Hálekel Tamás

1. $\underline{f(x) = (2x+1)^7}$ $\quad g(x) = 2x+1 \quad g'(x) = 2 \quad h(x) = x^7 \quad h'(x) = 7x^6$

$f(x) = \underbrace{h \circ g}_{= h(g(x))} \quad f'(x) = h'(g(x)) \cdot g'(x)$

$f'(x) = 7(2x+1)^6 \cdot 2 = \boxed{14(2x+1)^6}$

2. $\underline{f(x) = \sqrt{x^2-4} = (x^2-4)^{\frac{1}{2}}}$ $\quad g(x) = x^2-4 \quad g'(x) = 2x \quad h(x) = x^{\frac{1}{2}} \quad h'(x) = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$

$f(x) = h \circ g = h(g(x)) \quad f'(x) = h'(g(x)) \cdot g'(x)$

$\boxed{f'(x) = \frac{2x}{2\sqrt{x^2-4}}} \qquad \boxed{(\sqrt{x})' = \frac{1}{2\sqrt{x}}}$ (új „szabály")

3. $\underline{f(x) = \log_2(x^2-5x+6)}$ $\quad g(x) = \log_2(x) \quad g'(x) = \frac{1}{x \ln 2} \quad h(x) = x^2-5x+6$

$f(x) = g \circ h = g(h(x)) \quad f'(x) = g'(h(x)) \cdot h'(x) \qquad h'(x) = 2x-5$

$\boxed{f'(x) = \frac{2x-5}{(x^2-5x+6)\ln 2}}$

4. $\underline{f(x) = \ln(x^2+2)}$ $\quad g(x) = \ln(x) \quad g'(x) = \frac{1}{x} \quad h(x) = x^2+2 \quad h'(x) = 2x$

$f(x) = g \circ h = g(h(x)) \qquad f'(x) = g'(h(x)) \cdot h'(x)$

$\boxed{f'(x) = \frac{2x}{x^2+2}}$

5. $\underline{f(x) = e^{x^2+3x+6}}$ $\quad g(x) = e^x \quad g'(x) = e^x \quad h(x) = x^2+3x+6 \quad h'(x) = 2x+3$

$f(x) = g \circ h = g(h(x)) \qquad f'(x) = g'(h(x)) \cdot h'(x)$

$\boxed{f'(x) = e^{x^2+3x+6} \cdot (2x+3)}$

6. $\underline{f(x) = \sqrt{x^2+5x+6}}$ $\quad g(x) = \sqrt{x} = x^{\frac{1}{2}} \quad g'(x) = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}} \quad h(x) = x^2+5x+6$

$f(x) = g \circ h = g(h(x)) \quad f'(x) = g'(h(x)) \cdot h'(x) \qquad h'(x) = 2x+5$

$\boxed{f'(x) = \frac{2x+5}{2\sqrt{x^2+5x+6}}}$

7. $\underline{f(x) = \sqrt{1+e^x}}$ $\quad g(x) = \sqrt{x} = x^{\frac{1}{2}} \quad g'(x) = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}} \quad h(x) = 1 \quad h'(x) = 0$

$f(x) = g \circ (h+k) = g(h(x)+k(x)) \quad f'(x) = g'(h(x)+k(x)) \cdot (h'(x)+k'(x)) \quad k(x) = e^x \quad k'(x) = e^x$

$\boxed{f'(x) = \frac{e^x}{2\sqrt{1+e^x}}}$

8. $\underline{f(x) = \ln(1-\sqrt{x})}$ $\quad g(x) = \ln(x) \quad g'(x) = \frac{1}{x} \quad h(x) = 1-\sqrt{x} \quad h'(x) = -\frac{1}{2\sqrt{x}}$

$f(x) = g \circ h = g(h(x)) \qquad f'(x) = g'(h(x)) \cdot h'(x)$

$f'(x) = \frac{-1}{(1-\sqrt{x})2\sqrt{x}} = \frac{-1}{2\sqrt{x}-2x} = \boxed{\frac{1}{2x-2\sqrt{x}}}$

9. $\underline{f(x) = \ln(1-3x^2)}$ $\quad g(x) = \ln(x) \quad g'(x) = \frac{1}{x} \quad h(x) = 1-3x^2 \quad h'(x) = -6x$

$f(x) = g(h(x)) \quad f'(x) = g'(h(x)) \cdot h'(x) = \frac{-6x}{1-3x^2} = \boxed{\frac{6x}{3x^2-1}}$

(10.) $f(x) = e^{x^2} + \ln 7x$ $g(x) = e^x$ $g'(x) = e^x$ $h(x) = x^2$ $h'(x) = 2x$

$i(x) = \ln(x)$ $i'(x) = \frac{1}{x}$ $j(x) = 7x$ $j'(x) = 7$

$f(x) = g \circ h + i \circ j = g(h(x)) + i(j(x))$ $f'(x) = g'(h(x)) \cdot h'(x) + i'(j(x)) \cdot j'(x)$

$f'(x) = e^{x^2} \cdot 2x + \frac{7}{7x} = \boxed{e^{x^2} \cdot 2x + \frac{1}{x}}$ $\boxed{+\ (\ln(c \cdot x))' = \frac{1}{x}}$ (új szabály)

(11.) $f(x) = \sqrt{2 - 5x^2}$ $g(x) = \sqrt{x} = x^{\frac{1}{2}}$ $g'(x) = \frac{1}{2} x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$ $h(x) = 2 - 5x^2$ $h'(x) = -10x$

$f(x) = g \circ h = g(h(x))$ $f'(x) = g'(h(x)) \cdot h'(x)$

$f'(x) = \frac{-10x}{2\sqrt{2-5x^2}} = \boxed{\frac{-5x}{\sqrt{2-5x^2}}}$

(12.) $f(x) = (x^2 + 3) \cdot \log_5(4x - 2)$ $g(x) = x^2 + 3$ $g'(x) = 2x$

$i(x) = \log_5 x$ $i'(x) = \frac{1}{x \ln 5}$ $j(x) = 4x - 2$ $j'(x) = 4$

$f(x) = g \cdot i \circ j = g(x) \cdot i(j(x))$ $f'(x) = g'(x) \cdot i(j(x)) + g(x) \cdot i'(j(x)) \cdot j'(x)$

$\boxed{f'(x) = 2x \log_5(4x-2) + (x^2+3) \frac{4}{(4x-2)\ln 5}} = \boxed{2x \log_5(4x-2) + \frac{2(x^2+3)}{(2x-1)\ln 5}}$

(13.) $f(x) = \sqrt{2 + 5x^3} - e^{x^2+3} - \ln(9x - 1)$ $g(x) = \sqrt{x}$ $g'(x) = \frac{1}{2\sqrt{x}}$ $h(x) = 2 + 5x^3$ $h'(x) = 15x^2$

$i(x) = e^x$ $i'(x) = e^x$ $j(x) = x^2 + 3$ $j'(x) = 2x$

$k(x) = \ln x$ $k'(x) = \frac{1}{x}$ $l(x) = 9x - 1$ $l'(x) = 9$

$f(x) = g(h(x)) - i(j(x)) - k(l(x))$ $f'(x) = g'(h(x)) h'(x) - i'(j(x)) j'(x) - k'(l(x)) l'(x)$

$\boxed{f'(x) = \frac{15x^2}{2\sqrt{2+5x^3}} - e^{x^2+3} \cdot 2x - \frac{9}{9x-1}}$

