k+1} + \frac{\overbrace{2(k+1)-1}^{=(2k+1)}}{(k^2+2)(k+1)}\right] \cdot \left(\frac{k}{k+1}\right)^k = \lim_{k\to\infty} \left[\frac{1}{k+1} + 2 \cdot \frac{k+1}{(k^2+2)(k+1)} - \frac{1}{(k^2+2)(k+1)}\right] \cdot \left(\frac{k}{k+1}\right)^k =$

$= \lim_{k\to\infty} \left[\frac{1}{k+1} + 2 \cdot \frac{1}{k^2+2} - \frac{1}{(k^2+2)(k+1)}\right] \cdot \left(\frac{k}{k+1}\right)^k = 0 \Rightarrow$ konvergens

$\frac{1}{k+1} \to 0$, $\frac{1}{k^2+2} \to 0$, $\frac{1}{(k^2+2)(k+1)} \to 0$, $\left(\frac{k}{k+1}\right)^k \to \frac{1}{e}$