(14.) $f(x) = e^{7x} \cdot \lg 5x$ $g(x) = e^x$ $g'(x) = e^x$ $h(x) = 7x$ $h'(x) = 7$ új szabály $\boxed{(e^{c \cdot x})' = c \cdot e^{cx}}$

$i(x) = \lg(x)$ $i'(x) = \frac{1}{x \ln 10}$ $j(x) = 5x$ $j'(x) = 5$

$f(x) = g(h(x)) \cdot i(j(x))$

$f'(x) = g'(h(x)) \cdot h'(x) \cdot i(j(x)) + g(h(x)) \cdot i'(j(x)) \cdot j'(x)$

$f'(x) = e^{7x} \cdot 7 \cdot \lg 5x + e^{7x} \cdot \frac{5}{5x \ln 10} = 7 \cdot e^{7x} \cdot \lg 5x + \frac{e^{7x}}{x \cdot \ln 10} = \boxed{e^{7x}\left(7 \cdot \lg 5x + \frac{1}{x \cdot \ln 10}\right)}$

(15.) $f(x) = \ln\sqrt{x^2 - 2}$ $g(x) = \ln x$ $g'(x) = \frac{1}{x}$ $h(x) = \sqrt{x}$ $h'(x) = \frac{1}{2\sqrt{x}}$

$i(x) = x^2 - 2$ $i'(x) = 2x$

$f(x) = g(h(i(x)))$ $f'(x) = g'(h(i(x))) \cdot h'(i(x)) \cdot i'(x)$

$f'(x) = \frac{1}{\sqrt{x^2-2}} \cdot \frac{2x}{2\sqrt{x^2-2}} = \boxed{\frac{x}{x^2-2}}$

(16.) $f(x) = (x^2 + 5x - 3)^9$ $g(x) = x^2 + 5x - 3$ $g'(x) = 2x + 5$ $h(x) = x^9$ $h'(x) = 9x^8$

$f(x) = h(g(x))$ $f'(x) = h'(g(x)) \cdot g'(x)$

$\boxed{f'(x) = 9(x^2 + 5x - 3)^8 \cdot (2x + 5)}$

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(17.) $f(x)=\lg(2x+4)$ $\quad g(x)=\lg(x)$ $\quad g'(x)=\frac{1}{x\ln 10}$ $\quad h(x)=2x+4$ $\quad h'(x)=2$

$f(x)=g(h(x))$ $\quad f'(x)=g'(h(x))\cdot h'(x)=\frac{2}{(2x+4)\ln 10}=\boxed{\frac{1}{(x+2)\ln 10}}$

(18.) $f(x)=\ln(3x)$ $\quad \boxed{f'(x)=\frac{1}{x}}$ (ld: 10. feladat új módszer ~~megoldása~~)

(19.) $f(x)=e^{5x}$ $\quad \boxed{f'(x)=5\cdot e^{5x}}$ (ld: 14. feladat új módszer ~~megoldása~~)

(20.) $f(x)=\sqrt{1-4x^2}$ $\quad g(x)=\sqrt{x}$ $\quad g'(x)=\frac{1}{2\sqrt{x}}$ $\quad h(x)=1-4x^2$ $\quad h'(x)=-8x$

$f(x)=g(h(x))$ $\quad f'(x)=g'(h(x))\cdot h'(x)=\frac{-8x}{2\sqrt{1-4x^2}}=\boxed{\frac{-4x}{\sqrt{1-4x^2}}}$

(21.) $f(x)=\frac{1+e^{2x}}{1-e^{2x}}$ $\quad g(x)=1+e^{2x}$ $\quad g'(x)=2\cdot e^{x}$ $\quad h(x)=1-e^{2x}$ $\quad h'(x)=(-2)e^{x}$

$f(x)=\frac{g(x)}{h(x)}$ $\quad \cancel{f'(x)=\frac{f'(x)\cdot g(x)+f(x)\cdot g'(x)}{g^2(x)}}$ $\quad f'(x)=\frac{g'(x)\cdot h(x)-g(x)\cdot h'(x)}{h^2(x)}$

$f'(x)=\frac{2\cdot e^{x}\cdot(1-e^{2x})-(1+e^{2x})\cdot(-2)e^{x}}{(1-e^{2x})^2}=\frac{2\cdot e^{x}-2e^{3x}+2e^{x}+2e^{3x}}{(1-e^{2x})^2}=\boxed{\frac{4e^{2x}}{(1-e^{2x})^2}}$

(22.) $f(x)=\left(\frac{1+x}{1-x}\right)^2=\frac{1+2x+x^2}{(1-x)^2}$ $\quad g(x)=x^2+2x+1$ $\quad g'(x)=2x+2$ $\quad h(x)=1+x^2-2x$ $\quad h'(x)=(+2)x-2$

$(x+1)^2$ $\quad (x-1)^2$ $\quad 2x-2$

$f(x)=\frac{g(x)}{h(x)}$ $\quad f'(x)=\frac{g'(x)h(x)-g(x)h'(x)}{h^2(x)}=\frac{2(x+1)(x-1)^2-(x+1)^2(x-1)\cdot 2}{(x-1)^4}$

$f'(x)=2(x+1)\frac{(x-1)-(x+1)}{(x-1)^3}=2(x+1)\frac{-2}{(x-1)^3}=\boxed{\frac{-4(x+1)}{(x-1)^3}}$

(23.) $f(x)=\frac{1-\sqrt{x}}{1+\sqrt{x}}$ $\quad g(x)=1-\sqrt{x}$ $\quad g'(x)=\frac{-1}{2\sqrt{x}}$ $\quad h(x)=1+\sqrt{x}$ $\quad h'(x)=\frac{1}{2\sqrt{x}}$

$f(x)=\frac{g(x)}{h(x)}$ $\quad f'(x)=\frac{g'(x)h(x)-g(x)\cdot h'(x)}{h^2(x)}$

$f'(x)=\left(\frac{-(1+\sqrt{x})}{2\sqrt{x}}-\frac{1-\sqrt{x}}{2\sqrt{x}}\right)\cdot\frac{1}{(1+\sqrt{x})^2}=\frac{-1-\sqrt{x}-1+\sqrt{x}}{2\sqrt{x}}\cdot\frac{1}{(1+\sqrt{x})^2}=\boxed{\frac{-1}{(1+\sqrt{x})^2\sqrt{x}}}$

(24.) $f(x)=\frac{e^{3x}}{2+e^{3x}}$ $\quad g(x)=e^{3x}$ $\quad g'(x)=3\cdot e^{3x}$ $\quad h(x)=2+e^{3x}$ $\quad h'(x)=3\cdot e^{3x}$

$f(x)=\frac{g(x)}{h(x)}$ $\quad f'(x)=\frac{g'(x)h(x)-g(x)h'(x)}{h^2(x)}$

$f'(x)=\frac{3\cdot e^{3x}(2+e^{3x})-e^{3x}\cdot 3\cdot e^{3x}}{(2+e^{3x})^2}=\frac{6e^{3x}+3e^{6x}-3e^{6x}}{(2+e^{3x})^2}=\frac{6e^{3x}}{(2+e^{3x})^2}=\boxed{\frac{6e^{3x}}{(2+e^{3x})^2}}$

(25.) $f(x)=4x^2+e^{2x}+\ln 2x$ $\quad f'(x)=8x+2e^{2x}+\frac{1}{x}$ (a két újonnan levezetett szabály alapján)

(26.) $f(x)=10^{2x}-(x^2+7)^3$ $\quad g(x)=10^{x}$ $\quad g'(x)=10^{x}\ln 10$ $\quad h(x)=2x$ $\quad h'(x)=2$

$i(x)=x^2+7$ $\quad i'(x)=2x$ $\quad j(x)=x^3$ $\quad j'(x)=3x^2$

$f(x)=g(h(x))-j(i(x))$ $\quad f'(x)=g'(h(x))h'(x)-j'(i(x))i'(x)$

$f'(x)=10^{2x}\ln(10)\cdot 2-3(x^2+7)^2\cdot 2x=\boxed{2\cdot 10^{2x}\cdot\ln 10-6x(x^2+7)^2}$

(27.) $f(x)=\ln^2(x^2+1)$ $\quad g(x)=x^2$ $\quad g'(x)=2x$ $\quad h(x)=\ln x$ $\quad h'(x)=\frac{1}{x}$ $\quad i(x)=x^2+1$ $\quad i'(x)=2x$

$f(x)=g(h(i(x)))$ $\quad f'(x)=g'(h(i(x)))\cdot h'(i(x))\cdot i'(x)$

$f'(x)=2\ln(x^2+1)\frac{1}{x^2+1}\cdot 2x=\boxed{\frac{4x\ln(x^2+1)}{x^2+1}}$

$\cancel{f(x)}$ (28.) $\underline{f(x) = (x^2+4)^2}$ $g(x) = x^2$ $g'(x) = 2x$ $h(x) = x^2 + 4$ $h'(x) = 2x$

$f(x) = g(h(x))$ $f'(x) = g'(h(x)) \cdot h'(x) = 2(x^2+4) \cdot 2x = \boxed{4x(x^2+4)}$

(29.) $\underline{f(x) = (x^3+13)^5}$ $g(x) = x^5$ $g'(x) = 5x^4$ $h(x) = x^3 + 13$ $h'(x) = 3x^2$

$f(x) = g(h(x))$ $f'(x) = g'(h(x)) \cdot h'(x) = 5(x^3+13)^4 \cdot 3x^2 = \boxed{15x^2(x^3+13)^4}$

(30) $\underline{f(x) = \sqrt{4x^2 - 6x + 9}}$ $g(x) = \sqrt{x}$ $g'(x) = \frac{1}{2\sqrt{x}}$ $h(x) = 4x^2 - 6x + 9$ $h'(x) = 8x - 6$

$f(x) = g(h(x))$ $f'(x) = g'(h(x))\, h'(x) = \frac{8x-6}{2\sqrt{4x^2-6x+9}} = \frac{2(4x-3)}{2\sqrt{4x^2-6x+9}} = \boxed{\frac{4x-3}{\sqrt{4x^2-6x+9}}}$

(31.) $\underline{f(x) = \sqrt{x^5 + 5x^3}}$ $g(x) = \sqrt{x}$ $g'(x) = \frac{1}{2\sqrt{x}}$ $h(x) = \cancel{\sqrt{x^5 + 5x^3}}$ $h'(x) = 5x^4 + 15x^2$

$f(x) = g(h(x))$ $f'(x) = g'(h(x)) \cdot h'(x) = \boxed{\frac{5x^4 + 15x^2}{2\sqrt{x^5+5x^3}}}$

(32.) $\underline{f(x) = \frac{1}{3x^8 + 2x - x^{-3}}}$ $g(x) = 1$ $g'(x) = 0$ $h(x) = 3x^8 + 2x - x^{-3}$

$f(x) = \frac{g(x)}{h(x)}$ $f'(x) = \frac{\underbrace{g'(x)}_{=0} h(x) + \underbrace{g(x)}_{=1} h'(x)}{h^2(x)} = \frac{h'(x)}{h^2(x)}$ $h'(x) = 24x^7 + 2 + 3x^{-4}$

$\boxed{f'(x) = \frac{24x^7 + 2 + 3x^{-4}}{3x^8 + 2x - x^{-3}}}$

(33.) $\underline{f(x) = \frac{3{,}5\, x^{-\frac{4}{3}}}{\sqrt{4x^2-6x+9}}} = \frac{\frac{7}{2}\frac{1}{x^{\frac{4}{3}}}}{\sqrt{4x^2-6x+9}} = \cancel{\frac{7}{2x^{\frac{4}{3}}\sqrt{4x^2-6x+9}}}$

$g(x) = \frac{7}{2} \cdot x^{-\frac{4}{3}}$ $g'(x) = -\frac{7}{2} \cdot \frac{4}{3} x^{-\frac{7}{3}} = -\frac{14}{3} x^{-\frac{7}{3}}$

$h(x) = \sqrt{x}$ $h'(x) = \frac{1}{2\sqrt{x}}$ $i(x) = 4x^2 - 6x + 9$ $i'(x) = 8x - 6$

$f(x) = \frac{g(x)}{h(i(x))} \Rightarrow f'(x) = \frac{g'(x)\, h(i(x)) - g(x)\, h'(i(x))\, i'(x)}{(h(i(x)))^2}$ $h^2(i(x)) = (\sqrt{4x^2-6x+9})^2 = 4x^2 - 6x + 9$

$f'(x) = \frac{-\frac{14}{3}x^{-\frac{7}{3}} \cdot \sqrt{4x^2-6x+9} - \frac{7}{2}x^{-\frac{4}{3}} \cdot \frac{8x-6}{2\sqrt{4x^2-6x+9}}}{4x^2-6x+9} = \frac{\frac{-14\sqrt{4x^2-6x+9}}{3x^{\frac{7}{3}}} - \frac{7(8x-6)}{4x^{\frac{4}{3}}\sqrt{4x^2-6x+9}}}{4x^2-6x+9}$

Máshogyan megoldva:

$f(x) = \frac{3{,}5\, x^{-\frac{4}{3}}}{\sqrt{4x^2-6x+9}} = \frac{\frac{7}{2}\frac{1}{x^{\frac{4}{3}}}}{\sqrt{4x^2-6x+9}} = \frac{7}{2x^{\frac{4}{3}}\sqrt{4x^2-6x+9}}$ $g(x) = 7$ $g'(x) = 0$

$h(x) = 2x^{\frac{4}{3}}\sqrt{4x^2-6x+9} = i(x) \cdot j(k(x))$

$i(x) = 2x^{\frac{4}{3}}$ $i'(x) = \frac{8}{3}x^{\frac{1}{3}}$ $j(x) = \sqrt{x}$ $j'(x) = \frac{1}{2\sqrt{x}}$ $k(x) = 4x^2 - 6x + 9$ $k'(x) = 8x - 6$

$h'(x) = i'(x) \cdot j(k(x)) + i(x)\, j'(k(x))\, k'(x) = \frac{8\sqrt[3]{x}}{3}\sqrt{4x^2-6x+9} + 2x^{\frac{4}{3}}\frac{8x-6}{2\sqrt{4x^2-6x+9}}$

$f(x) = \frac{g(x)}{h(x)}$ $f'(x) = \frac{\cancel{g'(x)h(x)} - g(x)\, h'(x)}{h^2(x)}$

$f'(x) = \frac{-\left(\frac{56}{3}\sqrt[3]{x}\sqrt{4x^2-6x+9} + 14x^{\frac{4}{3}}\frac{8x-6}{2\sqrt{4x^2-6x+9}}\right)}{4x^{\frac{8}{3}}(4x^2-6x+9)} = -\frac{\sqrt[3]{x}\sqrt{\frac{1}{4x^2-6x+9}}\left(\frac{56}{3}(4x^2-6x+9) + 7x(8x-6)\right)}{4x^{\frac{8}{3}}(4x^2-6x+9)}$

$\cancel{f'(x) = \frac{\frac{1}{3}224x^2 - 112x + 168 + 56x^2 - 42x}{4x^{\frac{8}{3}} \cdot \frac{1}{x^{\frac{1}{3}}}(4x^2-6x+9)\sqrt{4x^2-6x+9}}} = \cancel{\frac{\frac{56}{3}x^2 - 70x + 168}{4x^{\frac{7}{3}}(4x^2-6x+9)^{\frac{3}{2}}}} = \cancel{\frac{\frac{28}{3}x^2 - 35x + 84}{2x^{\frac{7}{3}}(4x^2-6x+9)^{\frac{3}{2}}}} = \cancel{\frac{28x^2 - 35x + 84}{6x^{\frac{7}{3}}(4x^2-6x+9)^{\frac{3}{2}}}}$

$\cancel{f'(x) = 28x^2 - 35x + 84}$ $f'(x) = -\frac{\overbrace{56}^{2 \cdot 28}\overbrace{(4x^2-6x+9)}^{(4x^2-3x)+(-3x+9)} + 7x\overbrace{(8x-6)}^{2(4x-3)} \cdot 3}{3 \cdot \underset{2}{4} x^{\frac{8}{3}}(4x^2-6x+9) \cdot x^{-\frac{1}{3}} \cdot (4x^2-6x+9)^{\frac{1}{2}}}$

$f'(x) = \frac{28(4x^2-3x) + 28(-3x+9) + (4x^2-3x) \cdot 21}{6x^{\frac{7}{3}}(4x^2-6x+9)^{\frac{3}{2}}} = \frac{49(4x^2-3x) + 28(-3x+9)}{6x^{\frac{7}{3}}(4x^2-6x+9)^{\frac{3}{2}}} = \frac{7(7(4x^2-3x) + 4(-3x+9))}{6x^{\frac{7}{3}}(4x^2-6x+9)^{\frac{3}{2}}}$

$f'(x) = \frac{7(28x^2 - 21x + 12x + 36)}{6x^{\frac{7}{3}}(4x^2-6x+9)^{\frac{3}{2}}} = \boxed{\frac{7(28x^2 - 33x + 36)}{6x^{\frac{7}{3}}(4x^2-6x+9)^{\frac{3}{2}}}}$

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(34) $f(x)=\sqrt{x}(x^3+3x)$ $g(x)=\sqrt{x}$ $g'(x)=\frac{1}{2\sqrt{x}}$ $h(x)=x^3+3x$ $h'(x)=3x^2+3$

$f(x)=g(x)\cdot h(x)$ $f'(x)=g'(x)h(x)+g(x)h'(x)$

$f'(x)=\frac{x^3+3x}{2\sqrt{x}}+\sqrt{x}(3x^2+3)=\frac{(x^3+3x)+2x(3x^2+3)}{2\sqrt{x}}=\boxed{\frac{7x^3+9x}{2\sqrt{x}}}$

(35.) $f(x)=xe^x$ $g(x)=x$ $g'(x)=1$ $h(x)=e^x$ $h'(x)=e^x$

$f(x)=g(x)\cdot h(x)$ $f'(x)=g'(x)h(x)+g(x)\cdot h'(x)$

$f'(x)=e^x+x\cdot e^x=\boxed{e^x(1+x)}$

(36.) $f(x)=\frac{x^2-4}{x+7}$ $g(x)=x^2-4$ $g'(x)=2x$ $h(x)=x+7$ $h'(x)=1$

$f(x)=\frac{g(x)}{h(x)}$ $f'(x)=\frac{g'(x)h(x)-g(x)h'(x)}{h^2(x)}$

$f'(x)=\frac{2x(x+7)-(x^2-4)\cdot 1}{(x+7)^2}=\frac{2x^2+14x-x^2+4}{(x+7)^2}=\cancel{\frac{3x^2+14x-4}{(x+7)^2}}\boxed{\frac{x^2+14x+4}{(x+7)^2}}$

(37.) $f(x)=\sqrt{x}(x^3+3x)\to$ Ugyanaz, mint a 34. feladat

(38.) $f(x)=\frac{x^2-49}{x+1}$ $g(x)=x^2-49$ $g'(x)=2x$ $h(x)=x+1$ $h'(x)=1$

$f(x)=\frac{g(x)}{h(x)}$ $f'(x)=\frac{g'(x)h(x)-g(x)h'(x)}{h^2(x)}$

$f'(x)=\frac{2x(x+1)-(x^2-49)}{(x+1)^2}=\frac{2x^2+2x-x^2+49}{(x+1)^2}=\boxed{\frac{x^2+2x+49}{(x+1)^2}}$

(39.) $f(x)=\ln(x^2)$ $g(x)=\ln(x)$ $g'(x)=\frac{1}{x}$ $h(x)=x^2$ $h'(x)=2x$

$f(x)=g(h(x))$ $f'(x)=g'(h(x))h'(x)=\frac{2x}{x^2}=\boxed{\frac{2}{x}}$

(40.) $f(x)=e^{\sin(x)}$ $g(x)=e^x$ $g'(x)=e^x$ $h(x)=\sin(x)$ $h'(x)=\cos(x)$

$f(x)=g(h(x))$ $f'(x)=g'(h(x))\cdot h'(x)=\boxed{e^{\sin(x)}\cdot\cos(x)}$

(41.) $f(x)=x\cdot\sin(x)$ $g(x)=x$ $g'(x)=1$ $h(x)=\sin(x)$ $h'(x)=\cos(x)$

$f(x)=g(x)\cdot h(x)$ $f'(x)=g'(x)h(x)+g(x)h'(x)=\boxed{\sin(x)+x\cdot\cos(x)}$

(42.) $f(x)=\sqrt{x}e^x$ $g(x)=\sqrt{x}$ $g'(x)=\frac{1}{2\sqrt{x}}$ $h(x)=e^x$ $h'(x)=e^x$

$f(x)=g(x)\cdot h(x)$ $f'(x)=g'(x)h(x)+g(x)h'(x)=\frac{e^x}{2\sqrt{x}}+\sqrt{x}e^x=\boxed{\sqrt{x}e^x\left(\frac{1}{2x}+1\right)}=\boxed{\frac{e^x(2x+1)}{2\sqrt{x}}}$

(43.) $f(x)=e^x(x^3+3x)^5$ $g(x)=e^x$ $g'(x)=e^x$ $h(x)=x^3+3x$ $h'(x)=3x^2+3$ $i(x)=x^5$ $i'(x)=5x^4$

$f(x)=g(x)\cdot i(h(x))$ $f'(x)=g'(x)i(h(x))+g(x)i'(h(x))h'(x)$

$f'(x)=e^x(x^3+3x)^5+e^x\cdot 5(x^3+3x)^4(3x^2+3)=e^x(x^3+3x)^4[(x^3+3x)+5(3x^2+3)]$

$\boxed{f'(x)=e^x(x^3+3x)^4(x^3+15x^2+3x+15)}$

(44.) $f(x)=e^{x^2+19x+49}$ $g(x)=e^x$ $g'(x)=e^x$ $h(x)=x^2+19x+49$ $h'(x)=2x+19$

$f(x)=g(h(x))$ $f'(x)=g'(h(x))h'(x)=\boxed{e^{x^2+19x+49}(2x+19)}$

(45.) $f(x) = 6x^2 + \frac{3}{\sqrt[4]{x}} - \frac{x^2}{\sqrt[3]{x}} + \log_5 8 - 2\log_3 x$

$g(x) = 6x^2 \quad g'(x) = 12x \quad h(x) = \frac{3}{\sqrt[4]{x}} = 3 \cdot x^{-\frac{1}{4}} \quad h'(x) = -\frac{3}{4} x^{-\frac{5}{4}}$

$i(x) = \frac{x^2}{\sqrt[3]{x}} = x^2 \cdot x^{-\frac{1}{3}} = x^{\frac{6}{3}} \cdot x^{-\frac{1}{3}} = x^{\frac{5}{3}} \quad i'(x) = \frac{5}{3} x^{\frac{2}{3}} \quad j(x) = \log_5 8 \quad j'(x) = 0$

$k(x) = 2\log_3 x \quad k'(x) = \frac{2}{x \ln 3} \quad f(x) = g(x) + h(x) - i(x) + j(x) - k(x)$

$f'(x) = g'(x) + h'(x) - i'(x) + j'(x) - k'(x)$

$\boxed{f'(x) = 12x + \frac{3}{4} x^{-\frac{5}{4}} - \frac{5}{3} x^{\frac{2}{3}} + 0 - \frac{2}{x \ln 3}}$

(46.) $f(x) = \frac{\sin(x)}{4x^3 - 6} \quad g(x) = \sin(x) \quad g'(x) = \cos(x) \quad h(x) = 4x^3 - 6 \quad h'(x) = 12x^2$

$f(x) = \frac{g(x)}{h(x)} \quad f'(x) = \frac{g'(x)h(x) - g(x)h'(x)}{h^2(x)}$

$\boxed{f'(x) = \frac{\cos(x)(4x^3 - 6) - \sin(x)\, 12x^2}{(4x^3 - 6)^2}}$

(47.) $f(x) = e^x \ln(x) \quad g(x) = e^x \quad g'(x) = e^x \quad h(x) = \ln(x) \quad h'(x) = \frac{1}{x}$

$f(x) = g(x) \cdot h(x) \quad f'(x) = g'(x)h(x) + g(x)h'(x) = \boxed{e^x\left(\ln(x) + \frac{1}{x}\right)}$

(48.) $f(x) = \frac{2\cos(x^3 + 2x)}{x^2 + 6x + 3} \quad g(x) = 2\cos x \quad g'(x) = (-2)\sin(x) \quad h(x) = x^3 + 2x \quad h'(x) = 3x^2 + 2$

$f(x) = \frac{g(h(x))}{i(x)} \quad f'(x) = \frac{g'(h(x)) \cdot h'(x) \cdot i(x) - g(h(x))\, i'(x)}{i^2(x)} \quad i(x) = x^2 + 6x + 3 \quad i'(x) = 2x + 6 = 2(x+3)$

$f'(x) = \frac{(-2)\sin(x^3+2x)(3x^2+2)(x^2+6x+3) - 2\cos(x^3+2x)\overbrace{(2x+6)}^{2(x+3)}}{(x^2+6x+3)^2} \quad i(x) = x^2 + 6x + 3 = (x+3)^2$

$f'(x) = (-2)\frac{\sin(x^3+2x)(3x^2+2)(x+3)^2 + 2\cos(x^3+2x)(x+3)}{(x+3)^4}$

$\boxed{f'(x) = (-2)\frac{\sin(x^3+2x)(3x^3+9x^2+2x+6) + 2\cos(x^3+2x)}{(x+3)^3}}$

(49.) $f(x) = \sin\sqrt{10x^4 + 3x^2} \quad g(x) = \sin(x) \quad g'(x) = \cos(x) \quad h(x) = \sqrt{x} \quad h'(x) = \frac{1}{2\sqrt{x}}$

$i(x) = 10x^4 + 3x^2 \quad i'(x) = 40x^3 + 6x$

$f(x) = g(h(i(x))) \quad f'(x) = g'(h(i(x)))\, h'(i(x))\, i'(x)$

$f'(x) = \cos\sqrt{10x^4 + 3x^2} \cdot \frac{1}{2\sqrt{10x^4 + 3x^2}} \cdot (40x^3 + 6x) = \boxed{\cos\left(x\sqrt{10x^2 + 3}\right)\frac{(20x^2+3)}{\sqrt{10x^2+3}}}$

(50.) $f(x) = x\sin(x) \quad g(x) = x \quad g'(x) = 1 \quad h(x) = \sin(x) \quad h'(x) = \cos(x)$

$f(x) = g(x) \cdot h(x) \quad f'(x) = g'(x)h(x) + g(x)h'(x) = \boxed{\sin(x) + x \cdot \cos(x)}$

(51.) $f(x) = x \cdot \sin(3x) \quad g(x) = x \quad g'(x) = 1 \quad h(x) = \sin(x) \quad h'(x) = \cos(x) \quad i(x) = 3x \quad i'(x) = 3$

$f(x) = g(x) \cdot h(i(x)) \quad f'(x) = g'(x)h(i(x)) + g(x)h'(i(x))\, i'(x) = \boxed{\sin 3x + 3x\cos(3x)}$

(52.) $f(x) = \frac{\sin(x)}{2x^4 + 3} \quad g(x) = \sin(x) \quad g'(x) = \cos(x) \quad h(x) = 2x^4 + 3 \quad h'(x) = 8x^3$

$f(x) = \frac{g(x)}{h(x)} \quad f'(x) = \frac{g'(x)h(x) - g(x)h'(x)}{h^2(x)}$

$\boxed{f'(x) = \frac{\cos(x)(2x^4+3) - \sin(x)\, 8x^3}{(2x^4+3)^2}}$

(53.) $f(x) = \sqrt{\sin(x) + 3} \quad g(x) = \sqrt{x} \quad g'(x) = \frac{1}{2\sqrt{x}} \quad h(x) = x + 3 \quad h'(x) = 1 \quad i(x) = \sin(x) \quad i'(x) = \cos x$

$f(x) = g(h(i(x))) \quad f'(x) = g'(h(i(x)))\, \overbrace{h'(i(x))}^{=1}\, i'(x) = \boxed{\frac{\cos(x)}{2\sqrt{\sin(x)+3}}}$

Hakkel Tamás

54. $f(x) = x^4 \cdot \sin 5x \quad g(x) = x^4 \quad g'(x) = 4x^3 \quad h(x) = \sin(x) \quad h'(x) = \cos(x) \quad i(x) = 5x \quad i'(x) = 5$

$f(x) = g(x) \cdot h(i(x)) \qquad f'(x) = g'(x)\, h(i(x)) + g(x)\, h'(i(x))\, i'(x)$

új szabály

$$\boxed{\begin{aligned}(\sin(c\cdot x))' &= c\cdot \cos(c\cdot x)\\ (\cos(c\cdot x))' &= -c\cdot \sin(c\cdot x)\end{aligned}}$$

$f'(x) = 4x^3 \sin(5x) + 5x^4 \cos(5x) = \boxed{x^3\left(4\sin(5x) + 5x\cos(5x)\right)}$

55. $f(x) = x^3 \sqrt[4]{x^3} \sin(x) = x^{\frac{12}{4}} \cdot x^{\frac{3}{4}} \sin(x) = x^{\frac{15}{4}} \cdot \sin(x) \quad g(x) = x^{\frac{15}{4}} \quad g'(x) = \frac{15}{4} x^{\frac{11}{4}}$

$h(x) = \sin(x) \quad h'(x) = \cos(x)$

$f(x) = g(x)\, h(x) \qquad f' = g'(x)\, h(x) + g(x)\, h'(x)$

$f'(x) = \frac{15}{4} x^{\frac{11}{4}} \sin(x) + x^{\frac{15}{4}} \cdot \cos(x)$

$f'(x) = x^{\frac{11}{4}}\left(\frac{15\sin(x)}{4} + x\cos(x)\right) = \frac{x^3}{\sqrt[4]{x}}\left(\frac{15\sin(x)}{4} + x\cos(x)\right) = \boxed{\frac{1}{4} x^{\frac{11}{4}}\left(15\sin(x) + 4x\cos(x)\right)}$

56. $f(x) = (x^5 + \pi)^3 \cdot \sin 3x \quad g(x) = x^5 + \pi \quad g'(x) = 5x^4 \quad h(x) = x^3 \quad h'(x) = 3x^2 \quad i(x) = \sin 3x$

$i'(x) = 3 \cdot \cos 3x$

$f(x) = h(g(x)) \cdot i(x) \qquad f'(x) = h'(g(x))\, g'(x) \cdot i(x) + h(g(x)) \cdot i'(x)$

$f'(x) = 3(x^5 + \pi)^2 \cdot 5x^4 \cdot \sin(3x) + (x^5+\pi)^3 \cdot 3 \cdot \cos(3x)$

$\boxed{f'(x) = 3(x^5 + \pi)^2\left(5x^4 \sin(3x) + (x^5 + \pi)\cos(3x)\right)}$

57. $f(x) = \frac{x \cdot \cos(6x)}{x^2 - 1} \qquad g(x) = x \quad g'(x) = 1 \quad h(x) = \cos(6x) \quad h'(x) = (-6)\sin(6x) \quad i(x) = x^2 - 1 \quad i'(x) = 2x$

$f(x) = \frac{g(x)\cdot h(x)}{i(x)} \qquad f'(x) = \frac{[g'(x)h(x) + g(x)h'(x)] \cdot i(x) - g(x)h(x) \cdot i'(x)}{i^2(x)}$

$f'(x) = \frac{[\cos(6x) + x(-6)\sin(6x)](x^2-1) - x\cos(6x)\, 2x}{(x^2-1)^2}$

$f'(x) = \frac{1}{(x^2-1)^2}\left(x^2\cos(6x) - 6x^3\sin(6x) - \cos(6x) + 6x\sin(6x) - 2x^2\cos(6x)\right)$

$f'(x) = \frac{-1}{(x^2-1)^2}\left[(x^2+1)\cos(6x) + 6(x^3 - x)\sin(6x)\right] = \boxed{-\frac{(x^2+1)\cos(6x) + 6x(x^2-1)\sin(6x)}{(x^2-1)^2}}$

58.* $f(x) = \frac{x^3 \cdot \cos x^3}{\sin^3 x + 6} \qquad g(x) = x^3 \quad g'(x) = 3x^2 \quad h(x) = \cos(x) \quad h'(x) = -\sin(x)$

$j(x) = x^3 + 6 \quad j'(x) = 3x^2 \quad i(x) = \sin(x) \quad i'(x) = \cos(x)$

$f(x) = \frac{g(x)\, h(g(x))}{j(i(x))} \qquad (g(x)\, h(g(x)))' = g'(x)\, h(g(x)) + g(x)\, h'(g(x))\, g'(x)$

$(j(i(x)))' = j'(i(x))\, i'(x)$

$f'(x) = \frac{(g(x)h(g(x)))' j(i(x)) - g(x)h(g(x))(j(i(x)))'}{j^2(i(x))} = \frac{[g'(x)h(g(x)) + g(x)h'(g(x))g'(x)]\, j(i(x)) - g(x)h(g(x))\, j'(i(x))\, i'(x)}{j^2(i(x))}$

$f'(x) = \frac{[3x^2\cos(x^3) - x^3\sin(x^3)\cdot 3x^2](\sin^3 x + 6) - x^3 \cdot \cos x^3 \cdot 3\sin^2 x \cdot \cos(x)}{(\sin^3 x + 6)^2}$

$f'(x) = \frac{3x^2(\cos(x^3) - x^3\sin(x^3))}{\sin^3 x + 6} - \frac{3x^3 \cdot \cos(x^3)\cos(x)\cdot \sin^2(x)}{(\sin^3 x + 6)^2}$

$f'(x) = \frac{3x^2}{\sin^3 x + 6}\left((\cos(x^3) - x^3\sin(x^3)) - \frac{x\cos(x^3)\cos(x)\cdot\sin^2(x)}{\sin^3 x + 6}\right)$

$f'(x) = \frac{3x^2}{\sin^3 x + 6}\left(\cos(x^3)\left[1 - \frac{x\cos x\,\sin^2(x)}{\sin^3 x + 6}\right] - x^3\sin(x^3)\right)$

$f'(x) = \left(\frac{3x}{\sin^3 x + 6}\right)^2\left[(\cos(x^3) - x^3\sin(x^3))(\sin^3 x + 6) - (x\cos(x^3)\cos(x)\sin^2(x))\right]$

Hakkel James

(62) $f(x)=\frac{\sin 3x}{x^4+\sqrt{2}}+\cos^5(2x+1)$ $g(x)=\sin(3x)$ $g'(x)=3\cos(3x)$

$h(x)=x^4+\sqrt{2}$ $h'(x)=4x^3$ $i(x)=x^5$ $i'(x)=5x^4$ $j(x)=\cos(x)$

$j'(x)=-\sin(x)$ $k(x)=2x+1$ $k'(x)=2$

$f(x)=\frac{g(x)}{h(x)}+i(j(k(x)))$ $f'(x)=\frac{g'(x)h(x)-g(x)h'(x)}{h^2(x)}+i'(j(k(x)))j'(k(x))k'(x)$

$f'(x)=\frac{3\cos(3x)(x^4+\sqrt{2})-\sin(3x)4x^3}{(x^4+\sqrt{2})^2}+5\cos^4(2x+1)\sin(2x+1)\cdot 2$

$\boxed{f'(x)=\frac{3\cos(3x)}{x^4+\sqrt{2}}-\frac{4x^3\sin(3x)}{(x^4+\sqrt{2})^2}-10\cos^4(2x+1)\sin(2x+1)}$

(63.) $f(x)=\frac{\operatorname{arc\,tg}(2x^4)}{\cos^2(x)+5}$ $g(x)=\operatorname{arc\,tg}(x)$ $g'(x)=\frac{1}{1+x^2}$ $h(x)=2x^4$ $h'(x)=8x^3$

$i(x)=x^2+5$ $i'(x)=2x$ $j(x)=\cos(x)$ $j'(x)=-\sin(x)$

$f(x)=\frac{g(h(x))}{i(j(x))}$ $f'(x)=\frac{g'(h(x))h'(x)i(j(x))-g(h(x))i'(j(x))j'(x)}{i^2(j(x))}$

$f'(x)=\frac{\frac{8x^3(\cos^2(x)+5)}{1+4x^8}+\operatorname{arc\,tg}(2x^4)2\cos(x)\sin(x)}{(\cos^2(x)+5)^2}$

$\boxed{f'(x)=\frac{8x^3}{(1+4x^8)(\cos^2(x)+5)}+\frac{\operatorname{arc\,tg}(2x^4)\sin(2x)}{(\cos^2(x)+5)^2}}$

(64) $f(x)=\operatorname{arc\,tg}(2x^4)(\cos^2(x)+5)$ $g(x)=\operatorname{arc\,tg}(x)$ $g'(x)=\frac{1}{1+x^2}$ $h(x)=2x^4$ $h'(x)=8x^3$

$i(x)=x^2+5$ $i'(x)=2x$ $j(x)=\cos(x)$ $j'(x)=-\sin(x)$

$f(x)=g(h(x))i(j(x))$ $f'(x)=g'(h(x))h'(x)i(j(x))+g(h(x))i'(j(x))j'(x)$

$f'(x)=\frac{8x^3(\cos^2(x)+5)}{1+4x^8}+\operatorname{arc\,tg}(2x^4)\cdot 2\cos(x)\sin(x)$

$\boxed{f'(x)=\frac{8x^3(\cos^2(x)+5)}{1+4x^8}-\operatorname{arc\,tg}(2x^4)\cdot\sin(2x)}$

(65) $f(x)=\frac{1}{\sqrt[3]{x^2}}+\sqrt{x}=x^{-\frac{2}{3}}+\sqrt{x}$ $\boxed{f'(x)=\frac{-2}{3\sqrt[3]{x^5}}+\frac{1}{2\sqrt{x}}}=\boxed{\frac{3x^{\frac{7}{6}}-4}{6x^{\frac{5}{3}}}}$

(66.) $f(x)=(1-x^3)(x^2-2x)$ $g(x)=1-x^3$ $g'(x)=(-3)x^2$ $h(x)=x^2-2x$ $h'(x)=2x-2$
$(1-x)(1+x+x^2)$ $x(x-2)$ $2(x-1)$

$f(x)=g(x)h(x)$ $f'(x)=g'(x)h(x)+g(x)h'(x)=(-3)x^2(x^2-2x)+(1-x^3)(2x-2)$

$f'(x)=(-3)x^3(x-2)+2(1-x)^2(1+x+x^2)=(-3)x^4+6x^3+2x^3-2x^4+2x-2$

$\boxed{f'(x)=(-5)x^4+8x^3+2x-2}$

(67.) $f(x)=\frac{2x^2+3x}{1+2x}$ $g(x)=2x^2+3x$ $g'(x)=4x+3$ $h(x)=1+2x$ $h'(x)=2$

$f(x)=\frac{g(x)}{h(x)}$ $f'(x)=\frac{g'(x)h(x)-g(x)h'(x)}{h^2(x)}=\frac{(4x+3)(1+2x)-(2x^2+3x)\cdot 2}{(1+2x)^2}$

$f'(x)=\frac{8x^2+10x+3-4x^2-6x}{(1+2x)^2}=\boxed{\frac{4x^2+4x+3}{(1+2x)^2}}=1+\frac{2}{(1+2x)^2}$

(68.) $f(x)=\sin^2(x)-\sin(x^2)$ $g(x)=x^2$ $g'(x)=2x$ $h(x)=\sin(x)$ $h'(x)=\cos(x)$

$f(x)=g(h(x))-h(g(x))$ $f'(x)=g'(h(x))h'(x)-h'(g(x))g'(x)$

$f'(x)=2\sin(x)\cos(x)-\cos(x^2)2x=\boxed{\sin(2x)-2x\cos(x^2)}$

(69.) $f(x) = x^2 \cdot ctg\sqrt{x}$ $\quad g(x) = x^2 \quad g'(x) = 2x \quad h(x) = ctg(x) \quad h'(x) = \frac{1}{\sin^2 x} \quad i(x) = \sqrt{x} \quad i'(x) = \frac{1}{2\sqrt{x}}$

$f(x) = g(x) \cdot h(i(x)) \quad f'(x) = g'(x)h(i(x)) + g(x)h'(i(x))\, i'(x)$

$f'(x) = 2x\, ctg\sqrt{x} + x^2 \frac{1}{\sin^2\sqrt{x}\; 2\sqrt{x}} = \boxed{2x\, ctg\sqrt{x} + \frac{\sqrt{x^3}}{2\sin^2\sqrt{x}}}$

(70.) $f(x) = e^{-x} \cdot \cos(2x) \quad g(x) = e^{-x} \quad g'(x) = -e^{-x} \quad h(x) = \cos(2x) \quad h'(x) = (-2)\sin(2x)$

$f(x) = g(x)\,h(x) \quad f'(x) = g'(x)\,h(x) + g(x)\,h'(x) = (-1)e^{-x}\cos(2x) - e^{-x} \cdot 2 \cdot \sin(2x)$

$\boxed{f'(x) = -e^{-x}\left(\cos(2x) + 2\sin(2x)\right)}$

(71) $f(x) = tg^2\sqrt{x} + \frac{1}{\cos(\pi)} = tg^2\sqrt{x} - 1 \quad g(x) = x^2 \quad g'(x) = 2x \quad h(x) = tg\, x \quad h'(x) = \frac{1}{\cos^2 x}$

$f(x) = g(h(i(x))) - 1 \quad f'(x) = g'(h(i(x)))\,h'(i(x))\,i'(x) \quad i(x) = \sqrt{x} \quad i'(x) = \frac{1}{2\sqrt{x}}$

$f(x) = \cancel{2}\, tg\sqrt{x} \cdot \frac{1}{\cos^2\sqrt{x}\; \cancel{2} \cdot \sqrt{x}} = \boxed{\frac{tg\sqrt{x}}{\cos^2\sqrt{x} \cdot \sqrt{x}}}$

(72.) $f(x) = e^{ar\,sh(\frac{x}{2}+3)} \quad g(x) = e^x \quad g'(x) = e^x \quad h(x) = ar\,sh(x) \quad h'(x) = \frac{1}{\sqrt{x^2+1}} \quad i(x) = \frac{x}{2} + 3 \quad i'(x) = \frac{1}{2}$

$f(x) = g(h(i(x))) \quad f'(x) = g'(h(i(x)))\,h'(i(x))\,i'(x) = e^{ar\,sh(\frac{x}{2}+3)} \frac{1}{2\sqrt{(\frac{x}{2}+3)^2+1}}$

$f'(x) = \frac{e^{ar\,sh(\frac{x}{2}+3)}}{2\sqrt{\frac{x^2}{4} + 3x + 3 + 1}} = \boxed{\frac{e^{ar\,sh(\frac{x}{2}+3)}}{\sqrt{x^2 + 12x + 40}}}$

(73.) $f(x) = ctg\left(\frac{x}{x^2+1}\right) \quad g(x) = ctg(x) \quad g'(x) = -\frac{1}{\sin^2 x} \quad h(x) = x \quad h(x) - 1$

$f(x) = g\left(\frac{h(x)}{i(x)}\right) \quad f'(x) = g'\left(\frac{h(x)}{i(x)}\right) \frac{h'(x)\,i(x) - h(x)\,i'(x)}{i^2(x)} \quad i(x) = x^2 + 1 \quad i'(x) = 2x$

$f'(x) = \frac{-1}{\sin^2\left(\frac{x}{x^2+1}\right)} \cdot \frac{(x^2+1) - x \cdot 2x}{(x^2+1)^2} = \boxed{\frac{x^2 - 1}{\sin^2\left(\frac{x}{x^2+1}\right)(x^2+1)^2}}$

(74.) $f(x) = \ln\left(x + \sqrt{x^2 - 1}\right) \quad g(x) = \ln(x) \quad g'(x) = \frac{1}{x} \quad h(x) = x^2 - 1 \quad h'(x) = 2x$

$i(x) = \sqrt{x} \quad i'(x) = \frac{1}{2\sqrt{x}} \quad j(x) = x + \sqrt{x^2 - 1} \quad j(x) = x + i(h(x))$

$j'(x) = 1 + i'(h(x))\,h'(x) \quad j'(x) = 1 + \frac{2x}{2\sqrt{x^2-1}}$

$f(x) = g(j(x)) \quad f'(x) = g'(j(x))\,j'(x)$

$f'(x) = \frac{1 + \frac{2x}{2\sqrt{x^2-1}}}{x + \sqrt{x^2-1}} = \frac{1 + \frac{x}{\sqrt{x^2-1}}}{x + \sqrt{x^2-1}} = \frac{\sqrt{x^2-1} + x}{(x + \sqrt{x^2-1})\sqrt{x^2-1}} = \frac{\sqrt{x^2-1} + x}{x\sqrt{x^2-1} + x^2 - 1}$

$f'(x) = \frac{x^2 - 1 - x^2}{(x^2 + x\sqrt{x^2-1} - 1)(\sqrt{x^2-1} - x)} = \frac{-1}{[x(\sqrt{x^2-1} + x) - 1](\sqrt{x^2-1} - x)} = \frac{-1}{x((x^2-1) - x^2) - \sqrt{x^2-1} + x}$

$f'(x) = \frac{-1}{\cancel{x(-1)} - \sqrt{x^2-1} \cancel{+x}} = \boxed{\frac{1}{\sqrt{x^2-1}}}$

(75.) $f(x) = \ln(ch\sqrt{x}) \quad g(x) = \ln x \quad g'(x) = \frac{1}{x} \quad h(x) = ch(x) \quad h'(x) = sh(x)$

$f(x) = g(h(i(x))) \quad f'(x) = g'(h(i(x)))\,h'(i(x))\,i'(x) \quad i(x) = \sqrt{x} \quad i'(x) = \frac{1}{2\sqrt{x}}$

$f'(x) = \frac{\frac{sh\sqrt{x}}{2\sqrt{x}}}{ch\sqrt{x}} = \frac{sh\sqrt{x}}{ch\sqrt{x} \cdot 2\sqrt{x}} = \boxed{2\sqrt{x}\, th\sqrt{x}}$

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(76.) $f(x) = \operatorname{th}\left(\frac{\cos^2(2x)}{x^2-1}\right)$ $g(x) = \operatorname{th}(x)$ $g'(x) = \frac{1}{\operatorname{ch}^2(x)}$ $h(x) = x^2$ $h'(x) = 2x$

$i(x) = \cos(2x)$ $i'(x) = (-2)\sin(2x)$ $j(x) = x^2-1$ $j'(x) = 2x$

$f(x) = g\left(\frac{h(i(x))}{j(x)}\right)$ $f'(x) = g'\left(\frac{h(i(x))}{j(x)}\right)\frac{h'(i(x))\,i'(x)\,j(x) - h(i(x))\,j'(x)}{j^2(x)}$

$f'(x) = \frac{1}{\operatorname{ch}^2\left(\frac{\cos^2(2x)}{x^2-1}\right)} \cdot \frac{2\cos(2x)\cdot(-2)\sin(2x)(x^2-1) - \cos^2(2x)\cdot 2x}{(x^2-1)^2}$

$\boxed{f'(x) = (-2)\frac{\sin(4x)(x^2-1) + \cos^2(2x)\cdot x}{(x^2-1)^2}} = (-2)\left[\frac{\sin(4x)}{(x^2-1)} + \frac{x\cos^2(2x)}{(x^2-1)^2}\right]$

(77.) $f(x) = \operatorname{sh}(\sqrt{x}\cdot e^{-x})$ $g(x) = \operatorname{sh}(x)$ $g'(x) = \operatorname{ch}(x)$ $h(x) = \sqrt{x}$ $h'(x) = \frac{1}{2\sqrt{x}}$

$f(x) = g(h(x)\,i(x))$ $f'(x) = g'(h(x)\,i(x))(h'(x)\,i(x) + h(x)\,i'(x))$ $i(x) = e^{-x}$ $i'(x) = (-1)e^{-x}$

$f'(x) = \operatorname{ch}(\sqrt{x}\cdot e^{-x})\left(\frac{e^{-x}}{2\sqrt{x}} + \sqrt{x}\,e^{-x}\right) = \operatorname{ch}(\sqrt{x}\cdot e^{-x})\left(\frac{e^{-x}}{2\sqrt{x}} - \sqrt{x}\,e^{-x}\right)$

$f'(x) = \sqrt{x}\cdot e^{-x}\operatorname{ch}(\sqrt{x}\cdot e^{-x})\left(\frac{1}{2x} - 1\right) = \boxed{\frac{\sqrt{x}\cdot e^{-x}\operatorname{ch}(\sqrt{x}\cdot e^{-x})(1-2x)}{2x}}$

(78.) $f(x) = \sin^2(\arccos(x))$ $|x| < 1$ $g(x) = x^2$ $g'(x) = 2x$ $h(x) = \sin(x)$ $h'(x) = \cos(x)$

$i(x) = \arccos(x)$ $i'(x) = \frac{-1}{\sqrt{1-x^2}}$

$f(x) = g(h(i(x)))$ $f'(x) = g'(h(i(x)))\,h'(i(x))\,i'(x)$

$f'(x) = 2\sin(\arccos(x))\underbrace{\cos(\arccos(x))}_{x}\cdot\frac{-1}{\sqrt{1-x^2}}\left(-\frac{\sin(2\cdot\arccos(x))}{\sqrt{1-x^2}}\right)$

$\sin^2(\arccos(x)) = 1 - \underbrace{\cos^2(\arccos(x))}_{x^2} = 1 - x^2$

$\sin(\arccos(x)) = \sqrt{1-x^2}$

$f'(x) = 2x\,\frac{-\sqrt{1-x^2}}{\sqrt{1-x^2}} = \boxed{-2x}$

(79.) $f(x) = \log_2(\sqrt[3]{x^2}\cdot\operatorname{sh}(x)) + e^3$ $g(x) = \log_2(x)$ $g'(x) = \frac{1}{x\ln(2)}$

↳ konstans → deriváltja 0

$h(x) = \sqrt[3]{x^2} = x^{\frac{2}{3}}$ $h'(x) = \frac{2}{3}x^{-\frac{1}{3}} = \frac{2}{3\sqrt[3]{x}}$

$i(x) = \operatorname{sh}(x)$ $i'(x) = \operatorname{ch}(x)$

$f(x) = g(h(x)\,i(x)) + c$ $f'(x) = g'(h(x)\,i(x))(h'(x)\,i(x) + h(x)\,i'(x))$

$f'(x) = \frac{\frac{2\cdot\operatorname{sh}(x)}{3\cdot\sqrt[3]{x}} + \operatorname{ch}(x)\sqrt[3]{x^2}}{\sqrt[3]{x^2}\cdot\operatorname{sh}(x)\cdot\ln(2)} = \frac{2\,\operatorname{sh}(x)}{3\sqrt[3]{x}\sqrt[3]{x^2}\,\operatorname{sh}(x)\cdot\ln(2)} + \frac{\operatorname{ch}(x)}{\ln(2)\operatorname{sh}(x)}$

$f'(x) = \frac{2}{3x\ln(2)} + \frac{\operatorname{cth}(x)}{\ln(2)} = \frac{2+3x\operatorname{cth}(x)}{3x\ln(2)} = \boxed{\frac{2+3x\operatorname{cth}(x)}{x\ln(8)}}$